P3.10 ON SMALL-SCALE VARIABILITY OF RAINFALL FROM RADAR

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1. INTRODUCTION

Information about the small-scale (less than ~ 20 km) variability of rainfall is essential in many theoretical and practical applications. Ground-based radar data are increasingly used to study small-scale rainfall variability (Moszkowicz 2000). Despite their potential to give spatial patterns, radars are known to have a number of limitations. Radar-derived rainfall estimates are subject to a number of error sources (Austin 1987). Krajewski et al. (1996) showed through a simulation experiment how difficult it is to retrieve the statistical properties of the 'true' rain fields from the reflectivity fields. Because of these limitations, it is important to assess the accuracy of radar-derived spatial variability estimates.

The objective of this study is to assess how well the spatial variability of rainfall is captured by ground-based radars. We performed the assessment by comparing the radar-derived spatial structure with the corresponding estimates obtained from a high quality dense rain gauge network. The underlying premise of this study is that discrepancies between radar- and gauge-derived spatial variability estimates are mainly attributed to uncertainties in radar-derived estimates. This premise is reasonable because the instrumental and spatial representativeness errors, which are the most important error sources of gauge-rainfall, are greatly minimized in high quality dense rain gauge networks such as those used in this study.

2. DATA AND ANALYSIS PROCEDURE

The data used in this study were collected by rain gauge networks and ground-based radars deployed during two TRMM campaigns: Texas and Florida Underflight (TEFLUN-B) experiment and TRMM Large scale Biosphere-Atmosphere (TRMM-LBA). TEFLUN-B was conducted from 1 August through 27 September 1998 and focused on central Florida. TRMM-LBA took place in the southwest of the Amazon basin from 10 January through 28 February 1999. The radars used in this study comprised of the Weather Surveillance Radar-1988 Doppler (WSR-88D) operational radar during TEFLUN-B, and the S-band National Center for Atmospheric Research (NCAR) POLarimetric radar (S-POL) during TRMM-LBA. The radar-rainfall products evaluated for TEFLUN-B were the standard 2A-53 products provided by the Tropical Rainfall Measuring Mission (TRMM) Ground Validation Program (GVP). The radar-rainfall products evaluated for TRMM-LBA were created using the differential phase method of Ryzhkov et al. (2000) by the Colorado State University Radar Meteorology Group. As we are interested in the direct comparison of gauge- and radar-derived statistics, we defined our region of interest as a subset of the radar map which is centered over the gauge network (Figure 1).

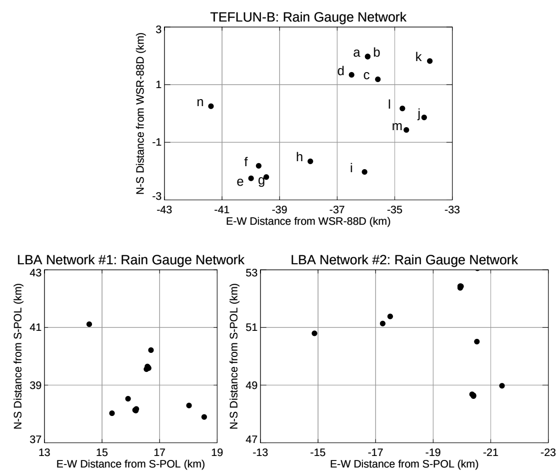

Figure 1. Diagram of the rain gauge networks deployed during (top) TEFLUN-B and (bottom) TRMM-LBA fields experiments, with respect to distances from the radar center.

For TEFLUN-B, the selected region has an area of 6 km $\times$ 10 km (Fig. 1, upper panel). The corresponding gauge network consisted of 14 gauges. For TRMM-LBA,

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our region of interest is a combination of a 6 km × 6 km area centered over Network #1 and a 6 km × 10 km area centered over Network #2. Networks #1 and #2 were spaced about 35 km apart. Networks #1 and 32 consisted of 14 and 13 gauges, respectively. From the radar maps, we subsetted 15, 9, and 15 pixels, corresponding to TEFLUN-B and TRMM-LBA Networks #1 and #2, respectively.

We used correlation structure and fractal characteristics to characterize the spatial structure. We used the transformation-based approach proposed by Habib et al. (2001) to estimate the sample correlations. To approximate the spatial correlation function over separation distance, we used the following three-parameter exponential model

$$r(h) = \theta_0 \exp[-(h/\theta_1)^{\theta_2}], \qquad (1)$$

where h is the Euclidean separation distance, θ_0 [the local decorrelation] is its value at $h = 0$, θ_1 is the correlation distance, and θ_2 is the shape factor. The difference 1 - θ_0 is known as the "nugget effect". We obtained the "best" fit parameters in the weighted least-squares sense; that is, the sum of the weighted squared difference between the model and data is minimized. To address the mismatch in scale between gauge- and radar-derived correlation functions, we derived equations for integrating the gauge point correlation over any specified area following the approach given by Vanmarcke (1983). We also examined the fractal characteristics of gauge- and radar-rainfall fields. Any random variable, which has the property of second-order stationarity, whose correlation structure can be represented by a power law

$$\rho(h) = mh^{-k}, \qquad (2)$$

obeys a scaling law (e.g., Yaglom 1987). Such a process canbe considered fractal, a concept introduced by Mandelbrot (1977). In (2), m and k are constants. The exponent k is related to the Hurst exponent. We estimated the scaling exponent k as a slope of the regression '$\log h$ vs. $\log \rho(h)$' obtained by log-transforming (2)

$$\log \rho(h) = \log m - k \log h. \qquad (3)$$

We applied the Durbin-Watson test (Neter and Wasserman 1974) to decide whether the rainfall fields satisfy the relation (3).

3. RESULTS

Figure 2 presents the area-correlation function derived form gauge- and radar-rainfall fields for (top) TEFLUN-B and (bottom) TRMM-LBA, at 5 and 15 minute rain accumulation times. The two correlation functions show considerable discrepancy. For distances shorter than about 5 km, the radar-derived correlation is always smaller than the gauge-derived one. Consider a separation distance of 2 km (the size of radar resolution) for TEFLUN-B, the correlation estimate is 0.72 (5 min) and 0.79 (15 min) for gauge-rainfall fields, while it is 0.55 for radar-rainfall fields. A summary of fitted parameters is given in Table 1. The largest discrepancy between gauge- and radar-derived correlation function model lies in the shape parameter: the radar-derived shape parameter is about 0.5 to 0.6 times that of the gauge-derived value.

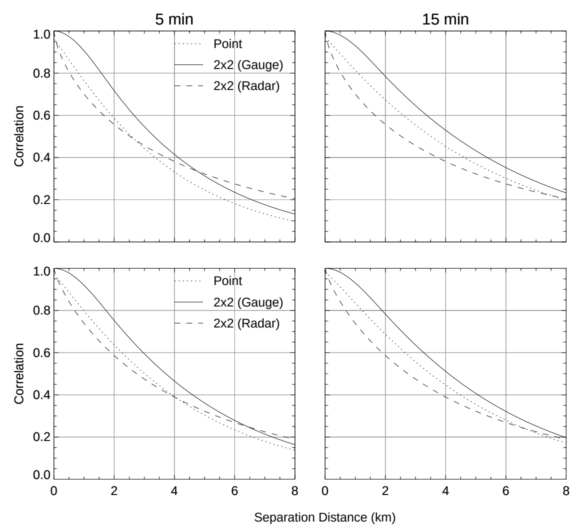

Figure 2. Spatial correlation functions obtained from gauge- and radar-rainfall fields for (top) TEFLUN-B and (bottom) TRMM-LBA. Dotted lines represent correlations of point intensity for (left) 5 min and (right) 15 min gauge accumulations. The correlation function of these point intensities are integrated to yield the area-correlation function over an area of 2 × 2 km^2 (solid lines). Dashed lines represent the correlation function obtained from radar-rainfall fields.

Table 1. Comparison of the fitted spatial correlation function parameters estimated by (i) integrating the correlation function of gauge point intensities over 2 × 2 km^2, and (ii) calculating directly from radar rain field.

Sensor	Scale (minute)	θ_0	θ_1, km	θ_2
TEFLUN-B				
Gauge	5	1.00	4.24	1.47
	15	1.00	5.61	1.37
Radar	Snapshot	1.00	4.21	0.72
TRMM-LBA				
Gauge	5	1.00	5.00	1.40
	15	1.00	5.47	1.41
Radar	Snapshot	1.00	4.30	0.82

Figure 3 shows a log-log plot of the sample correlations. The gauge-derived sample correlations show a large scatter, particularly for TEFLUN-B. We tested the linearity in the log-log dependency of the correlation on separation distance, ranging from 2 to 8 km, using the Durbin-Watson test. In Table 2, *D* is the Durbin-Watson test statistic and d_L and d_U are the lower and upper bounds, respectively, such that a value of D outside these bounds leads to a definite decision. As can be seen from Table 2, there is no definite decision regarding the linearity in TRMM-LBA rainfall fields. In principle, this means more data are required. For TEFLUN-B, the radar-rainfall exhibited a linear pattern, while the gauge-rainfall fields did not.

Table 2. Test of linearity pattern in log-log dependency of the correlation on distance observed in gauge- and radar-rainfall fields, based on the Durbin-Watson statistic test at 95% confidence level.

Sensor	Scale (minute)	D	d_L	d_u
TEFLUN-B				
Gauge	5	1.81	1.58	1.64
	15	1.85	1.58	1.64
Radar	Snapshot	0.61	1.08	1.36
TRMM-LBA				
Gauge	5	1.57	1.57	1.63
	15	1.58	1.57	1.63
Radar	Snapshot	1.35	1.08	1.36

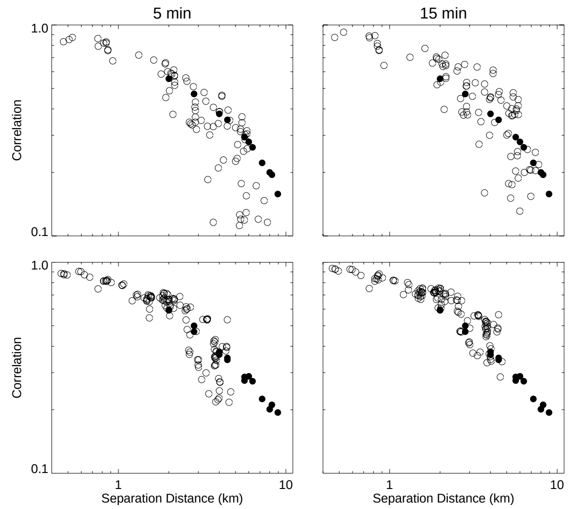

Figure 3. Log-log plot of correlations estimated from (filled circles) radar-rainfall and (open circles) gauge-rainfall, for (top) TEFLUN-B and (bottom) TRMM-LBA.

3. APPLICATION-BASED EVALUATION

Above we showed the magnitude of the discrepancy in correlation functions derived from gauge- and radar-rainfall fields. One of the applications where the correlation function input is required is the quantification of the gauge sampling error. In this section, we examine quantitatively to what degree the error in the radar-derived correlation function affects the gauge sampling error estimates. To define the sampling error, consider a single gauge located within a square area of size A. Let $\hat{R}$ be the rain rate measured with a gauge over period t, and say this measurement is used to estimate the rain rate R averaged over area A and period t. The estimation error is $\varepsilon = \hat{R} - R$. Our interest is in the sampling error σ_ε, which is defined here as the standard deviation of ε. Morrissey et al. (1995) derived an equation, which requires the correlation function, to estimate σ_ε. Let us rewrite the equation as

$$\sigma_\varepsilon = \sigma_R \phi[\rho(d_{i,j})]. \quad (4)$$

ϕ represents the impact on sampling error by the correlation function. ϕ^2 is sometimes called the variance reduction factor (Gebremichael et al. 2003).

Let $\hat{\phi}$ and ϕ denote the phi-functions estimated using radar- and gauge-derived correlation functions, respectively. Setting the parameter area A = 4 km^2 (the size of the radar-rainfall resolution), we calculated $\hat{\phi}$ and ϕ for every gauge shown in Fig. 1. Table 1 shows the results for TEFLUN-B. Results show that the ratio $\hat{\phi}/\phi$ ranges from 1.0 to 1.09 for TEFLUN-B, and 0.96 to 1.09 for TRMM-LBA. Therefore, the error introduced in gauge sampling error estimates due to the use of radar-derived correlation function is within 10%.

Table 3. Comparison of the parameter ϕ obtained using the correlation function derived from 15 min gauge-rainfall with the parameter $\hat{\phi}$ estimated using the correlation function derived from radar-rainfall, for each gauge location during TEFLUN-B shown in Fig. 1 with respect to the corresponding radar pixel.

Label	Official ID	$\hat{\phi}$	ϕ	$\hat{\phi}/\phi$
a	101a	0.34	0.37	1.09
b	101b	0.34	0.37	1.09
c	102	0.48	0.48	1.00
d	103	0.46	0.46	1.00
e	108a	0.35	0.38	1.08
f	108b	0.36	0.39	1.07
g	108c	0.40	0.42	1.04
h	109	0.36	0.39	1.07
i	110	0.34	0.37	1.09
j	112	0.34	0.38	1.09
k	113	0.36	0.38	1.08
l	114	0.44	0.45	1.02
m	115	0.45	0.46	1.01
n	116	0.42	0.43	1.03

5. CONCLUSIONS

We assessed the accuracy of radar-rainfall products in capturing the small-scale spatial variability of rainfall in terms of the spatial correlation function and fractal characteristics. We used data from high quality dense rain gauge networks deployed during TEFLUN-B and TRMM-LBA field campaigns to perform the assessment. Our results revealed that:

1) At small separation distances (< 5 km), the radar-rainfall field gives smaller correlations than the corresponding gauge-rainfall;
2) The error introduced in gauge sampling error estimates due to the use of radar-derived correlation function is within 10%; and
3) Larger sample size is required to study fractal characteristics.

5. REFERENCES

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