P2B.1 CONSTRAINED GAMMA DROP SIZE MODEL FOR POLARIMETRIC RADAR RAIN ESTIMATION: JUSTIFICATION AND DEVELOPMENT

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1. INTRODUCTION

Accurate rain estimation from radar measurements has been a difficult task due to the variation of raindrop size distribution (DSD), lack of accurate axis ratio model, measurement error, clutter, and so forth. Previously, rain estimation from weather radars has been largely dependent upon empirical relations such as R-Z relations. The development of polarimetric radar makes accurate rain DSD retrieval and rain rate estimation possible. Polarimetric radar observables: Z_{DR} and K_{DP} depend on the shape of raindrops while the raindrop shape is directly related to drop size, and hence contain the information about rain DSD, and are used to retrieve rain DSDs and improve rain rate estimation.

Recently, the constrained Gamma rain DSD retrieval was developed [Brandes et al., 2002&2003a; Vivekanandan et al., 2003; Zhang et al., 2001]. Since K_{DP} has large measurement error and poor range resolution, the constrained Gamma DSD method uses radar measured Z, Z_{DR}, and an observed relation between the shape and slope parameters to retrieve the parameters of Gamma rain DSD, and then calculate rain parameters such as rain rate and median volume diameter.

In this paper, we summarize the development of the constrained gamma DSD model for rain estimation from polarimetric radar measurements. Justification for using the constrained Gamma DSD retrieval is provided. The verification and application of the retrieval algorithm is described by Brandes et al. (2003a&b). It is shown that the constrained Gamma DSD retrieval improves rain estimations from a pair of remote measurements, such as, reflectivity and differential reflectivity, and it reduces the bias and standard error in retrieved rain parameters.

2. FORMULATION AND JUSTIFICATION

Polarization radar measurements include horizontal reflectivity (Z_H), vertical reflectivity (Z_V), differential reflectivity (Z_{DR}), specific differential phase (K_{DP}), linear depolarization ratio (LDR), and co/cross-correlation coefficients. Among them, reflectivity and differential reflectivity are the most important in rain estimation. They depend on rain DSD and the scattering amplitudes as follows:

$$Z_{H,V} = \frac{4\lambda^4}{\pi^4 |K_w|^2} \int_{D_{min}}^{D_{max}} |f_{a,b}(D)|^2 N(D)dD \quad (1)$$

$$Z_{DR} = 10\log\left(\frac{Z_H}{Z_V}\right) \quad (2)$$

where $f_{a,b}$(D) are backscattering amplitudes in major (a) and minor (b) axis (corresponding to horizontal and vertical polarizations), K_w is the dielectric factor of water, λ is the wavelength, D is the equivolume diameter and N(D) is the particle size distribution. Because the difference between the scattering amplitudes at the two polarizations depends on the raindrop shape which is related to raindrop size, two issues for accurate rain estimation are (i) the raindrop axis ratio relation and (ii) the rain DSD model.

2.1 Raindrop Axis Ratio

Recent studies have shown that raindrop shape is more spherical than the previously used equilibrium shape. A number of the direct measurement of raindrop shape were fitted with a smooth polynomial function (Brandes et al., 2002) to obtain

$$r=-0.000249D^4+0.00503D^3-0.0364D^2+0.0251D+0.995 \quad (3)$$

where r is the drop axis ratio and D is the equi-volume drop diameter measured in mm. The solid line in Figure 1 shows the proposed axis ratio relation plotted as a function of the equi-volume drop diameter, and the dashed line is the corresponding equilibrium axis ratio $r = 1.03 - 0.062D$ proposed by Pruppacher and Beard (1970). Obviously, the linear relation cannot capture the non-linear nature of the axis ratio for raindrops, especially for those size between 1 to 2 mm, that mainly contribute to rain rate.

Instead of using a variable slope, we write the axis ratio as a sum of its mean represented by (3) and a fluctuation r_1 as

$$\hat{r} = r + r_1. \quad (4)$$

It is reasonable to assume the fluctuation is proportional to the difference of the mean shape from a sphere (1-r). A standard deviation of 0.2 corresponds to a uniform distribution between ± 0.35. A range of the axis ratio of $r \pm 0.35(1-r)$ is also shown in the figure that covers most observations.

A Taylor series expansion of the scattering amplitudes yields

$$\hat{f}_{a,b} = f_{a,b}(1 + g_{a,b} r_1) \quad (5)$$

where $g_{h,v}$ is an oscillation correction term whose simple form can be derived based on Rayleigh

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scattering. Then, the relative errors of $|\hat{f}_{a,b}|^2$ and the ratio are estimated and shown in Fig. 2. It shows that the errors of radar reflectivity and differential reflectivity caused by (20%) axis ratio error are small. Considering that the oscillation in raindrop shape may occur in various directions and the large number of drops in a radar sample volume, oscillation effects tend to cancel each other. Therefore, the error due to the axis ratio fluctuation can be neglected as long as the mean axis ratio is correctly modeled.

2.2 Rain DSD Model

Ulbrich (1983) suggested using the Gamma distribution:

$$N(D) = N_0 D^{\mu} \exp(-\Lambda D) \qquad (6)$$

to represent rain DSD. The Gamma DSD with three parameters (*N₀*, μ, and Λ) is capable of describing variety of raindrop size distributions and has been widely accepted by the radar meteorology community. The problem is how to determine the DSD parameters from limited remote measurements such as Z and Z_{DR} and accurately estimate rain.

Analysis of DSD data collected in Florida during the summer of 1998, as shown in Fig. 3, revealed a high correlation between μ and Λ, suggesting that a useful μ - Λ relation could be derived [Brandes et al., 2003a; Vivekanandan et al., 2003; Zhang et al., 2001], given as

$$\Lambda = 0.0365\mu^2 + 0.735\mu + 1.935 \,. \qquad (7)$$

The relation also holds for DSD observations collected in Oklahoma. The relation simplifies the three-parameter Gamma DSD to a two-parameter model that is easy to retrieve from remote measurements. It has been shown (Zhang et al., 2003) that the relation (7) is not purely error effect (Chandrasekar and Bringi, 1987).

The μ -Λ relation suggests that a characteristic size parameter such as *D_0* and the width of mass distribution (σ_m) are also related. In Fig. 4, we plotted σ_m versus D_0 for the DSD measurements. The converted relation from the (7) relation is also shown as well as that for fixed μ values (0, 3, 6). The converted relation from the μ - Λ relation agrees well with the directly fitted σ_m - D_0 relation. The results from a fixed μ values agree with the measurements only for a certain range of DSDs. The results for μ =0 agree with DSD measurements with a large D_0 but don't agree with that for small drops. Results for μ = 6 agree with that for small drops but not for large drops. Certainly μ = 3 (currently used by many algorithms) is a good mean value, but it is not as good as the μ - Λ relation for the whole range of observations.

2.3 Derived Rain Estimators

With the constrained relation (7) for the Gamma DSD and the fixed shape-size relation (3), the DSD shape parameter μ uniquely determines Z_{DR}. Hence, μ can be retrieved from measured Z_{DR} , and Λ and N_0 are calculated subsequently from the relation (7) and Z_H. [Zhang et al., 2001]. Rain rate and median volume diameter are then easily calculated from the retrieved rain DSD.

For convenience in retrieving rain parameters, simple relations are derived based on the constrained Gamma DSD model. We calculate rain parameters and radar measurements (Z_H and Z_{DR}) for Λ in a range from 0.5 to 13 with a fixed N_0. Ratios of N_t and Z_H are uniquely determined by Z_{DR} for a constrained Gamma rain DSD. After taking the logarithm of the ratio and applying a polynomial fit, as shown in Fig. 5, we obtain

$$N_t = 2.08 \times Z_H \times 10^{0.728 Z_{DR}^2 - 2.066 Z_{DR}} \qquad (8)$$

Similarly, we have

$$R = 7.60 \times 10^{-3} \times Z_H \times 10^{-0.165 Z_{DR}^2 + 0.897 Z_{DR}} \qquad (9)$$

$$D_0 = 0.171 Z_{DR}^3 - 0.725 Z_{DR}^2 + 1.479 Z_{DR} + 0.717 \qquad (10)$$

$$\mu = 6.084 D_0^2 - 29.85 D_0 + 34.64 \,. \qquad (11)$$

It is noted that the above relations are fundamentally different from that fitted from simulated or measured datasets. Eq. (8) - (11) are unique for the given axis ratio relation (3) and the issue of data selection or rain DSD simulations was not involved in the derivation.

3. RAIN DSD RETRIEVAL

Data used to compare the direct solution of the DSD parameters Eqs. (1), (2), (3), (6) and (7) and that from polynomial relations (8)-(11) were collected in east-central Florida during the summer of 1998 in a special experiment (PRECIP98) to evaluate the potential of polarimetric radar for estimating rain in tropical environment as described by Brandes et al. (2002).

Instead of using statistical comparisons such as scatter plots or histograms, we show results with instantaneous comparison. Fig. 6 shows a time series comparison between video-disdrometer measurement and radar retrievals using the constrained Gamma model. Both the retrievals by direct solution for DSD parameters and that from the derived estimators (8)-(11) are shown. Both retrievals agree with the in-situ measurements well not only for rain rate but also for other DSD parameters N_t, μ, D_0. Further comparisons and complete verification are given by Brandes et al. (2003 a&b).

4. SUMMARY AND DISCUSSIONS

We developed a constrained-Gamma rain DSD retrieval algorithm based on (i) a fixed raindrop axis ratio relation (3) and (ii) a μ - Λ relation (7) for the Gamma DSD model. These constraining relations allow stable and reasonable retrievals of rain DSD parameters from radar measurements. Simple rain parameter estimators are derived using polynomial fitting. The radar estimated rain rate, raindrop size and

DSD parameters (N_t and μ) agree well with the disdrometer measurements. Justification for using the constrained Gamma approach is as follows: The mean axis ratio relation (3) is needed to capture the nonlinear nature and more spherical shape for raindrops. The oscillation effect r_1 is believed to be small in a radar sample volume. Nevertheless, the oscillation correction term can be included in the model calculation and rain DSD retrieval when r_1 is known from different phase (ϕ_{DP}). The μ - Λ relation simplifies and improves rain DSD retrieval. As seen from its converted σ_m - D_0 relation, the μ - Λ relation has physical base rather than purely statistical error.

Previously, rain DSDs were simulated by randomly generating Gamma DSD parameters within certain ranges. With the μ - Λ relation (7), more realistic rain DSDs can be simulated with less uncertainty as follows: (i) construct mean values of μ and Λ using (7) in a μ range of (-1, 9); (ii) randomize μ (or Λ, or them both) with a standard deviation (say 0.7 corresponding to a uniform distribution of $\pm$1.2 shown in Fig. 3); (iii) generate lognormal distributed rain rate R and calculate the third DSD parameter N_0 (or N_t or N_w).

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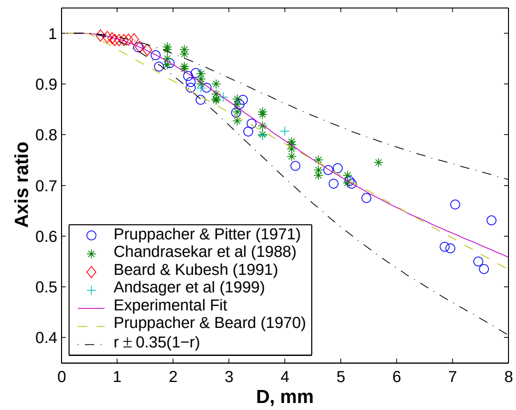

Figure 1. Raindrop axis ratio as a function of equi-volume diameter derived from in-situ measurements.

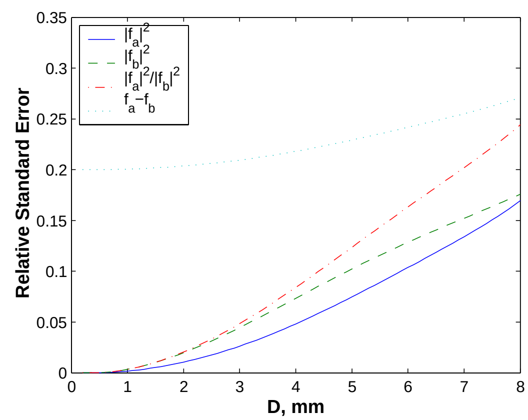

Figure 2. Relative standard error of the parameters corresponding to radar measureables due to 20%(1-r) fluctuation of axis ratio.

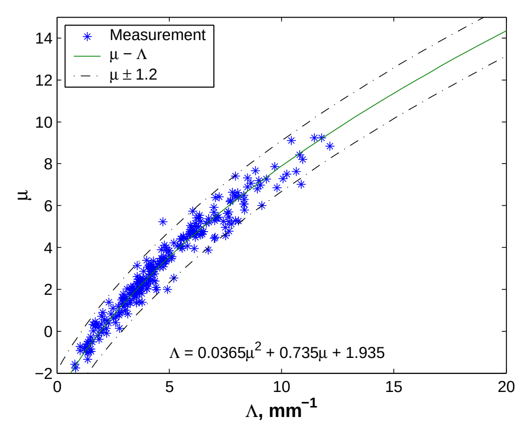

Figure 3: Scatter plots of μ - Λ values obtained using the moment method with filtering of rain rate R >5mm/hr and total counts C_T > 1000.

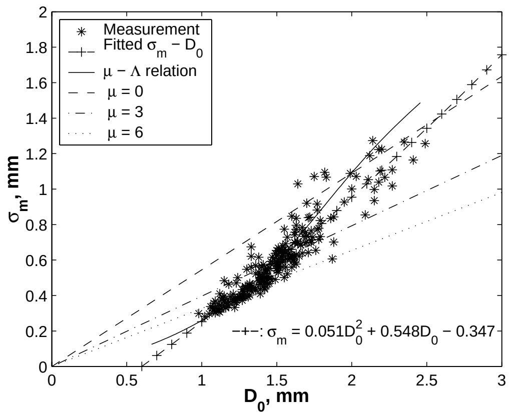

Figure 4. Dependence of σ_m on D_0 for the μ - Λ relation and fitted values of μ =0, 3, and 6.

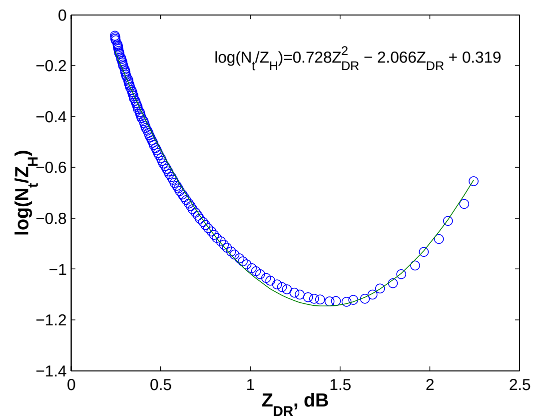

Figure 5: Dependences of $\log(N_t/Z_H)$ on differential reflectivity (Z_{DR}) for constrained Gamma DSD.

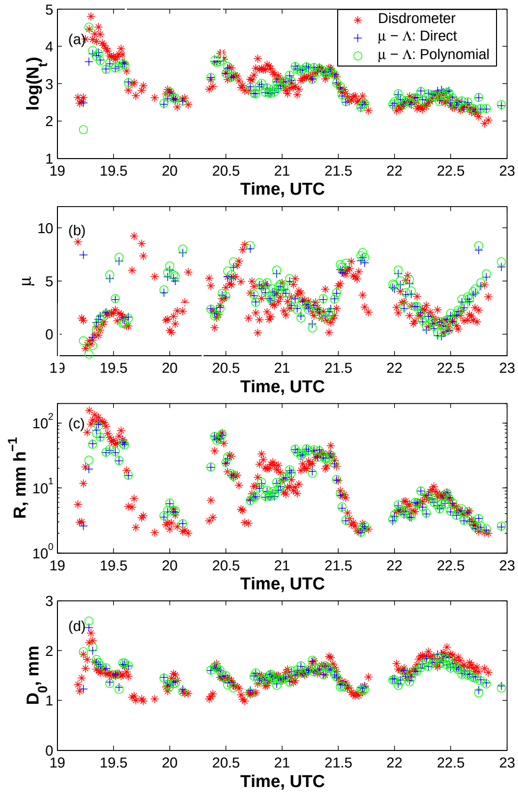

Figure 6: Comparison of DSD parameters retrieved from polarimetric radar measurements using the two approaches as compared with disdrometer measurement. (a) $\log(N_t)$, (b) μ, (c) rain rate (R), and (d) median volume diameter (D_0).