5B.6

An Analysis of Dual-Polarized Radar Measurables for Rainfall Measurement

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1 Introduction

The models of the rain drop size distribution (DSD), drop axis ratio and drop canting form the physical basis for rain rate estimation techniques. A recent formulation proposed by Testud et.al. (2001) uses the shape parameter μ, intercept parameter N_w and medium volume diameter D_0:

$$N(D) = N_w f(\mu) \left(\frac{D}{D_0}\right)^{\mu} exp[-(3.67+\mu)\frac{D}{D_0}], \quad (1)$$

where,

$$f(\mu) = \frac{6}{3.67^4}\frac{(3.67+\mu)^{\mu+4}}{\Gamma(\mu+4)}. \quad (2)$$

The drop axis ratio model used in this article is the linear axis ratio model proposed by Gorgucci et al. (2000):

$$r = 1.03 - \beta D. \quad (3)$$

The β model accounts for the effects of drop shape changes due to drop oscillations. One property of the β model is that for any nonlinear shape model, it is possible to find an “equivalent” linear model with certain β. The canting angle (ϕ) follows a Gaussian distribution with 0 mean and σ_ϕ standard deviation (Bringi and Chandrasekar 2001).

2 Approximation Results for Rayleigh Scattering

In the Rayleigh-Gans scattering case, following the results in the book of Bringi and Chandrasekar (2001), the elements of the backscattering matrix $[\mathbf{S}]_{\mathbf{BSA}}$ can be written as:

$$S_{hh} = \frac{k_0^2}{4\pi\epsilon_0}\left(\alpha_z \sin^2\phi + \alpha\cos^2\phi\right), \quad (4)$$

$$S_{vv} = \frac{k_0^2}{4\pi\epsilon_0}\left(\alpha_z \cos^2\phi + \alpha\sin^2\phi\right), \quad (5)$$

$$S_{vh} = S_{hv} = \frac{k_0^2}{4\pi\epsilon_0}\left[\frac{(\alpha_z-\alpha)}{2}\sin 2\phi\right], \quad (6)$$

where,

$$\alpha = \frac{1}{6}\pi D^3\epsilon_0\Lambda_h \quad (7)$$

$$\alpha_z = \frac{1}{6}\pi D^3\epsilon_0\Lambda_v \quad (8)$$

Λ_h and Λ_v are defined as:

$$\Lambda_h = \frac{\epsilon_r - 1}{1+\frac{1}{2}(1-\lambda_z)(\epsilon_r-1)} \quad (9)$$

$$\Lambda_v = \frac{\epsilon_r - 1}{1+\lambda_z(\epsilon_r-1)} \quad (10)$$

$$\lambda_z = \begin{cases} \frac{1-e^2}{e^2}\left(-1+\frac{1}{2e}\ln\frac{1+e}{1-e}\right); & e^2 = 1-\left(\frac{a}{b}\right)^2, \quad 0<\frac{a}{b}\le 1 \\ \frac{1+f^2}{f^2}\left(1-\frac{1}{f}\arctan f\right); & f^2 = \left(\frac{a}{b}\right)^2-1, \quad \frac{a}{b}\ge 1 \end{cases} \quad (11)$$

Assuming the DSD and canting angle distribution are independent, radar measurables are calculated from the backscattering matrix:

$$\begin{aligned} Z_h = & \frac{1}{9|K_w|^2}(<\cos^4\phi>\cdot<D^6\cdot|\Lambda_h|^2>+ \\ & <\sin^4\phi>\cdot<D^6\cdot|\Lambda_v|^2>+<2\sin^2\phi\cos^2\phi> \\ & \cdot<D^6\cdot\Re e\{\Lambda_h\Lambda_v^*\}>) \end{aligned} \quad (12)$$

$$\begin{aligned} Z_v = & \frac{1}{9|K_w|^2}(<\sin^4\phi>\cdot<D^6\cdot|\Lambda_h|^2>+ \\ & <\cos^4\phi>\cdot<D^6\cdot|\Lambda_v|^2>+<2\sin^2\phi\cos^2\phi> \\ & \cdot<D^6\cdot\Re e\{\Lambda_h\Lambda_v^*\}>) \end{aligned} \quad (13)$$

$$\begin{aligned} \Re e\{R_{co}\} = & \frac{1}{9|K_w|^2}(<\sin^2\phi\cos^2\phi>\cdot<D^6 \\ & \cdot|\Lambda_h|^2>+<\sin^2\phi\cos^2\phi>\cdot<D^6 \\ & \cdot|\Lambda_v|^2>+(<\cos^4\phi+\sin^4\phi>)\cdot \\ & <D^6\cdot\Re e\{\Lambda_h\Lambda_v^*\}>) \end{aligned} \quad (14)$$

$$P_x = \frac{1}{9|K_w|^2}<\sin^2\phi\cos^2\phi>\cdot<D^6\cdot|\Lambda_h-\Lambda_v|^2> \quad (15)$$

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$$K_{dp} \propto < \cos^2\phi - \sin^2\phi > \cdot < D^3 \cdot \Re e\{\Lambda_h - \Lambda_v\} > . \tag{16}$$

When the linear axis ratio model (β model) is used to analyze the Rayleigh-scattering problem, a second-order polynomial approximation to $|\Lambda_h|^2$, $|\Lambda_v|^2$, $\Re e\{\Lambda_h \Lambda_v^*\}$ and $\Re e\{\Lambda_h - \Lambda_v\}$ results in the following:

$$|\Lambda_h|^2 \approx a_0 + a_1\beta D + a_2(\beta D)^2 \tag{17}$$

$$|\Lambda_v|^2 \approx b_0 + b_1\beta D + b_2(\beta D)^2 \tag{18}$$

$$\Re e\{\Lambda_h \Lambda_v^*\} \approx c_0 + c_1\beta D + c_2(\beta D)^2 \tag{19}$$

$$\Re e\{\Lambda_h - \Lambda_v\} \approx d_0 + d_1\beta D + d_2(\beta D)^2 \tag{20}$$

The values of a_i, b_i and c_i solely depend on the permittivity ϵ_r for the case of Rayleigh scattering (i.e. the EM wavelength is far greater than the drop diameter). Following the gamma DSD assumption,

$$< D^6|\Lambda_h|^2 > \approx < D^6 > (a_0 + a_1 D_T + a_2 D_T^2) \tag{21}$$

$$< D^3 \Re e\{\Lambda_h - \Lambda_v\} > \approx < D^3 > (d_0 + d_1 D_k + d_2 D_k^2) \tag{22}$$

where

$$D_T = \beta \frac{< D^7 >}{< D^6 >} = \frac{\mu + 7}{\mu + 3.67}\beta D_0, \tag{23}$$

$$D_k = \beta \frac{< D^4 >}{< D^3 >} = \frac{\mu + 4}{\mu + 3.67}\beta D_0. \tag{24}$$

Similar approximations hold for $< D^6|\Lambda_h|^2 >$ and $< D^6 \Re e\{\Lambda_h \Lambda_v^*\} >$. With these approximate formulas, radar measureables Z_h,Z_v,$\Re e\{R_{co}\}$,K_{dp} and P_x can be well approximated by polynomials of D_T or D_k, and the coefficients of the polynomials are only determined by σ_ϕ.

When the canting angle standard deviation is small, a new β estimator can be derived from these approximate forms as,

$$\hat{\beta} = C \frac{\left(\frac{K_{dp}}{Z_h}\right)^{\frac{1}{3}} (\xi_{dr} - 1)^{\frac{1.09}{3}}}{e^{-4.18\sigma_\phi^2/3}} \tag{25}$$

where C is a constant, and ξ_{dr} is the linear Z_{dr}. This formulation follows from the fact that $\frac{K_{dp} < D^6 >}{Z_h < D^3 >}$ can be approximately treated as a linear function of D_T, whereas 2nd order term of D_T in the polynomial formulation of ξ_{dr} contributes most to $\xi_{dr} - 1$. The uncertainty of μ is not essential for this estimator, but it does cause a maximally around 10% bias for the β estimation. Under the Rayleigh-Gans scattering with the permittivity $\epsilon_r = 80 - j20$, $\sigma_\phi = 0$ and the constant $C = 3$, the estimation accuracy is shown in the Fig. 1.

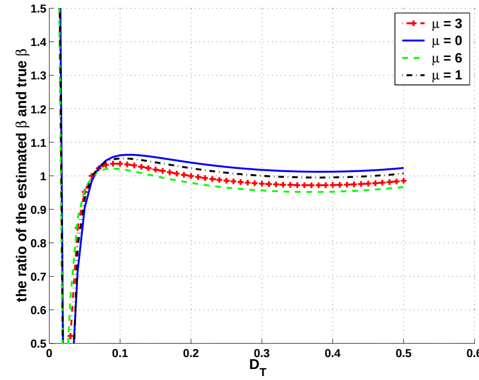

Figure 1: The estimation accuracy of β versus D_T. Note that with the approximation forms proposed in this article, if μ is aprioi known, then all measurables Z_h,Z_{dr} and K_{dp} can be well approximated as polynomials of D_T scaled by the DSD moments (in real rain fall cases, D_T usually ranges from 0.05 to 0.3)

Now comparing the new β estimator with the one previously proposed by Gorgucci et al.(2000)

$$\tilde{\beta} = a K_{dp}^b Z_h^{-c} \xi_{dr}^d, \tag{26}$$

we find that when $b = c = \frac{1}{3}$ and when ξ_{dr} is not too small, the β estimator proposed by Gorgucci et al. (2000) is similar to $\hat{\beta}$ because in the β estimator proposed by Gorgucci et al. (2000), d is close to 1. Note that when ξ_{dr} is not small (i.e.,> 1.5), $(\xi_{dr} - 1)^{\frac{1.09}{3}}$ can be well approximated by a linear function of ξ_{dr}. The canting effects in (25) can be handled effectively by setting σ_ϕ to be a constant. Therefore, the β estimator proposed by Gorgucci et al. (2000) has a correct form when a, b, c, d are chosen properly. Simulations show that $\tilde{\beta}$ can be estimated to an accuracy of around 5% provided the Z_h and Z_{dr} data are accurately calibrated and 'smoothed' in range to approximately 'match' the K_{dp} data. The β estimation works well in homogeneous sections of rain. It is also recognized that it is difficult to completely separate out the effects of β changes due to drop oscillations (if the underlying shape model is non-linear) from changes to the DSD itself.

3 Discussions about the β model

To understand the "physical" meaning of β, we studied the heavy rain case with strong down-draft on Feburary 17, 1999 during the TRMM LBA campaign in Brazil. This case was studied in detail by Atlas and Williams (2003). Due to the strong down-draft and the intensive concentration of water mass in a small rain cell in this case, it is most likely to observe the equivalent β change due to the collisional forcing of drop oscillations. The β estimator proposed by Gorgucci et al. (2000), of the form

$$\beta = 2.08 Z_h^{-0.365} K_{dp}^{0.38} \xi_{dr}^{0.965} \tag{27}$$

has been used to estimate β.

Radar RHI scans at 17:11,17:18 and 17:28 are selected to study the evolution of the storm, the DSD and axis ratio change. The radar data processing algorithm used in the comparison follows the one described in Bringi et al.(2002a), i.e. spatial filtering is applied to smooth Φ_{dp} and other radar measurables. DSD parameters D_0 and N_w are estimated based on Gorgucci et al. (2002). Rain rates are estimated according to the "pol-based" Z-R algorithm proposed by Bringi et al.(2002b).

Vertical profiles of Z_h, Z_{dr} and K_{dp} at 17:11, 17:18 and 17:28 are shown in Fig.2. From the analysis of Atlas and Williams (2003), the cell centered at the range of $42\ km$ was in its vigorous growth phase at 17:11 and 17:18, and in its downdraft phase at 17:28. In particular the Z_{dr} values are much smaller and more homogeneous in the rain cell at 17:28 as compared with the structure at 17:11 and 17:18. The vertical profile of β, D_0, N_w and R through the center of the rain cell is shown in Fig. 3-6. At 17:28, the $D_0 \approx 2\ mm$ throughout the rain cell with $log_{10}(N_w) \approx 4.3$ or $N_w \approx 20,000\ mm^{-1}m^{-3}$ reflecting an equilibrium-type of DSD during the downdraft stage. The D_0 values are much larger during the vigorous growth phase. In Fig. 3, the β values systematically decrease (below $2\ km$ height) from 0.055 to 0.05 to $0.047\ mm^{-1}$ at 17:11, 17:18 and 17:28, respectively, and inversely the rain rate increases from 50, 75 to $120\ mm/h$. This decrease in β with increasing rain rate may be indicative of collisional forcing of drop oscillations (Beard et al. 1983). There is other indirect evidence that in tropical rainfall the mean drop axis ratio versus D relation needs to be adjusted from equilibrium shapes to account for drop oscillations (May et al. 1999). We plan to use direct measurements of drop shape, canting and DSD using a 2D-video disdrometer and coordinated polarimetric radar data to validate the β hypothesis.

4 Acknowledgements

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References

[1] Atlas, D., Williams, C. R. 2003: The Anatomy of a Continental Tropical Convective Storm. *J. Atmos. Sci.*,**60**:3-15.

[2] Beard, K. V., Johnson, D. B. and Jameson, A. R.,1983: Collisional Forcing of Raindrop Oscillations. *J. Atmos. Sci.*, **40**:455–462.

[3] Bringi, V. N. and Chandrasekar, V.,2001: *Polarimetric Doppler weather radar : Principles and applications.* Cambridge Univ. Press, Cambridge, UK.

[4] Bringi, V. N., Huang, G. and Chandrasekar, V.,2002a: A Methodology for Estimating the Parameters of a Gamma Raindrop Size Distribution Model from Polarimetric Radar Data: Application to a Squall-Line Event from the TRMM/Brazil Campaign. *J. Atmos. Oceanic Technol.*,**19**:633–645.

[5] Bringi, V. N., Tang, T. and Chandrasekar, V.,2002b: Evaluation of a new polarimetrically-tuned Z-R relation. *Second European Conf. on Radar Meteor.*, pages 217–221.

[6] Gorgucci, E., Scarchilli, G., Chandrasekar, V. and Bringi, V. N.,2000: Measurement of Mean Raindrop Shape from Polarimetric Radar Observations. *J. Atmos. Sci.*,**57**:3406–3413.

[7] Gorgucci, E., Chandrasekar, V., Bringi, V. N. and Scarchilli, G.,2002: Estimation of Raindrop Size Distribution Parameters from Polarimetric Radar Measurements *J. Atmos. Sci.*,**59**:2373–2384.

[8] May, P. T., Keenan, T. D., Zrnić, D. S., Carey, L. D. and Rutledge, S. A.,1999: Polarimetric Radar Measurements of Maritime Rain at a 5-cm Wavelength. *J. Appl. Meteor.*,**38**:750–765.

[9] Testud. J., Oury. S, Black, R. A., Amayenc, P., and Dou, X.,2001: The Concept of "Normalized" Distribution to Describe Raindrop Spectra: A Tool for Cloud Physics and Cloud Remote Sensing. *J. Appl. Meteor.*,**40**:1118–1140.

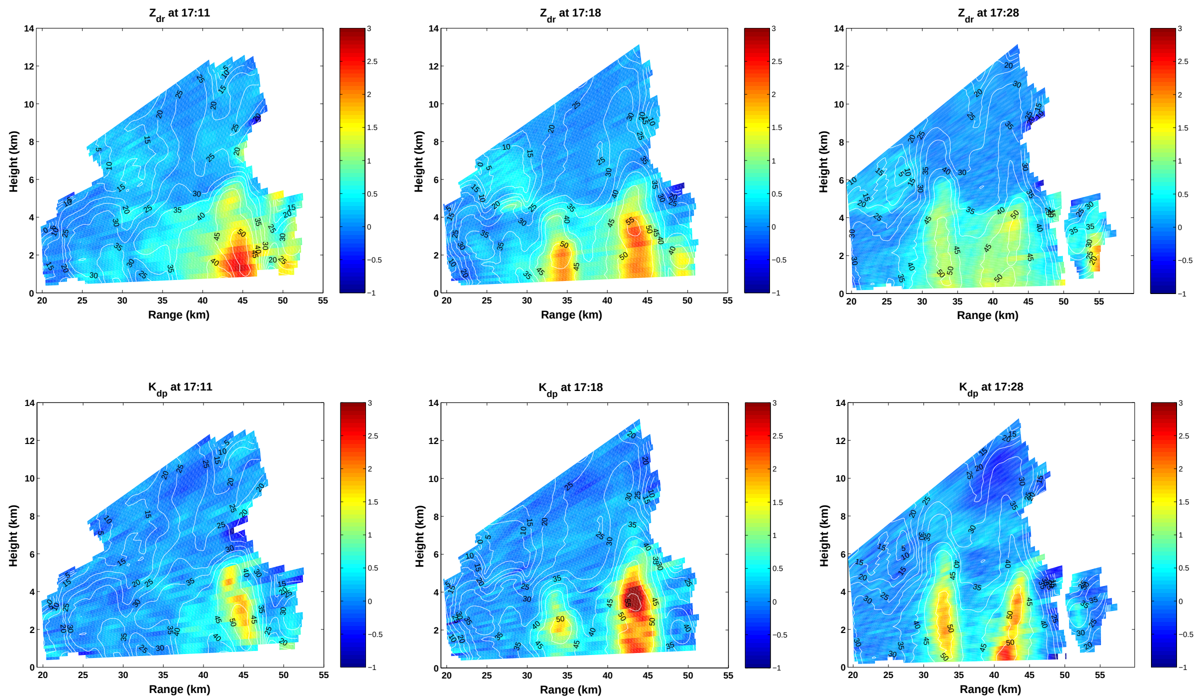

Figure 2: TRMM LBA Brazil RHI radar profiles at 17:11, 17:18 and 17:28 on February 17, 1999.

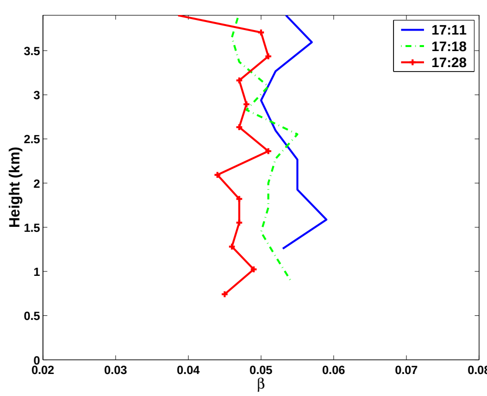

Figure 3: averaged β at the center of the storm

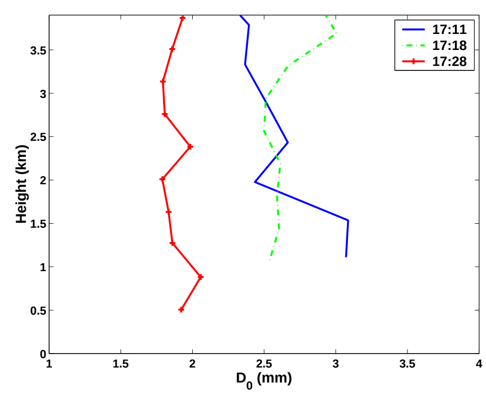

Figure 4: averaged D_0 at the center of the storm

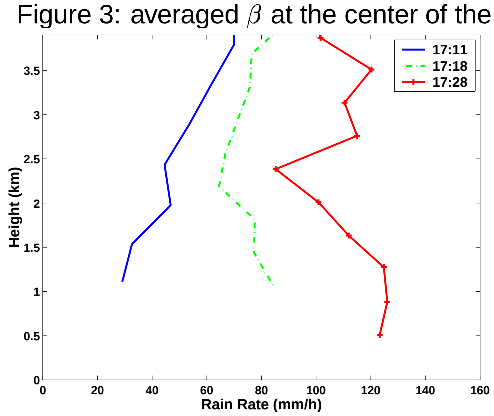

Figure 5: averaged rain rate at the center of the storm

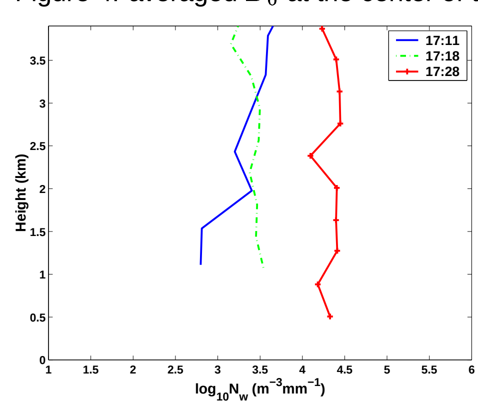

Figure 6: averaged N_w at the center of the storm