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1. INTRODUCTION

Most current quantitative precipitation nowcasting (QPN) methodologies are deterministic in nature and convey little realistic information regarding the uncertainty in the forecast. For flood and streamflow forecasting applications it is important to understand and accurately represent this uncertainty (Smith and Austin, 2000). This will allow a greater deal of confidence in the use of precipitation forecasts for hydrological purposes. This paper presents results obtained using a stochastic Bayesian precipitation nowcasting scheme.

The nowcast methodology utilises an integro-difference equation (IDE) methodology and, as such, allows the use of other observed meteorological parameters (e.g. Doppler winds) as a constraint on the stochastic process. This produces a physically based statistical nowcast. The scheme produces a range of nowcasting solutions using a series of initial fields of radar reflectivity, resulting in a distribution of short-period forecasts. This paper examines the nature and interpretation of the distribution of QPNs. Areal precipitation forecasts may be used as input to a simple lumped hydrological models to investigate the impact of the nowcast distribution on streamflow forecasts. This investigation will provide information on how forecast uncertainty can be propagated through such models, and how one may best handle distributions of QPNs of this nature in a hydrological context.

2. METHODOLOGY

The nowcast is based upon a Stochastic Bayesian scheme with an integro-difference equation (IDE) methodology. This allows the production of a distribution of nowcast fields through the stochastic parameterization whilst retaining physically realistic constraints by using the IDE formulation.

The methodology is described briefly in Fox et al (2003) and fully in Xu et al. (2003).

Wikle et al. (2001) and Wikle (2003) showed that modeling complicated processes in such a framework can be made more efficient if one utilizes scientific information such as those suggested by governing partial differential equations (PDEs). That is, one uses the PDEs to inform the structure of the underlying process and parameter models in the hierarchical framework. This may be done in physical space as demonstrated by Wikle (2003) for the problem of modeling the spread of invasive species. Alternatively, in high dimensional contexts, the use of PDE priors in the spectral domain lead to computational simplifications.

As shown by Wikle et al. (2001) for modeling tropical wind fields, such methods work well when there is a sound fundamental scientific basis for the choice of the PDE model. However, in many cases, such as the thunderstorm nowcasting problem, one knows that there are physically-based spatial-temporal features such as diffusion and propagation, but the set of governing PDEs is either unknown or too complicated to be efficiently utilized in the stochastic framework. Thus, one seeks to consider alternative ways to efficiently model known qualitative dynamical features without direct specification of the underlying governing equations. The IDE framework provides such a specification.

Although the PDE and IDE framework share the notion of continuous space, the IDE framework differs in that it is formulated in discrete time. There is a substantial literature on deterministic IDEs in the mathematical ecology literature (e.g., Kot et al. 1996). Although the IDE model has been considered in the statistical modeling of spatio-temporal processes, as described by Wikle and Cressie (1999) and Brown et al. (2000, 2001), it is only recently that it has been recognized that such a framework can efficiently model relatively complicated dynamical processes. The key to modeling dynamical processes is the redistribution kernel. Kot et al. (1996) showed that such a framework can model diffusive wave fronts. Specifically, the shape and speed of diffusion

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depends on the kernel width and tail behavior. In fact, Kot et al. (1996) showed that the deterministic IDE framework yields the same analytical solution as Fisher's (1937) solution to a reaction-diffusion PDE for modeling the speed of propagation for a diffusive wave. More recently, Wikle (2001, 2002) demonstrated that the IDE framework can also model non-diffusive propagation (such as one would see with propagating features) via the relative displacement (i.e., translation) of the kernel. That is, as shown schematically in Figure 1, wider kernels imply greater diffusion. Furthermore, if the kernel is shifted (translated) relative to its initial location, then the propagation at that location at the next time is in the opposite direction of the shift and the speed of propagation is related to the magnitude of the shift (e.g., see Wikle 2002).

Examples of shifting kernels are shown in figure 1. In this figure the upper panel shows the estimated propagation orientation suggested by the spatially varying kernels shown in the lower panel.

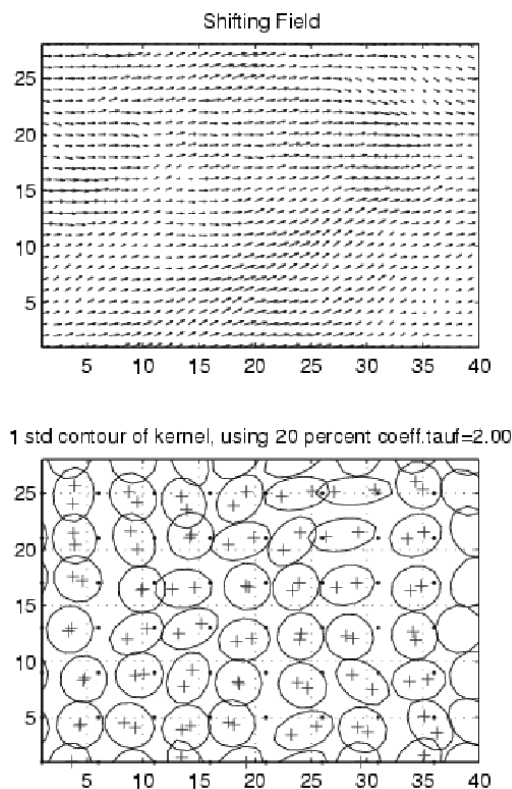


Figure 1: Example of the shifting field (top panel) generated from the spatially varying kernels (bottom panel) for case shown in figure 2.

3. PRELIMINARY RESULTS

The scheme uses six time-steps to initialise the nowcast model and then produces 1000 nowcast sequences. This large number of nowcasts fields then allows the computation of the mean, or best-guess, nowcast and the variance of the mean. Thus, for each pixel, there is an estimated forecast of precipitation and an associated error.

An example nowcast sequence is shown in figure 2. This nowcast was produced from 2.5km CAPPIS from 3rd November 2000 during the Sydney 2000 Forecast Demonstration Project (Fox et al. 2001). The example shows the forecast progression of a supercell thunderstorm as it approached the Sydney metropolitan area. This storm produced large hail, tornadoes and heavy rain leading to localised flooding. The nowcast retains high intensity features whilst providing an indication of the error associated with the predicted radar reflectivity. This could be converted to a rainfall intensity field. As can be seen, the uncertainty in the northern cell is locational, in that the largest variances are situated at the edge of the cell. However, the cell to the south has little uncertainty attached, and the cell just moving into the southern edge of the domain has a large uncertainty in the intensity. This latter is most likely due to a lack of information for the cycle of the system's initialisation as the cell moved into the model domain.

Assuming that the distribution of nowcast precipitation rates for each pixel is normal then we can assign a complete probability distribution of predicted precipitation rates for each pixel at each time increment. In particular we can determine, for example, the 90% confidence interval and provide forecasters with an effective range of possible precipitation rates at a point. An example of this would be the peak reflectivity in the T+40 nowcast.

Using a standard convective rainfall Z-R relationship the nowcast provides a 90% confidence interval of the rainfall intensity being between 26.7 mm h^{-1} and 34.9 mm h^{-1} , with a mean of 30.5 mm h^{-1} . Although the validation of the nowcast probabilities is yet to be completed one can see the potential of such products for decision making in event oriented nowcast applications.

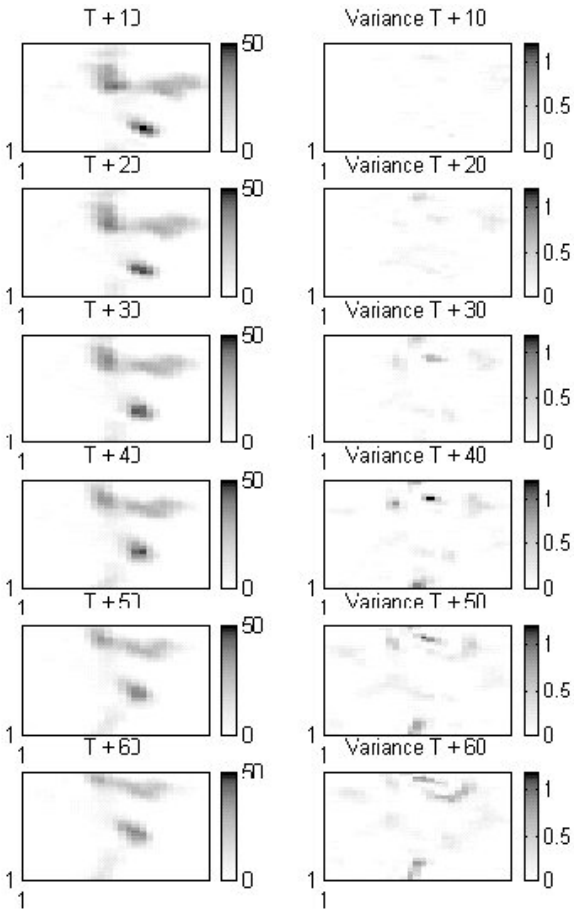


Figure 2: Example of nowcast product. The left hand set of panels show the mean of the nowcast distribution for 10 minute time-steps out to T+60. Right hand panels show the variance of the mean nowcast field. The domain shown is 100km by 70km and the grayscales are in dBZ.

4. DISCUSSION

There are many other questions remaining to be explored. The assumption of normally distributed nowcasts at individual pixels may or may not be good, and may depend upon the structure and possible motion of the cell or cells being tracked. For example, if slight variations in cell position in early time-steps lead to divergent outcomes at later times then it is possible to produce skewed or perhaps even bimodal precipitation nowcasts.

While point precipitation nowcasting is one aspect of the application of these nowcasts, the most powerful use should be hydrological. Using the pixel values over a catchment one can estimate, again, a best-guess and associated error in catchment rainfall. If one wishes to perform a

simple lumped model assessment of future streamflow suitable for a small catchment, then one can use the best-guess catchment total precipitation and 90% confidence interval (or any other probability) to obtain the possible range of streamflows. Clearly one can aggregate nowcast precipitation over time in a similar manner, but this ignores any hydrological model induced errors. However, it leads to a mechanism by which these errors can be assessed and incorporated into a hydrological forecast.

Using such probabilistic precipitation forecasts in a distributed hydrological model is not so straightforward. This is because errors in one grid box can propagate to another implying that the individual errors are not independent and therefore cannot be summed in the simplistic way as above.

The authors intend to explore these issues in further work involving prolonged case studies and the more effective implementation of the IDE methodology.

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