8B.5 An Algorithm for Estimating the Variance of the Canting Angle Distribution Using Polarimetric Covariance Matrix Data

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1. INTRODUCTION

The measurement of rainfall is an important application of weather radar. There are many rainfall estimation algorithms using radar measurements; these can be roughly classified into two categories, physically-based and statistical/engineering-based (Bringi and Chandrasekar 2001). Physically-based methods rely fundamentally on the precipitation model, namely models for the drop size distribution (dsd), drop shape and drop orientation (or canting). Especially, the drop shape and drop orientation can play an important role on the physically-based methods using linear dual-polarization radar measurements, such as Z_{dr} and K_{dp}.

Basically, the drop shape is close to oblate spheroidal. In this case, the falling raindrops without oscillation will have their symmetric axis near vertical. The drop orientation due to turbulence is known to be Gaussian with zero mean and standard deviation less than 5^o (Beard and Jameson 1983). However, falling drops will also exhibit steady oscillations. Figure 1 shows two fundamental oscillation modes, axisymmetric oscillation and transverse oscillation, as well as their combination (multimode oscillation). Axisymmetric oscillations affect the shape of the drop but do not affect the orientation because they oscillate along the symmetry axis. However, transverse oscillations as well as multi-mode oscillations will exhibit an orientation distribution which is different from the traditional Gaussian assumption of orientation (based on drop turbulence). Since raindrops will be subject to both multi-mode oscillations as well as turbulence, a simple way to account for both effects is via an effective canting angle distribution which is Gaussian with mean of 0^o and with unknown but larger standard deviation (σ_{eff}).

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In this paper, we propose an algorithm to estimate the effective standard deviation of canting angle (σ_{eff} or σ_β) using the covariance matrix. We apply this algorithm to an intense rain event using data from the CSU-CHILL radar. Based on simulations, we also show that the bias in rainfall estimation can be reduced significantly by using the estimate of σ_β. This methodology may be considered as an alternate to the method proposed by Gorgucci et al. (2002) to account for drop oscillations. The method herein does not use the K_{dp} data, but does use LDR and ρ_{co}.

2. Theory

In order to study the impact of canting angle on radar measurements, Hendry et al. (1987) proposed a canting parameter, ρ_4, which directly related to canting angle distribution and defined as,

$$\rho_4 = \int_{-\pi/2}^{\pi/2} \cos 4(\beta-\beta_o)p(\beta-\beta_o)d(\beta-\beta_o) \quad (1)$$

where β_o is the mean canting angle and $p(\beta)$ is the pdf of canting angle which is assumed to be symmetric about β_o. The ρ_4 can be computed from the ratio of maximum and minimum cross-polar power (P_{cr}) as,

$$\frac{(P_{cr})_{max}}{(P_{cr})_{min}} = \frac{1+\rho_4}{1-\rho_4} \quad (2)$$

For a Gaussian canting angle distribution, ρ_4 is a function of standard deviation of canting angle (σ_β) as,

$$\rho_4 = \exp(-8\sigma_\beta^2) \quad (3)$$

Hendry et al. (1987) also showed that the relation between ρ_4 and σ_β is not sensitive to the precise form for the pdf of β except that the pdf be symmetric. The CSU-CHILL radar can measure the full (3x3) covariance matrix. The covariance matrix defined by Tragl (1990) is,

$$\mathbf{\Sigma} = \begin{bmatrix} \langle|S_{hh}|^2\rangle & \sqrt{2}\langle S_{hh}S_{hv}^*\rangle & \langle S_{hh}S_{vv}^*\rangle \\ \sqrt{2}\langle S_{hv}S_{hh}^*\rangle & 2\langle|S_{hv}|^2\rangle & \sqrt{2}\langle S_{hv}S_{vv}^*\rangle \\ \langle S_{vv}S_{hh}^*\rangle & \sqrt{2}\langle S_{vv}S_{hv}^*\rangle & \langle|S_{vv}|^2\rangle \end{bmatrix} \quad (4)$$

where "*" refer to conjugate and $\langle . \rangle$ denotes time average. The unitary polarization transformation matrix is defined as,

$$\mathbf{T}(\chi) = \frac{1}{1+\chi\chi^*}\begin{bmatrix} 1 & \sqrt{2}\chi & \chi^2 \\ -\sqrt{2}\chi^* & (1-\chi\chi^*) & \sqrt{2}\chi \\ (\chi^*)^2 & -\sqrt{2}\chi^* & 1 \end{bmatrix} \quad (5)$$

where χ is the polarization ratio defined as,

$$\chi = \frac{\cos(2\tau)\sin(2\theta_t) + j\sin(2\tau)}{1+\cos(2\tau)\cos(2\theta_t)} \quad (6)$$

where τ is ellipticity angle and θ_t is tilt angle. Once we measure the covariance matrix in one polarization basis (e.g. H-V basis), we can transfer to any other polarization basis by applying basis transformation as,

$$\mathbf{\Sigma}' = \mathbf{T}(\chi)\cdot\mathbf{\Sigma}\cdot\mathbf{T}^{T*}(\chi) \quad (7)$$

The canting angle (β) is defined as the angle between local vertical (refer as $\hat{z}$ direction) and the projection of the symmetry axis on the polarization plane. Since there is no evidence that symmetry axis projected on the horizontal X-Y plane will tend to any special direction, we usually assume that drop orientation along the azimuthal direction is uniformly distributed. Therefore, the distribution of β is symmetric along the $\hat{v}$ direction (it also means that the mean of β is zero). In this case, the maximum and minimum cross-polar power should be at $\theta_t = 45^o$ and $\theta_t = 0^o$. For the covariance measured at linear basis, the ellipticity angle is 0^o. Therefore, the maximum and minimum cross-power are,

$$\begin{aligned} (P_{cr})_{min} \propto \mathbf{\Sigma}'(2,2)|^{\tau=0^o}_{\theta_t=0^o} &= 2\langle|S_{hv}|^2\rangle && (8a) \\ (P_{cr})_{max} \propto \mathbf{\Sigma}'(2,2)|^{\tau=0^o}_{\theta_t=45^o} &= \frac{1}{2}\langle|S_{hh}|^2\rangle + \frac{1}{2}\langle|S_{vv}|^2\rangle \\ &\quad -Re\left[\langle S_{hh}S_{vv}^*\rangle\right] && (8b) \end{aligned}$$

From (8a,b), we can compute ρ_4 as well as σ_β.

The upper panel of Figure 2 shows the scatter plot of estimated σ_β versus Z_{dr} in a convective rain cell on 11 June, 2000 of the STEPS project. In the lower panel, we divide Z_{dr} into several equal intervals from 0.5 dB to upper bound of Z_{dr} (each interval is 0.5 dB). The results show that σ_β decreases as Z_{dr} increases and reflects the fact that larger drops are more stably oriented as compared to small-sized drops. Moreover, polarimetric-based rainfall algorithms need to take account of this behavior of σ_β versus Z_{dr} instead of assuming that σ_β is fixed at $5-10^o$. The rain rate algorithm proposed herein first estimates σ_β using ρ_4, and then estimates the dsd parameters following Tang (2003).

3. Simulation and Discussion

As discussed above, the shape of rain drops are usually assumed to be oblate spheroidal. Therefore, the backscattering matrix elements (assuming Rayleigh scatter) with canting angle of β can be expressed as,

$$\begin{aligned} S_{hh} &= \frac{k_o^2}{4\pi\epsilon_o}[\alpha\cos^2\psi + \sin^2\psi(\alpha_z\sin^2\beta \\ &\quad +\alpha\cos^2\beta)] && (9a) \\ S_{vv} &= \frac{k_o^2}{4\pi\epsilon_o}[\alpha\cos^2\psi + \sin^2\psi(\alpha_z\cos^2\beta \\ &\quad +\alpha\sin^2\beta)] && (9b) \\ S_{hv} &= \frac{k_o^2}{4\pi\epsilon_o}(\frac{\alpha_z-\alpha}{2}\sin^2\psi\sin^2\beta) && (9c) \end{aligned}$$

where k_o is the wave number in free space, ψ is the angle between the symmetry axis and the direction of incident wave, and α and α_z are the polarizability elements. For the Gaussian canting angle distribution with zero mean and $\sigma_\beta < 10^o$ ($\psi \approx 90^o$), the intrinsic Z_h, Z_v and the real part of R_{co} (copolar covariance, $S_{hh}S_{vv}^*$) can be expressed as (Tang 2003),

$$\begin{bmatrix} Z_h & Z_v & Re[R_{co}] \end{bmatrix}^T = \mathbf{P}^{-1}\mathbf{m} \quad (10)$$

where $\mathbf{P}$ is the angle moment operating matrix which can be expressed as,

$$\mathbf{P} = \begin{bmatrix} \langle\cos^4\beta\rangle & \langle\sin^4\beta\rangle & 2\langle\sin^2\beta\cos^2\beta\rangle \\ \langle\sin^4\beta\rangle & \langle\cos^4\beta\rangle & 2\langle\sin^2\beta\cos^2\beta\rangle \\ \langle\sin^2\beta\cos^2\beta\rangle & \langle\sin^2\beta\cos^2\beta\rangle & \langle\cos^4\beta\rangle + \langle\sin^4\beta\rangle \end{bmatrix} \quad (11)$$

and $\mathbf{m}$ is,

$$\mathbf{m}^T = \begin{bmatrix} Z_h^m & Z_v^m & Re[R_{co}]^m \end{bmatrix} \quad (12)$$

where the superscript "m" refer to "measured" value. If the dsd is the gamma distribution suggested by Testud et al. (2001), the reflectivity-weighted mean size (D_g) is,

$$D_g = \frac{\langle D^7\rangle}{\langle D^6\rangle} = \frac{7+\mu}{3.67+\mu}D_o \quad (13)$$

and the relation between D_g and differential reflectivity in linear scale (ξ_{dr}) follows a power law as,

$$\xi_{dr} - 1 = \frac{Z_h - Z_v}{Z_v} = 0.0353 D_g^{2.0996} \quad (14)$$

If the relation between drop terminal velocity and drop size follows a power law (Atlas and Ulbrich 1977), the rainfall rate is,

$$
\begin{aligned}
R &= (0.6 \times 10^{-3}\pi)(3.78) N_w f(\mu) \\
&\quad \Gamma(4.67+\mu)\frac{D_o^{4.67}}{(3.67+\mu)^{4.67+\mu}}
\end{aligned} \tag{15}
$$

where $f(\mu)$ is

$$
f(\mu) = \frac{6}{3.67^4}\frac{(3.67+\mu)^{\mu+4}}{\Gamma(\mu+4)} \tag{16}
$$

Since the reflectivity factor is the 6th moment of dsd, Tang (2003) showed that the Z-R relation in terms of D_g is,

$$
R = \frac{Z_h}{F(\mu)(\frac{\mu+3.67}{\mu+7})^{2.33} D_g^{2.33}} \tag{17}
$$

where $F(\mu)$ is a known function of μ.

In our simulation, we used Joss disdrometer dsd data from Darwin, Australia and assumed that the mean axis ratio is that suggested by Andsager et al. (1999) for $1 \leq D \leq 4\ mm$ and by Beard and Chuang (1987) for $D < 1$ and $D > 4\ mm$. The canting angle (β) is assumed to be Gaussian distribution with zero mean and $\sigma_\beta = 2$, 10 and 20^o. Using these conditions, we simulate the radar measurements and corresponding covariance matrix. The "true" rainfall rate is assumed to be given by (15). Since μ is very difficult to estimate in the real case, we assume that $\mu = 3$. Figure 3 shows the estimated rainfall rate versus "true" rainfall rate at $\sigma_\beta = 2^o$. The estimated rainfall rate is using (17). The uncorrected case (circle marks) directly uses "measured" ξ_{dr} to computed D_g. For the corrected cases, we use σ_β to calculate the angle moment matrix ($\mathbf{P}$), and then compute the intrinsic Z_h, Z_v and D_g. There are two versions of corrected rainfall rate. The first one (dot marks) uses simplified ρ_4 method (using (8a,b)) to estimate σ_β. The second one (diamond marks) uses the known σ_β which we use to simulate the radar measurements (in this case $\sigma_\beta = 2^o$). Since σ_β is very small, the three estimators are almost the same as expected. Figure 4 shows the estimated rainfall rate versus "true" rainfall rate at $\sigma_\beta = 10^o$. The results show that σ_β correction reduces the normalized error from 9.19% to 3.90%, and reduces the normalized bias from 9.01% to 0.65%. Figure 5 shows an extreme case. The bias in the corrected rainfall rate is due to large σ_β ($> 10^o$, our rainfall algorithm is only valid for $\sigma_\beta < 10^o$). However, σ_β correction still reduces the normalized bias from 51.08% to 9.56%.

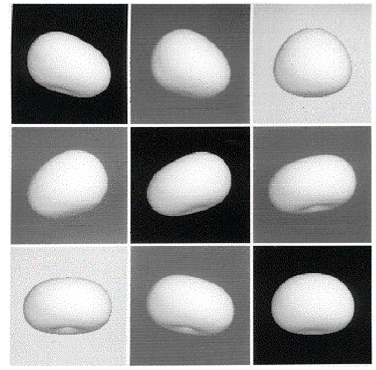

Figure 1: A computer-generated oscillation sequence. The lower right black panel is the equilibrium shape of a 5 mm drop, two other black panels (the diagonal panels) show the transverse oscillation mode, two white panels (upper right and lower left) show the axisymmetric oscillation mode, and the four grey panels show the mixed oscillation mode. Courtesy of Prof. Ken Beard, University of Illinois.

Our results indicate the need to take account of effective drop canting (due to multi-mode oscillations and/or turbulence) for intense rainfall. The proposed methodology differs from that in Gorgucci et al. (2002) where the slope of a linear axis ratio model ($r = 1 - \beta D$) is estimated based on Z_h, Z_{dr} and K_{dp} data. Note that drop oscillations as well as turbulence-induced canting will tend to decrease this slope relative to the equilibrium axis ratio model. Our method is an alternate approach which uses LDR and ρ_{co} data (as opposed to K_{dp}) to achieve the same end goal; however, it requires very good antenna polarimetric performance to achieve high quality LDR measurements. From the simulation, we found that even within traditional assumption ($\sigma_\beta = 10^o$), the drop canting still can bias the rainfall upto 10%. Usually, we can tune the coefficients of our algorithms by simulation or rain gage data to overcome this impact. However, it is better to take account the effect of canting angle in theoretical models and "tune" the algorithms. Moreover, the results also show that our two versions of the "correction" algorithm are very close. It means that the simplified ρ_4 method can estimate σ_β accurately,and therefore, account for multi-mode drop oscillations and/or canting due to turbulence.

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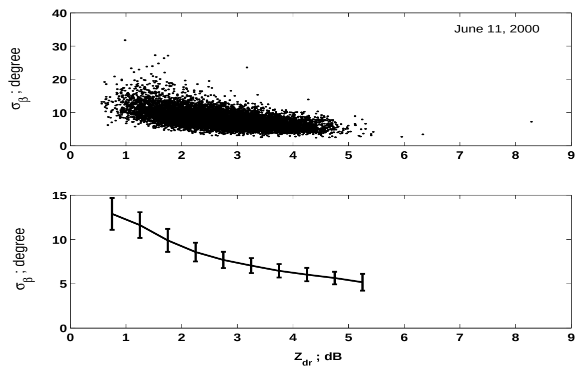

Figure 2: Estimated σ_β versus Z_{dr}. The upper panel is a scatter plot. In the second panel, we divide Z_{dr} into 8 equal intervals, each interval being 0.5 dB. We calculate the mean and standard deviation of corresponding σ_β in each interval. The error bar is one standard deviation.

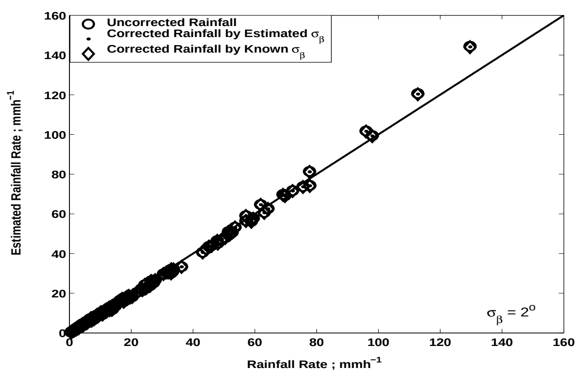

Figure 3: The estimated rainfall versus "true" rainfall at $\sigma_\beta = 2^o$. The circle-marks are the uncorrected rainfall estimations, the dot-marks are the rainfall estimations corrected by estimated σ_β, and the diamond-marks are the rainfall estimation corrected by the known $\sigma_b eta$.

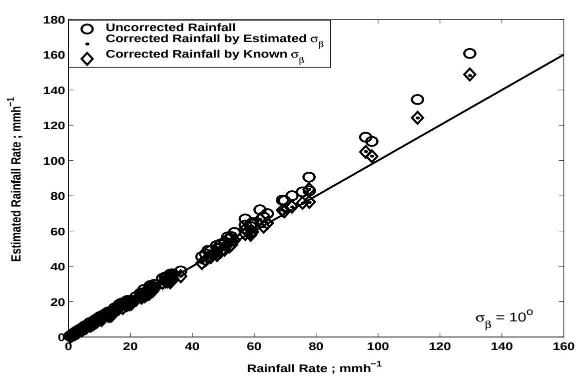

Figure 4: As in Figure 3 except $\sigma_\beta = 10^o$.

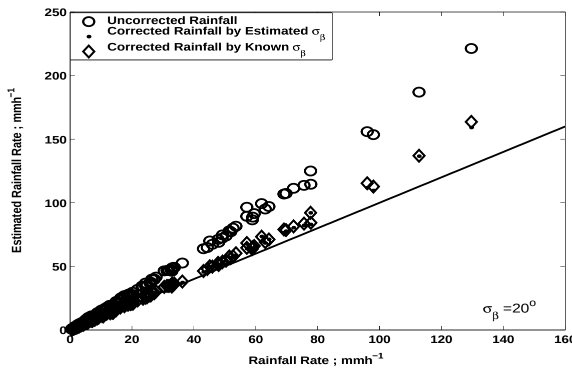

Figure 5: As in Figure 3 except $\sigma_\beta = 20^o$.

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