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1. INTRODUCTION

Synthesized rainfall fields with higher resolution than observed would be a useful tool in quantifying the errors of rainfall estimation by weather radar. The most applied procedures are modelling the rainfall as fractional noise and decomposing the rainfall field using a wavelet transform. Here both procedures are compared to determine which one reproduces the best the rainfall variability. To simplify our study, it has been done over one-dimensional rainfall time series measured by a Doppler disdrometer.

2. MODELLING THE RAINFALL AS FRACTIONAL NOISE

Fractional noises like fractional Brownian motion (fBm) or fractional Gaussian noise (fGn) have been broadly applied in modelling fractal signals and textures (Mandelbrot and Ness, 1968). The reason lies in the fact that they could be easily built from white noises just imposing a power law to white Fourier spectra. Because observed rainfall time series seem to exhibit Fourier spectra of the form $\Gamma(f) \propto f^{-\beta}$, a similar procedure is proposed for simulating rainfall fields. In order to do this, an average β -exponent is obtained from the Fourier analysis of a set of measured rainfall sequences. Besides, the supposed 'white noise support' is extracted from each sequence and accumulated to obtain an experimental pdf. This pdf is used later to generate the initial white noise in the synthesis process. Since the rainfall intensity is a magnitude always positive, the time series which are processed as fractional noises are not directly series of rainfall intensity but series of the natural logarithm of rainfall intensity.

3. APPLYING A WAVELET MODEL

A wavelet transform, in particular the Haar transform, decomposes the discrete rainfall field observed at a certain scale to both the average component and the fluctuation component of the rainfall field at the next larger scale. The proposed wavelet model is based on the fact that the experimental pdf's of the standardized rainfall fluctuation component exhibit zero-mean gaussianity with power-law decaying variance σ_m^2

over successive fine to coarse scales, as expressed by equation (1) (Perica, Fofu-Georgiou, 1996; see figure 1a and figure 1b).

$$\sigma_m \propto 2^{m \cdot H} \quad (1)$$

with m the index scale and H constant.

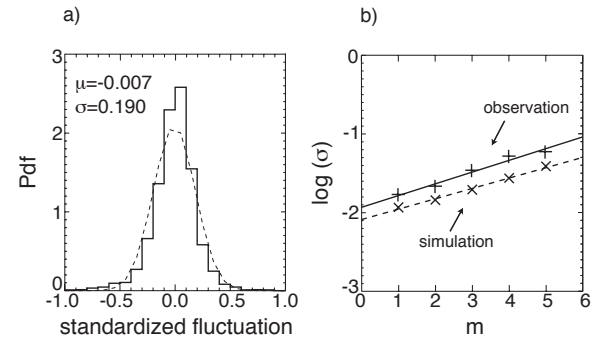


Figure 1. a) Experimental probability density function of the standardized rainfall fluctuation component at the 4 minutes resolution scale (the continuous line is the computed histogram and the dashed line represents the fitted normal pdf). It has been obtained from a set of measured rainfall time series by a Doppler disdrometer (data provided by the J.S. Marshall Radar Observatory). b) Linear regression of $\log(\sigma)$ respect to the m index scale (observed rainfall in crosses and continuous line; simulated rainfall as fractional noise in x's and dashed line)

According to this, rainfall sequences are synthesized by generating random values of each fluctuation component from the corresponding zero-mean normal pdf. These values have been distributed over the time support as they were uncorrelated. The assumption of no correlation between the values of the fluctuation component is not entirely unrealistic since $1/f$ processes become nearly uncorrelated in the wavelet domain (Abry et al., 1995).

4. NECESSITY OF A COMPARATIVE ANALYSIS OF THE PROPOSED PROCEDURES

Each proposed model for rainfall simulation focuses in one aspect of the rainfall variability: Fourier spectrum obeying a power decaying law in case of rainfall synthesized as fractional noise; and simple scales law relative to the amplitude of rainfall fluctuations between close positions in case of the wavelet model. Due to this, each procedure will reproduce the variability feature on which it is based but it may not be able to recreate the rest of them. Therefore, a

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comparative analysis of simulated rainfall sequences by both procedures becomes a first necessary step to assess their reliability.

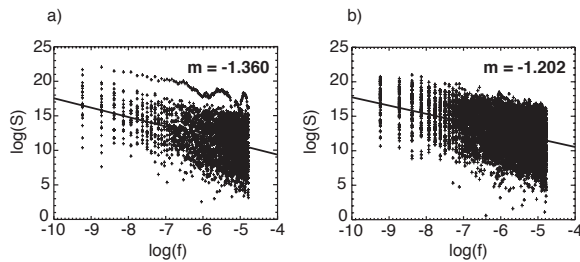


Figure 2. a) Fourier spectra of a set of observed rainfall sequences. b) Fourier spectra of a set of simulated rainfall sequences by applying a simple wavelet model. The linear regression is also showed in both cases.

5. WAVELET ANALYSIS OF SIMULATED RAINFALL AS FRACTIONAL NOISE

By applying a wavelet transform to simulated fractional rainfall sequences of the same resolution than observed, the pdf of the fluctuation component associated to each scale has been obtained. As can be seen from figure 1b, the standard deviation of pdf's from simulation is always lower than the corresponding standard deviation resulting from the wavelet analysis of real data. This fact may indicate that modelling rainfall as fractional noise is not able to reproduce the highest rainfall fluctuations between close positions.

6. FOURIER ANALYSIS OF SIMULATED RAINFALL USING A WAVELET MODEL

In figure 2 Fourier spectra of synthesized rainfall by using wavelets and Fourier spectra of real precipitation are presented. Comparing both spectra shows that the energy of the lowest frequency components is higher in case of observed rainfall. This means that simulated rainfall by using the proposed wavelet model is less correlated for long distances than real rainfall, which also is suggested by looking at both sets of rainfall sequences (see figures 3a and 3b).

7. CONCLUSIONS

Two different procedures for simulating rainfall fields have been tested and compared to determine which one reproduces the best the rainfall variability. However, both exhibit limitations when analyzed from the point of view of the other procedure. In this sense, synthesized rainfall applying a wavelet model shows weaker long-range correlation than observed rainfall. On the other hand, the standardized fluctuation component of rainfall simulated as fractional noise presents systematically lower standard deviation than real rainfall over all analyzed scales. Moreover, probably because it flows as a multiplicative cascade from coarse to fine scales, the wavelet model seems to recreate better rare bursty events. Therefore, it may be suggested that a wavelet model capturing more accurately the correlation structure of rainfall wavelet components would be an interesting improvement in synthesizing rainfall fields.

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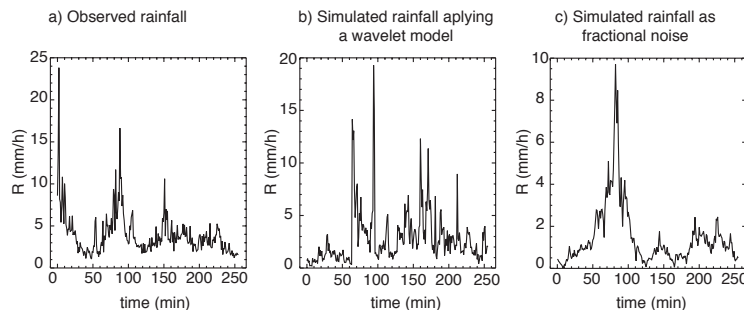


Figure 3. Representative time series of: a) observed rainfall, b) simulated rainfall applying a wavelet model, and c) simulated rainfall as fractional noise.