1A.4 IMPROVED RADAR RAINFALL ESTIMATES: WHAT INDEX TO USE IN THE GAMMA FUNCTION FOR THE DROP SPECTRUM?

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1 INTRODUCTION

Rainfall estimates from radar reflectivity of rain are accurate to factor of two or so, largely because of the unknown variability of the raindrop size spectrum. The natural variability of rain drop spectra is well captured by the normalized gamma function:

$$N(D) = N_w f(\mu) \left(\frac{D}{D_0}\right)^{\mu} \exp\left(-\frac{(3.67+\mu)D}{D_0}\right) \quad (1)$$

where

$$f(\mu) = \frac{6}{(3.67)^4} \frac{(3.67+\mu)^{\mu+4}}{\Gamma(\mu+4)} \quad (2)$$

with three independent parameters, N_w, the normalised concentration, D_o, the median volumetric drop diameter, and μ a shape factor for the width of the spectrum. The normalization factor $f(\mu)$ is chosen so that for a given N_w the value of the liquid water is indepenent of μ. The numerical factor ensures that when $\mu = 0$ it reduces to the conventional exponential form $N(D) = N_w exp(-3.67)D/D_0$.

In this paper we examine how different values of μ affect rainfall retrievals from radar and show that when polarisation radar techniques are used, then the unknown value of μ is the factor limiting the accuracy of the inferred rainfall rate. We then present evidence that the value of μ in naturally occurring rain is close to 5; previous retrievals have assumed a value near to 1 and we suggest that this could bias the rainfall rates inferred from polarisation radar.

2 ERRORS IN Z-R RELATIONS IN RAIN.

A conventional Z-R relationship is equivalent to assuming that as R becomes heavier, D_0 increases, but N_w remains constant. Integration over suitably weighted values of Eqn(1) yields a relation of the form $Z = aR^b$ with $b = 1.5$ with a varying as $1/\sqrt{N_w}$. The oft quoted factor of two error in the value of R arises because in natural rain N_W varies by up to a factor of ten.

This error can be reduced if an estimate of drop size can be made. Differential reflectivity, Z_{dr}, essentially measures raindrop shape and hence drop size which is related to D_0. Because Z_{dr} is a ratio it is independent of N_w amd so for a given Z_{dr} (constant D_0) both Z and R will scale with N_w The values of Z for R=1mm/hr as a function of Z_{dr} assuming Goddard et al (1995) raindrop shapes are plotted in Fig 1 for various svalues of μ. If we have an observed reflectivity (Z_{obs}) which is a factor x above the R=1mm/hr line in the figure, then the rainfall rate is x mm/hr. Assuming for the moment that the radar observations have no errors, then the technique essentially uses Z and Z_{dr} to infer the value of N_w and D_0 in Equn (1). The remaining error in the rainfall rate arises from the unknown value of μ. The separation of the μ curves in Fig 1 indicates that assuming μ is zero if in reality it was 5 would lead to an overestimate of rainfall rates by about 33%. A similar error arises from the unknown value of μ when estimating rainfall rate from specific differential phase shift (K_{dp}). We now examine the evidence for the range of μ in naturally occurring rain.

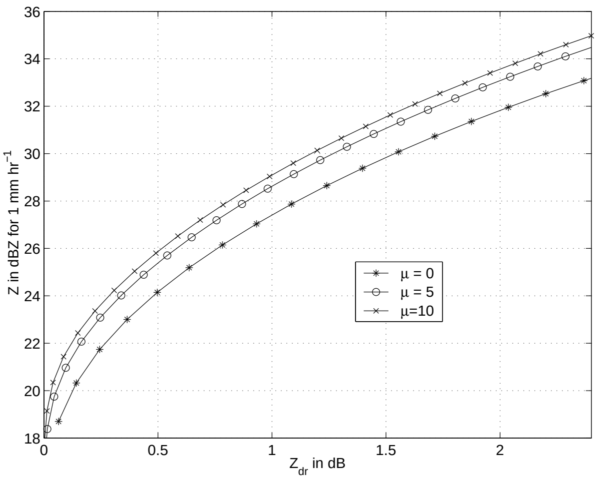

Figure 1: The value of Z for a rainfall rate of 1mm/hr as a function of observed Z_{dr} for different values of μ. Assuming μ=0 rather than 5 will lead to overestimate of rainfall by 33%.

3 ESTIMATING μ FROM DROP SPECTRA

The value of μ in naturally occuring rain can be derived by fitting the observed spectra to the normalized spectrum in Eqn (1). A least squares fit is not appropriate because it gives equal weight to small and large rain drops, whereas for radar and rainfall the larger drops are much more important. Kozu and Nakamura (1991) and Illingworth and Johnson (1999) both equated the sixth, fourth and third moments of observed raindrop size distributions to the appropriately weighted integral of the gamma function and deduced values of μ in the range 0 to 15 with a mean value of about 5 or 6.

These high values of μ derived from fitting the higher moments have been criticised because they are very de-

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pendent upon the largest raindrops in the spectrum which are poorly sampled by disdrometers. The values obtained depend upon the moments chosen for the fit. Testud et al (2001) and Bringi and Chandrasekar (2001) infer μ values closer to unity; this may be because they first derive N_0 and D_0 in an exponential spectrum by fitting moments, but then choose μ to minimise a least squares fit to the observed spectrum. This procedure for fixing μ assigns equal weight to drops of all sizes and may lead to values of μ which are inappropriate for the higher moments involved in radars studies. Ulbrich and Atlas (1998) used truncated moments by limiting the experimental spectra to D_{max} and found that this reduced μ from about 4 to a median value of 0 with a mean of 1.6.

4 THE VALUE OF μ FROM DIFFERENTIAL DOPPLER VELOCITY, DDV

Radar measurements themselves have a much larger sampling volume than disdrometers and do not suffer from the poor sampling of the larger drops which are present in low concentrations. Wilson et al (1997) reported values of μ derived from the 'Differential Doppler Velocity', DDV, the difference in the Doppler velocity of rain for horizontally and vertically polarised radiation with the radar beam dwelling at a finite elevation. They showed that the value of DDV as a function of Z_{DR} depends upon the value of μ and found a range of μ between 2 and 10 with a mean value of 5.

5 ESTIMATES OF μ FROM DOPPLER SPECTRAL WIDTH AT VERTICAL INCIDENCE

A second radar based technique which should provide a direct estimate of the breadth of the drop size spectrum is to examine the Doppler spectral width at vertical incidence. We would expect the Doppler width to reduce as the value fo μ rises and the drop spectrum becomes more monodispersed. Theoretical curves of the Doppler width due to the spread in terminal velocity as a function of rainfall rate for various values of N_w and μ at 35GHz are plotted in Fig.2.

The Doppler width is determined by the spread of terminal velocities, which is greater for the small drops but it is also weighted by their diameter to the sixth power. For light rain the width increases with rainfall rate, until two effects cause it to fall. First, most drops become large and so have a similar terminal velocity, and, second, the largest drops tend to Mie scatter so have less weighting. We can now explain the dependency of Doppler width with N_w in Fig 2. For a given rainfall rate a low value of N_w implies a higher D_0 so as N_w falls the maximum value of Doppler width will occur at a lower rainfall rate.

We now consider other contributions to the Doppler width apart from the spread in terminal velocites, such as the broadening from wind shear and turbulence within the beam. The broadening of the spectrum due to horizontal wind (θ) is given by Sloss and Atlas (1968) to be: $\sigma_b = 0.3u\theta$. For the MMCR used in this study θ is

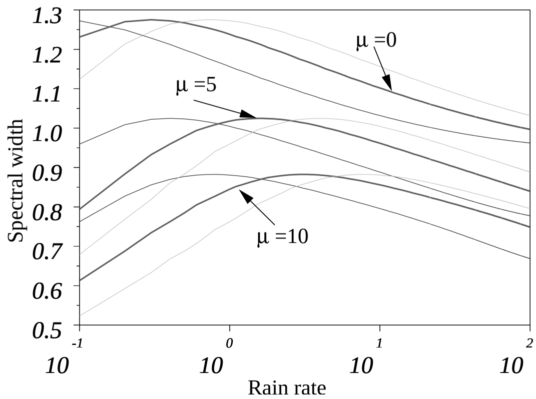

Figure 2: Spectral width at vertical incidence as a function of rainfall rate for various values of μ and with N_w = 20000 (upper dotted line), 8000 (solid line) and 2000 (dashed line) in units of $m^{-3}mm^{-1}$.

only 0.29degs, so limiting observations to winds less than 15m/s the maximum broadening is a negligible 0.025m/s. It is difficult to determine the contribution of turbulence. All we can say is that the spread in terminal velocities we infer is a maximum value, and if turublence was signficant our inferred values of μ would be even higher. One final aspect to consider is atmsopheric pressure. The terminal velocities should increase as $1/\rho^{0.4}$ where ρ is the density. Accordingly as the altitude increases and the presure falls, the values of spectral width and rainfall rate in Fig 2 should be scaled appropriately.

6 OBSERVATIONS

We have analysed the spectral width observations made at the Manus ARM site in the tropical Pacific. The data comprise Doppler spectra taken every ten seconds with a vertical resolution of 90m and rain rate data every minute, but no observations of drop spectra or N_w. Two vertical profiles of spectral width with rainfall rates close to 1 mm/hr are displayed in Fig 3 and demonstrate the effect of the correction for density which reduces the spectral width at 3.5km from about 1.2m/s to 1m/s; the effect is much less at 1km height with the spectral wdith remaining close to 0.8m/s. Comparison with Fig 2 suggests at 3.5km heigth the minimum value of μ is 5, and at 1km height μ is closer to 10.

The results from a longer period of rain are presented in Fig 4 where the mean value of inferred μ for the three different values of N_w have been calculated from 563 profiles with rain rates in the range 0.2 to 6mm/hr. No correction has been made for the time the rain takes to fall to the ground, but a day with low winds has been chosen to minimise any wind drift.

The bright band and melting layer is at 4.5km. The values of μ do appear to increase in the km below the melting layer. The origin of this is not clear. Possibly there is enhanced turbulence in the region where melting which leads to localised cooling. Alternatively, the higher spec-

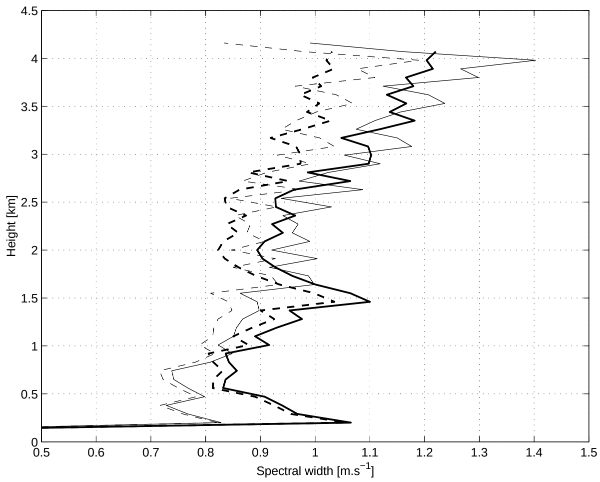

Figure 3: Profile of Doppler width from Manus for R = 1mm/hr with and without the air density correction. 8 Aug 2000. Bold line 18:35:00, thin line for 16:18:31 UTC.

tral width could result from the presence of large drops which have an ice core which has not completely melted, and so will have a large contribution to Z but a lower terminal velocity. A third suggestion is that there is some evaporation of the smaller droplets which will occur in the 1km below cloud base; the distance for evaporation is approximately proportional to the cube of the drop size,

If we consider the data in the rain below 3.5km altititdue, then the message from this data and that examined on many other days is quite clear. For all realistic assumptions of N_w the inferred values of μ are indeed at least 5.

7 CONCLUSIONS

The analysis of drop spectra recorded at the ground by the method of moments suggest that the value of μ in rain is close to 5. Two radar based technique which do not suffer from the small sampling volume of ground based disdrometers both support this value of μ. The first method relies on the difference in the Doppler velocity measured with vertical and horizontal polarisation at finite elevation. The second method uses the Doppler width observed at vertical incidence as a measure of the spread of raindrop velocities and hence sizes which are present within the beam.

Polarisation radars will have similar sample volumes to those used in the radar techniques above, and hence the drop spectra characteristics inferred from the radar should be the appropriate ones to use in interpreting the polariasation data. We therefore urge the use of μ =5 when deriving rainfall rates using using Z_{dr} or K_{dp}. Many reports in the literature have used a value of μ much closer to zero or unity. We suggest that this choice could bias retrieved rainfall rates by up 33%.

ACKNOWLEDGEMENTS. We thank the ARM program for access to the Manus data and support of NERC grant NER/T/S/1999/00105.

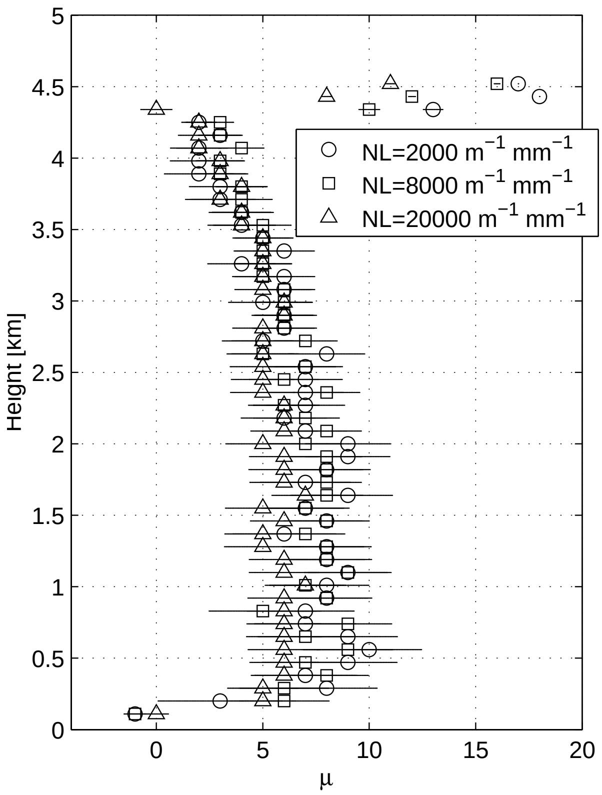

Figure 4: Observations of the values of μ inferred from the Spectral width at vertical incidence from Manus.

8 REFERENCES

Bringi, V. N. and Chandrasekar, V., 2001: *Polarimetric Doppler Weather Radar*, CUP, 636pp.

Goddard, J.W.F. et al., 1995: Dual wavelength polarisation measurements in precipitation using the CAMRA and Rabelais radars. Preprints *27th Int Conf on Radar Meteorology*, Vail, CO, Amer Meteor. Soc., 196-198.

Illingworth, A.J. and Johnson, M.P.,1999: The role of raindrop shape and size spectra in deriving rainfall rates using polarisation radar. *29th Int Conf on Radar Meteorology*, Montreal, Quebec, Amer Meteor. Soc., 301-304.

Kozu, T and Nakamura, K., 1991: Rainfall parameter estimation from dual-radar measurement combining relflectivity profile and path-integrated attenuation *J. Atmos. Oceanic Technol.*, **8**, 259-271.

Sloss, P.W. and Atlas, D., 1968: Wind shear and reflectivity gradient effects on Doppler radar spectra. *J. Atmos. Sci.*, **25** , 1080-1089.

Testud, J., Bouar, E., Obligis, E., and Ali-Mehenni, M., 2000: The rain profiling algorithm applied to polarimetric weather radar. *J. Atmos. Sci.*, **17**, 332-356.

Ulbrich, C.W., and Atlas, D., 1998: Rainfall microphysics and radar properties: analysis methods for drop size spectra. *J. Appl. Meteorol.*, **37**, 912-923.

Wilson, D.R., Illingworth, A.J., and Blackaman, T.M., 1997: Differential Doppler Velocity: A radar parameter for characterising hydrometeor size distributions *J. Appl/ Meteor.*, **36**, 649-663.