P1A.16 **A SIMULATION STUDY OF THE EFFECTS OF "SMALL SAMPLE" PROBLEM IN THE USE OF DISTROMETER DATA FOR POWER LAW RELATIONS INVOLVING RADAR MEASUREABLES**

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1 Introduction

The $Z-R$ relation of the form $Z = aR^b$ has been long used for radar-based rain rate estimation, and are usually developed based on the distrometer data, which usually contains inadequate number of rain drop samples. Kostinski and Jameson (2002) observed from simulations, that (an initial assumed linear $Z-R$), after regression would give an exponent b that decreases as the average number of drops per sample increases, and pointed out that the spurious exponents are produced by inadequate sampling. The retrieval of dsd parameters (using dual-polarized radar data) also involves power law relations of the form $Z_h/N_w = aD_0^b$,$D_0 = cZ_{dr}^d$ or $Z_h/N_w = a(R/N_w)^b$. We study the effects of the "small sample problem" on the exponent of these relations when power law regression is used to analyze the sample data.

2 Methodology and Results

A Monte Carlo simulation procedure similar to the one in Kostinski and Jameson(2002) is conducted to study the "small sample problem". In this study, a gamma drop size distribution of the form:

$$N(D) = N_t \frac{(3.67+\mu)^{\mu+1}}{\Gamma(\mu+1)D_0} \left(\frac{D}{D_0}\right)^{\mu} e^{-\frac{(3.67+\mu)D}{D_0}} \quad (1)$$

is used to generate the DSD, where N_t is the number of drops per unit volume, D_0 is the medium volume diameter, and μ is the DSD shape factor.

Fixing the sample volume to $1\ m^3$, we can control the range of N_t, and vary the parameters: μ and D_0 uniformly (D_0: 0.5 to $2.5mm$ and μ: -1 to 6) for each N_t range. The full range of $log_{10}N_t$ (from 1 to 4.5) in this study is divided into small intervals. For a given set of N_t,μ and D_0, a standard procedure is used to simulate gamma random variables (Chandrasekar and Bringi 1987). Subsequently for a given N_t interval, 1000 gamma dsds are simulated with parameter values uniformly ranging over the intervals specified earlier. The radar measurables are simulated based on Rayleigh scattering and the axis ratio model assumption: the combined model of Andsager-Beard (1999) and Beard-Chuang (1987). From the actual DSD, we can estimate the parameters: D_0 and N_w based on DSD moments, and obtain the actual rain rate and radar measurables.

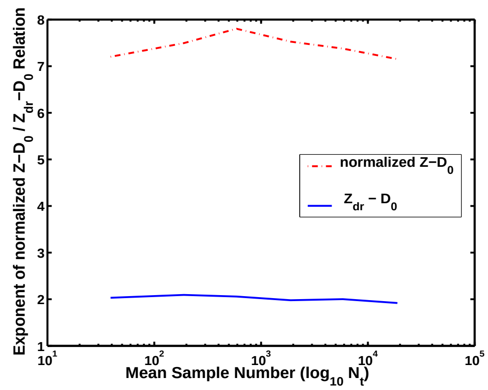

Figure 1: Power law exponent (of the form $Z_h/N_w = aD_0^b$ and $D_0 = cZ_{dr}^d$) versus mean sample number

Power law regressions of the forms $Z_h/N_w = aD_0^b$, $D_0 = cZ_{dr}^d$ (direct nonlinear fit instead of log linear fit) are applied to the estimated DSD parameters $\hat{D_0}$, $\hat{N_w}$ and measurables Z_{dr}, Z_h for each N_t interval. Fig.1 shows that the exponent of normalized $Z-D_0$ relation changes within 7 to 8 (roughly 10% variation). As long as the sample number is high enough, the exponent stablizes to around 7, but the actual number may depend on the DSD parameters D_0 and μ. The exponent of $Z_{dr}-D_0$ is consistently close to 2, and is immune to the change of sample number. These results suggest that the "small sample" problem is not as important for the exponent of normalized relations: $Z_h/N_w = aD_0^b$ and $D_0 = cZ_{dr}^d$ when power law regressions are applied to find these exponents.

To study the effects of "small sample problem" on

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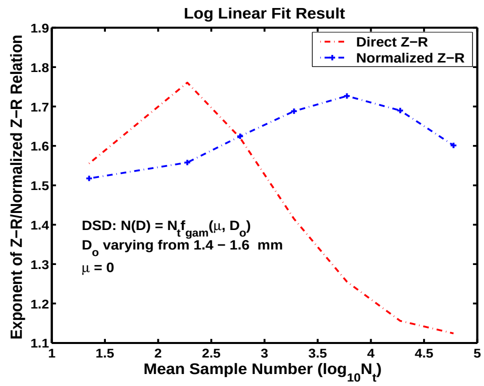


Figure 2: Normalized Z-R power law / direct Z-R power law exponent versus mean sample number for each N_t interval

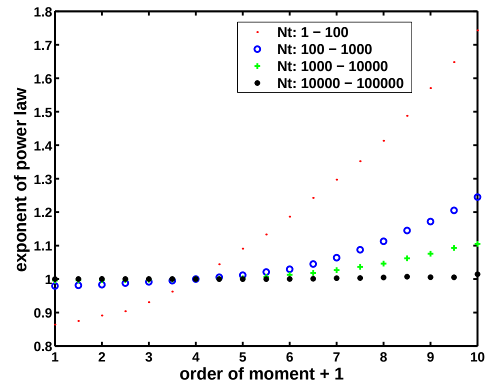


Figure 3: Power law exponent versus order of moment for β estimation

the normalized Z-R relation of the form $Z_h/N_w = a(R/N_w)^b$, a Monte Carlo simulation is conducted in the same way as the previous study for $Z_h - N_w$ and $Z_{dr} - D_0$ relations except that the DSD parameters μ and D_0 are chosen to be fixed, and Z is only treated as the $6th$ moment of DSD. Fig.2 shows that the exponent of the normalized Z-R relation varies between 1.5 and 1.75, whereas the direct $Z - R$ power law exponent changes in the range from 1 to 1.75, which is generally in agreement with the results of Kostinski and Jameson (2002). Thus, the normalized $Z - R$ relation exponent is shown to be less sensitive to the change of sample size than the direct Z-R relation exponent.

From the previous studies, we find that the power law relations normalized with respect to N_w are not very sensitive to the "small sample" problem. It is of interest to study the effects of the "small sample" problem on other normalization or 'scaling' laws. Sempere-Torres et al. (1994) have proposed a general formulation for $N(D)$ based on a scaling law:

$$N(D;W) = W^{\alpha} g\left(\frac{D}{W^{\beta}}\right) \qquad (2)$$

where α and β are constants, which satisfy $4\beta+\alpha = 1$, W is the water content and g is a general distribution function. The evaluation of β is based on various moments of the measured DSD. Power law fittings are applied to fit the DSD moments and the reference variable W. The exponents of these relations and the orders of moments form a 1-1 mapping. The β is considered to be the the slope of the exponent versus the order of moment function.

Assuming gamma DSD specified in (1) with varying N_t and fixed μ, D_0, a Monte Carlo simulation is conducted to study the problem. Following the procedure for estimating β, the exponent versus moment number curves for the N_t intervals are shown in Fig.3. In theory, when fixing μ and D_0, DSD moments are all linear functions of N_t, therefore, the slope of the curves should be uniformly 0. From the figure, we can see that the slope of these curves decreases as sample number increases, and approaches 0 only when the sample number is sufficiently large (i.e. 10000 - 100000 in the plot). Therefore, the evaluation of β based on DSD moments and power law fitting is significantly affected by the "small sample" problem.

3 An Alternative Approach

As we know, the distrometer-based analysis will be more or less affected by the "small sample" problem from the previous study. We prefer to adopt an analytical approach to analyze radar measureables based on some model assumptions, and derive new DSD retrieval relations so that we can use these relations to verifiy or adjust the empirical relations like the ones proposed by Bringi et al. (2002).

In this paper, we use the three parameter drop size distribution model, the combined shape model (Andsager and Beard (1999) and Beard and Chuang model (1987)), and the Gaussian canting angle distribution model (mean 0 and standard deviation σ_ϕ) to model the rainfall process. First, the case of $\sigma_\phi = 0$, i.e. no canting case is studied. For a spheroid with volume-equivalent diameter D, under Rayleigh-Gans scattering, the scattering amplitude for both horizontal and vertical polarizations can be well approximated as second-order polynomials of D. Both Z_h and Z_v can be treated as integrals of these approximate polynomials weighted by D^6. Therefore, we define a new parameter D_g as,

$$D_g = \frac{< D^7 >}{< D^6 >}. \tag{3}$$

With the parameter D_g, the ξ_{dr} (i.e. Z_{dr} in linear scale) can be well approximated solely as a function of D_g. Note that the new parameterization of ξ_{dr} as a function of D_g suppresses the effects of the uncertainty of μ.

The $D_g - \xi_{dr}$ relation is given as follows,

$$\xi_{dr} - 1 = 0.0353 D_g^{2.0996}. \tag{4}$$

This relation can be used for radar-based DSD parameter retrieval. Comparing the relation with the $D_0 - Z_{dr}$ relation proposed by Bringi et al. (2002), we find that since $\xi_{dr} - 1$ is very close to $Z_{dr}/10$, the $D_0 - Z_{dr}$ relation takes a similar power law form as the $D_g - \xi_{dr}$ relation.

When σ_ϕ is small (usually $\leq 10°$), the $D_g - \xi_{dr}$ relation is adjusted as,

$$\xi_{dr} - 1 = e^{-2\sigma_\phi^2} \cdot 0.0353 D_g^{2.0996}. \tag{5}$$

To validate the D_g estimator, we simulate radar measureables with the 2-D video distrometer drop spectra from Florida, Graz and Papua New Guinea, and assuming drop axis ratios follow the combined Andsager and Beard, and Beard and Chuang model, with $\sigma_\phi = 10°$. The simulated ξ_{dr} is applied to estimate D_g. The retrieval accuracy of D_g is shown in Fig.4.

The D_g-rain rate relation can be written as,

$$R = \frac{Z \cdot F_g(\mu)}{D_g^{2.33}} \tag{6}$$

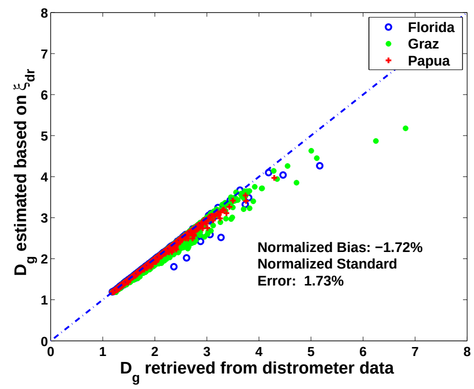
where

$$F_g(\mu) = 3.78 \times 0.6 \times 10^{-3} \pi \frac{\Gamma(\mu + 4.67) \cdot (\mu + 7)^{2.33}}{\Gamma(\mu + 7)}. \tag{7}$$

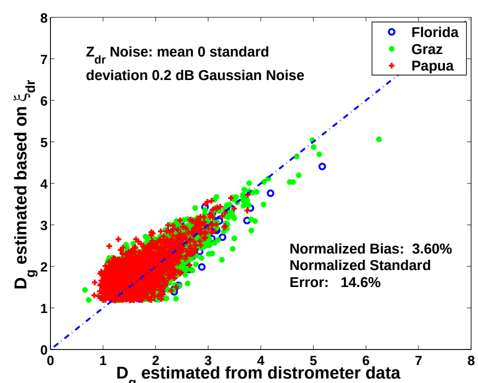
Therefore, rain rate can be estimated from D_g with the above $R - Z$ relation.

4 Summary

In this paper, the effects of the "small sample" problem on the normalized power law relations of the form $Z_h/N_w = aD_0^b$,$D_0 = cZ_{dr}^d$ or $Z_h/N_w = a(R/N_w)^b$ are studied, and it is concluded to that the exponents are not sensitive to the "small sample" problem. Intuitively speaking, the normalization with N_w makes use of the $3rd$ moment of DSD, and



(a)



(b)

Figure 4: (a)the performance of the D_g estimator without noise (b) the performance of the D_g estimator in the presence of random noise. Note that in both plots, D_g's are retrieved when $Z_{dr} > 0.2\ dB$.

partly "cancels" the statistical fluctuation of other moments of the DSD. From Monte Carlo simulations, we also show that the evaluation of β for Sempere-Torres's DSD scaling law can be significantly affected by the "small sample" problem. A new parameter D_g is defined as the ratio of the $7th$ and the $6th$ moments of DSD, and a $D_g - \xi_{dr}$ relation is proposed and verified via simulation.

5 Acknowledgements

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