1. INTRODUCTION

Measurement errors need to be specified so that radar observations can be properly assimilated for numerical weather prediction. There are two related aspects to this problem: (1) errors in the original measurements within each radar pulse volume, and (2) representativeness of the radar data estimates used in the assimilation process. For radial velocities, the first error source depends on the strength of the return signal and the spread or width of the Doppler velocity spectrum. Spectral width in turn depends mainly on reflectivity and velocity gradients within and across the pulse volume and turbulence within the pulse volume (Doviak and Zrnić 1984). Estimation of these errors is complicated by the fact that the components needed for reliable error estimation are themselves only measured and, therefore, have inherent uncertainties.

Because of its dependence on reflectivity and velocity gradients, spectral width should exhibit significant correlation over substantial distances, similar to those expected for the reflectivity and velocity fields. Such strong correlation will diminish its utility as an estimator of error in the mean radial velocity to be used in the observational error covariance matrix that is needed within the data assimilation procedure (Sun and Miller 2003).

We will show that all three measurands: reflectivity, radial velocity, and spectral width have correlation lengths that are comparable to the characteristic dimensions of the storm that is being modeled. We will present two alternatives for possible use as estimates of the radial velocity errors. Both will be shown to exhibit two important and desirable characteristics: (1) their amplitudes are essentially Gaussian distributed and (2) each is a spatially uncorrelated variable.

2. CASE STUDY STORM

We will use radar data collected during the Severe Thunderstorm Electrification and Precipitation Study (STEPS 2003) field program conducted near the Colorado-Kansas border in summer 2000 to illustrate our findings. The storm under consideration occurred on June 29 and was observed by three Doppler radars: two research polarimetric radars from NCAR (S-Pol) and CSU (CSU-CHILL) and the NWS operational radar at Goodland Kansas. The storm began ~2100 UTC in northeast Colorado and moved southeastward, remained in a multicellular phase for nearly two hours before making a 35-deg right turn as it became more supercellular. At this time (~2325 UTC) storm size and reflectivity increased dramatically, an F1 tornado first touched ground, and positive cloud-to-ground lightning ac-

![Figure 1. S-Pol radar measurements after interpolation to Cartesian (x,y) locations on the original conic (elevation angle = 4.3 deg) scan surfaces. The fields shown are: (a) reflectivity (dBZ), (b) radial velocity (m/s), and (c) spectral width (m/s) following the gray scales on the right. Reflectivity 15 and 45 dBZ contours (thick lines) are overlaid for reference. East-west and north-south distances are km relative to the Goodland WSR-88D radar.](image-url)
activity increased. The tornado was on the ground for about 16 min after which time the storm continued as a copious hail-producer with 70 dBZ reflectivities, finally diminishing in severity some 3.5 hours after it began.

Since NWS WSR-88D datasets are thresholded at fairly conservative signal strengths before archival, we will use unthresholded data from the NCAR S-Pol in order to help understand the characteristics of the data in regions with weak as well as strong signal strength. The radar data assimilation results reported by Sun and Miller (2003) used the Goodland (KGLD) datasets.

Figure 1 shows examples of reflectivity, radial velocity, and spectral width as measured by the NCAR S-Pol radar (near the left edge of the figure). These data were interpolated to Cartesian locations in the original conic scan surface (elevation angle of 4.3 deg) using the SPRINT interpolation software (Mohr and Vaughn 1979; Miller et al. 1986). These fields have been smoothed for presentation (CEDRIC, Mohr et al. 1986); however, spatial correlation functions were computed using the original linearly interpolated and unsmoothed datasets.

3. CHARACTERISTICS OF THE RADAR DATA

Modern radars commonly use the pulse-pair method (Rummler 1968) for computing the moments of the Doppler velocity spectrum. The variance for the mean radial velocity ($\hat{V}$) and spectral width ($\hat{\sigma}$) estimators are (Doviak and Zrnić 1984):

$$\sigma^2(\hat{V}) = \frac{\lambda}{8\sqrt{\pi}MT} W_i$$

and

$$\sigma^2(\hat{\sigma}) = \frac{3\lambda}{64\sqrt{\pi}MT} W_i$$  \hspace{1cm} (2)

where $\lambda$ is the radar wavelength, $W$ is the true spectral width, $M$ is the number of equally-spaced pulses, and $T$ is the time between pulses. The maximum unambiguous (Nyquist) velocity is $V_n = \lambda/4T$.

Equations 1 and 2 are appropriate for high signal-to-noise power ratios (SNR), Gaussian velocity spectra in white noise, narrow spectra ($W << 2V_n$), and large numbers of samples. Further, their use as error estimators also requires knowledge of $W$. In our opinion, these constraints make Eqs. 1 and 2 rather impractical as estimates of the measurement errors. Therefore, we have sought error estimators for radial velocity that can be obtained from the measurements themselves.

The distributions of radial velocity and spectral width are shown in Fig. 2 as scatter plots, with the received power as the independent variable. These scatter plots include mean velocity and spectral width estimates from all measurement locations, particularly those where there is no meteorological signal and those where the received signal is weak. In principle, radial velocities computed with the pulse-pair algorithm should be randomly distributed between plus and minus the Nyquist velocity, which in this case was 25.6 m/s. These noise-only estimates are the narrow band of velocities
at received power values between -123 and -117 dBm. It is noteworthy that spectral width is largely dependent on the signal power until about -90 dBm (a SNR of about 20 dB) as seen in the declining spectral widths as the received power increases from the minimum received power (-130 dBm). The only reliable spectral width estimates associated with the weather signal must be those with received powers in excess of about -90 dBm. As can be seen in Fig. 2a, however, there are clearly very useful velocity estimates at received signal strengths between about -115 and -90 dBm, a 20 dB or so band of weak signals.

Figure 3 shows spatial correlation functions for those fields plotted in Fig. 1. Several points are evident: (1) reflectivity is strongly correlated (1/e) to lag distances of nearly 15 km along its major axis (Fig. 1a), (2) strong velocity correlation extends well beyond the 15-km lag limits (Fig. 3b), and (3) spectral width has a 1/e correlation distance of 3-5 km and is elongated most along the reflectivity major axis (Fig. 1a). This broad region of spectral width correlation means that it is not a purely random variable. If spectral width were to be used in the error covariance matrix, the off-diagonal elements would need to be calculated. There are strong indications that spectral width covariances would also be flow-dependent as seen in the similarities between reflectivity and spectral width correlations. There is also a hint of elongation along the major axis of the velocity correlation (compare Fig. 3b and 3c).

4. ALTERNATIVE VELOCITY ERROR ESTIMATORS

The SPRINT interpolation is bilinear and uses only measured values from the two ranges, two azimuths, and two elevations surrounding the output grid point (Mohr and Vaughn 1979). In the case of two-dimensional interpolation to the original conic scan surface, only two ranges and two azimuths are used. In addition to the interpolated
radial velocity values, the standard deviation of all velocity estimates used in the interpolations to each output grid point is computed (Miller et al. 1986). Figure 4 shows the local standard deviation corresponding to the radial velocity field shown in Fig. 1b. It is readily seen that this local standard deviation is more nearly an uncorrelated random variable than is the spectral width (compare Figs. 3c and 4b). Therefore, we propose it as an alternative to spectral width as an error estimator for radial velocity.

Any measured quantity can be represented as the sum of its “true” value and a measurement error. If the original measurements are spatially filtered a modest amount (in this case 3 ranges by 3 azimuths), we can assume that this gives us the “true” value. Therefore, an estimate of the error will be \( \text{Error} = \text{Measurement} - \text{Filtered} \). This is equivalent to assuming that all localized fluctuations in the measurements are a result of random measurement error and are not meteorological in nature.

We tested this hypothesis by using a two-dimensional (range-azimuth) triangular filter on the original radar measurements. The residuals or “errors” in radial velocities are plotted in Fig. 5. A Gaussian distribution having the same variance and mean as the sample error histograms is overlaid for comparison. It seems quite clear that the error amplitudes found as residuals are essentially Gaussian distributed. Further, spatial spectra (not shown) of the errors were found to be uniform across all scales (white noise spatial spectra which result from correlations that are essentially delta functions).

5. SUMMARY CONCLUSIONS

- Reflectivity, radial velocity, and spectral width exhibit significant correlations (> 1/e) over distances in excess of several grid points.
- Because of its significant correlation, spectral width is questionable as an error estimator for radial velocity.
- Local standard deviations of the velocity measurements used for interpolations to each of the output grid points exhibit much less spatial correlation when compared to spectral width.
- Reduced spatial correlation of local standard deviations makes them more acceptable as error estimators for velocity.
- The difference between the original measured radial velocity and a modestly filtered one shows the most promise as an error estimator since its amplitude is Gaussian distributed, and it exhibits a white-noise spatial spectrum. As a consequence, the only non-zero elements in the observation error covariance matrix will be the measurement error variances along the diagonal.

6. ACKNOWLEDGEMENTS

We acknowledge the dedicated efforts during operations of the following: NCAR/ATD staff for S-Pol, CSU radar group for CSU/CHILL, and the NWS/WFO at Goodland for KGLD.

7. REFERENCES


