8B.6 TRANSFORMATION OF THE POLARIMETRIC COVARIANCE MATRIX FOR IMPROVING HYDROMETEOR CLASSIFICATION

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1. Introduction

Multiparameter radar enables detection and estimation of the types of precipitation by exploring the relations between polarimetric measurables and hydrometeor characteristics such as size, shape, orientation and concentration of the particles. In linear polarization, the polarimetric measurables used for classification purpose include reflectivity factor (Z_h), differential reflectivity (Z_{dr}), specific differential phase (K_{dp}), linear depolarization ratio (LDR) and co-polar correlation coefficient (ρ_{co}). By combined rules based on these variables, the knowledge on precipitation types can be obtained from radar measurements (Liu and Chandrasekar 2000). However, all the physical factors of particles jointly affect the linear polarimetric measurables so that there exists ambiguity in classification (Straka et al. 2000). For example, LDR depends on mean canting angle as well as shape irregularity and canting dispersion; ρ_{co} shows the degree of decorrelation which results from the factors such as non-zero backscatter differential phase shift, shape irregularity, mixture of different precipitation types, etc.

Compared to linear polarization basis, the circular polarimetric measurables can separate some physical factors. Circular depolarization ratio (CDR) is dominated by particle shape and independent of particle orientation while the estimator ρ_4 ($\rho_4 = \mathbf{E}[cos4\beta]$ where $\mathbf{E}$ stands for expectation over the pdf of the canting angle β) is only related to the latter. With the availability of the full covariance matrix, it is feasible to implement basis transformation. We can take advantage of circular basis transformation to separate the orientation and shape factors.

However, it is well known that propagation effects will corrupt the circular radar measurements while the linear measurements are much less sensitive to such effects (Jameson and Davé 1988, Torlaschi and Holt 1993). In this paper, propagation effects will be corrected in linear measurement and basis transformation will be employed to retrieve hydrometeor properties on orientation as well as on shape for improving precipitation classification.

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2. Polarimetric Covariance Matrix and Basis Transformation

In linear polarization basis, the scattering matrix can be expressed as

$$\mathbf{S} = \begin{bmatrix} S_{hh} & S_{hv} \\ S_{vh} & S_{vv} \end{bmatrix} \quad (1)$$

To study randomly distributed medium, we employ the second order moments and get the covariance matrix in linear basis as

$$\Sigma_l = \begin{bmatrix} \langle |S_{hh}|^2 \rangle & \sqrt{2}\langle S_{hh}S_{hv}^* \rangle & \langle S_{hh}S_{vv}^* \rangle \\ \sqrt{2}\langle S_{hh}^* S_{hv} \rangle & 2\langle |S_{hv}|^2 \rangle & \sqrt{2}\langle S_{vv}^* S_{hv} \rangle \\ \langle S_{hh}^* S_{vv} \rangle & \sqrt{2}\langle S_{hh}^* S_{hv} \rangle & \langle |S_{vv}|^2 \rangle \end{bmatrix} \quad (2)$$

The linear dual-polarization radar such as CSU-CHILL is able to measure this full 3X3 covariance matrix. The covariance matrix in circular basis ($\mathbf{\Sigma}_c$) can be obtained from the transformation of $\mathbf{\Sigma}_l$ in Eq. (2) as,

$$\mathbf{\Sigma}_c = \mathbf{T}\mathbf{\Sigma}_l\mathbf{T}^H \quad (3)$$

where $\mathbf{T}$ is defined as,

$$\mathbf{T} = \frac{1}{2}\begin{bmatrix} 1 & j\sqrt{2} & -1 \\ \sqrt{2} & 0 & \sqrt{2} \\ 1 & -j\sqrt{2} & -1 \end{bmatrix} \quad (4)$$

To study the effect of particle orientation, the hydrometeor is regarded as possessing an axis of symmetry and its orientation can be described by ψ, the angle between incidence direction and the symmetry axis, and β, the angle between horizontal polarization and the projection of the symmetry axis in the polarization plane. Applying the concept of basis transformation, the covariance transformation matrix incurred by canting angle in linear basis is:

$$\mathbf{T}_l(\beta) = \begin{bmatrix} \cos^2\beta & \frac{\sin 2\beta}{\sqrt{2}} & \sin^2\beta \\ -\frac{\sin 2\beta}{\sqrt{2}} & \cos 2\beta & \frac{\sin 2\beta}{\sqrt{2}} \\ \sin^2\beta & -\frac{\sin 2\beta}{\sqrt{2}} & \cos^2\beta \end{bmatrix} \quad (5)$$

Similarly, in circular basis the covariance transformation matrix is

$$\mathbf{T}_c(\beta) = \frac{1}{2}\begin{bmatrix} e^{-j2\beta} & j\sqrt{2}e^{-j2\beta} & -e^{-j2\beta} \\ \sqrt{2} & 0 & \sqrt{2} \\ e^{j2\beta} & -j\sqrt{2}e^{j2\beta} & -e^{j2\beta} \end{bmatrix} \quad (6)$$

In order to correct the propagation effects, generally the net mean canting angle along the propagation path is assumed to be zero (in the absence of electrification). Once such assumption holds, the propagation matrix in linear polarization is diagonal

$$\mathbf{P}_l = \begin{bmatrix} e^{\lambda_1 r} & \\ & e^{\lambda_2 r} \end{bmatrix} \tag{7}$$

So is the corresponding covariance transformation matrix imposed by propagation effects:

$$\mathbf{T}_l(P) = e^{2\lambda_1 r} \begin{bmatrix} 1 & & \\ & e^{-u} & \\ & & e^{-2u} \end{bmatrix} \tag{8}$$

$$\text{where } u = (\lambda_1 - \lambda_2) r = -\frac{r}{2}\left(\frac{A_{dp}}{8.686} + jK_{dp}\right)$$

However, the propagation matrix in circular basis is not diagonal and its covariance transformation matrix is rather complicated as shown in the symmetric matrix:

$$\mathbf{T}_c(P) = e^{(\lambda_1+\lambda_2)r} \cdot$$
$$\begin{bmatrix} \cosh^2\frac{u}{2} & \sqrt{2}\cosh\frac{u}{2}\sinh\frac{u}{2} & \sinh^2\frac{u}{2} \\ - & \cosh^2\frac{u}{2} + \sinh^2\frac{u}{2} & \sqrt{2}\cosh\frac{u}{2}\sinh\frac{u}{2} \\ - & - & \cosh^2\frac{u}{2} \end{bmatrix} \tag{9}$$

Eq. (6) implies that it is convenient to retrieve the information on canting angle from circular covariance matrix. On the contrary, Eq. (8) implies that it is feasible to correct the propagation effect in linear polarization basis. Note that for correcting propagation effects the same assumption is considered here as in other related literature (Jameson and Davé 1988, Torlaschi and Holt 1993).

3. Data Analysis

In the Severe Thunderstorm Electrification and Precipitation Study (STEPS, 2000) project, a strong convective storm developed in the dual-doppler coverage area of CSU-CHILL and S-Pol radars from 2143UTC to 2245UTC on 11 June 2000. At 2225UTC, the storm was developing toward the baseline of the two radars and CSU-CHILL took a RHI scan along 338^o azimuth angle. Fig. (1) illustrates a ray of data at low elevation with conventional linear radar measurables. At 30 km along range, the reflectivity reaches 60 dB and Z_{dr} shows a large reading around 4 dB as well as rapidly increasing Φ_{dp}. There is no apparent increase in ρ_{xh}, which means the net mean canting angle along the propagation path is close zero according to Ryzhkov (2001). Furthermore, LDR suggests the canting angle is distributed with a finite width.

Before applying polarization transformation, the constructed covariance matrix in linear polarization basis must be calibrated carefully, especially for Z_{dr}, LDR and

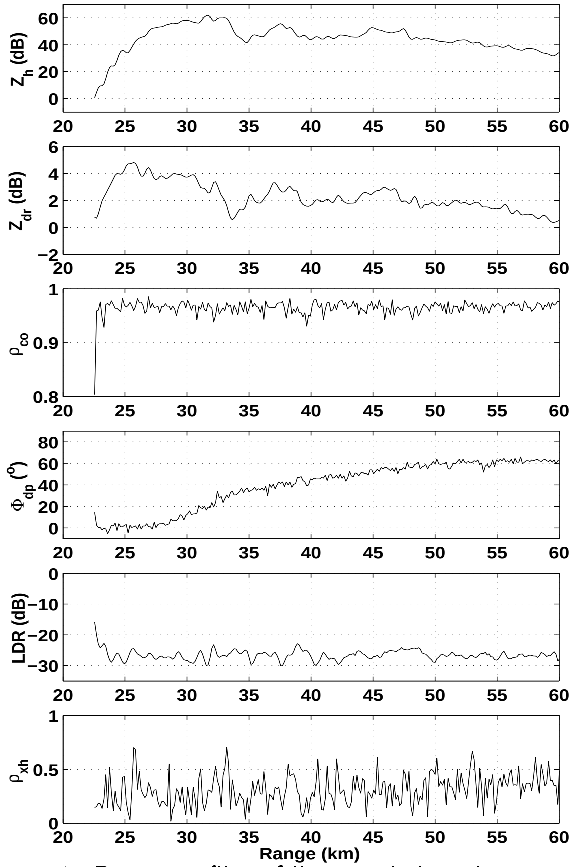

Figure 1: Range profiles of linear polarimetric measurements on 11 June 2000; EL=0.76^o, AZ=338^o.

the co-to-cross covariance phases. Hubbert et al. (2003) addressed this issue and gave a detailed methodology, which is utilized here to first construct the calibrated covariance matrix.

The antenna may also introduce error in polarization states and result in incorrect interpretation after polarization transformation. Assuming the precipitation is homogenous, the antenna-induced polarization error can be represented by another transform matrix and for CSU-CHILL it was estimated by Hubbert and Bringi (2003b).

What is left is the intrinsic covariance matrix with the propagation part intact. With the assumption of zero net mean canting angle, the transformation matrix is simply a diagonal matrix shown in Eq. (8) with two parameters: Φ_{dp} and A_{dp}. The estimation on Φ_{dp} is obtained by adaptively filtering the measured differential phase along the propagation path (Hubbert and Bringi 1995), and the estimation of A_{dp} is based on the Φ_{dp} constraint algorithm described by Bringi et al. (2001). For this single RHI sweep, a linear relation between A_{dp} and K_{dp} is assumed and the averaged coefficient is applied to the whole sweep.

Once these steps are done, Eq. (3) is implemented to transform the covariance matrix from linear basis to circular basis and the radar measuables in circular po-

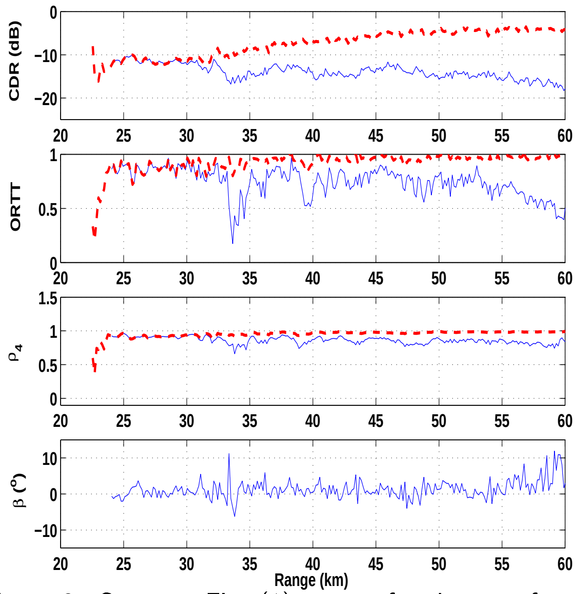

Figure 2: Same as Fig. (1) except for the transformed circular polarimetric measurements. The variables before propagation correction are ploted in dash lines, while those after correction are plotted in solid lines.

larization are retrieved. Fig. (2) shows both the circular measuables that are retrieved before and after the propagation correction. Note the dramatic distortion caused by propagation effects on CDR, $ORTT$ and ρ_4. Mean canting angle is also estimated from the circular covariance matrix for every resolution bin and its reading shows close zero.

4. Improvement on Precipitation Classification

In circular basis, CDR is independent of hydrometeor orientation and ρ_4 is only dependent on the orientation. Hydrometeor classification procedure may be able to take advantage of such property. To study this possibility, a variety of storm types are selected for heavy rain (as analyzed above), rain hail mixture (2105UTC 29 Aug. 2002), hail (2115UTC 4 Jun. 2001) and snow (2139UTC 17 Mar. 2003). The scatterplots versus reflectivity are given in Fig. (3).

Z_{dr} is able to present excellent separation among the precipitation types in the sense of mean shape. However, LDR looks similar for heavy rain and rain-hail mixture. Even for rain and hail, there is still some overlapping regions in the scatterplot. In general, LDR depends on particle shape, its orientation and the decorrelation. Many factors could contribute to decorrelation, which makes it hard to interpret the reading of ρ_{co}. Much overlap can also be found except that part of rain-hail mixture leads to fairly low ρ_{co} (below 0.95).

As for CDR comparing to Z_{dr}, even though it is independent of orientation distribution, it introduces more

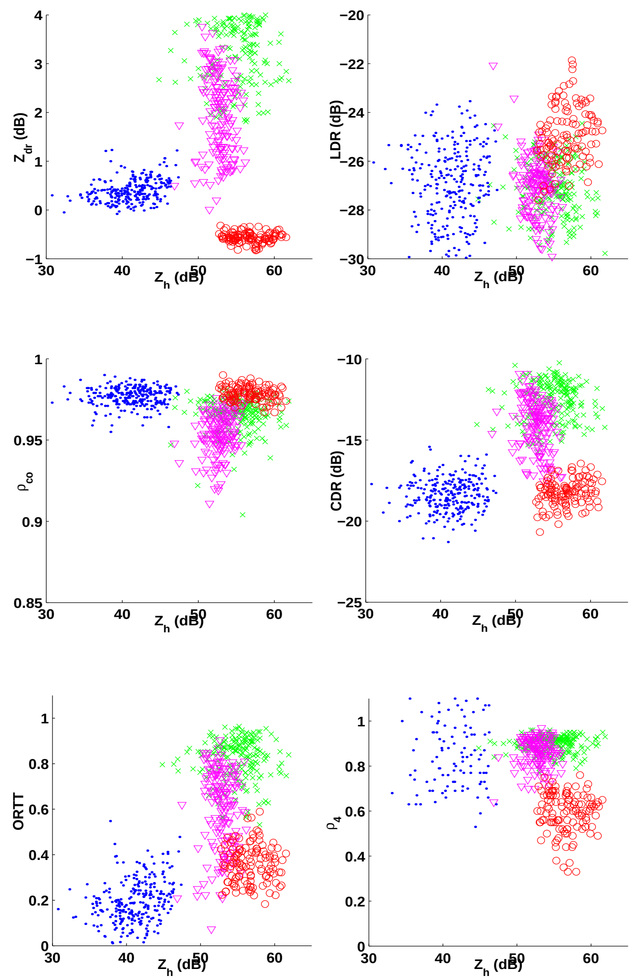

Figure 3: Scatterplots of (a) Z_{dr}, (b) LDR, (c) ρ_{co}, (d) CDR, (e) $ORTT$, (f) ρ_4 vs Z_h (in the sequence of left to right, top to bottom), where marker 'x' represents data for the rain case, marker 'o' for the hail case, marker '·' for snow and '△' for rain-hail mixture.

ambiguity than Z_{dr}. Al-Jumily et al. (1991) illustrated the ambiguity of distinguishing between rain and hail by CDR with real circular polarization radar measurement. Here we will investigate the reason as follows. From Eq. (6) we can express propagation-corrected CDR by the variables in the principle plane as:

$$CDR = 10\log_{10}\left(\frac{\langle|S_{11}-S_{22}|^2\rangle}{\langle|S_{11}+S_{22}|^2\rangle}\right) = 10\log_{10}\left(\frac{\sqrt{zdr}+1/\sqrt{zdr}-2\Re\rho_{12}}{\sqrt{zdr}+1/\sqrt{zdr}+2\Re\rho_{12}}\right) \quad (10)$$

where zdr refers to the differential reflectivity in principle plane in linear scale and ρ_{12} refers to correlation coefficient between S_{11} and S_{22}. Therefore, CDR will fluctuate with decorrelation besides mean shape. Even without decorrelation, CDR will give same value for both negative Z_{dr} and positive Z_{dr} as shown here for hail (Z_{dr} around -0.5 dB) and snow (Z_{dr} around 0.5 dB), i.e., CDR cannot distinguish between oblate and prolate shapes at low elevation angles.

Similarly, $ORTT$ could be expressed as:

$$
\begin{aligned}
ORTT &= \langle\cos 2(\beta')\rangle \frac{|\langle(S_{11}+S_{22})^*(S_{11}-S_{22})\rangle|}{\sqrt{\langle|S_{11}+S_{22}|^2\rangle\langle|S_{11}-S_{22}|^2\rangle}} \\
&= \rho_2 \frac{|\sqrt{zdr}-1/\sqrt{zdr}+j2\Im\rho_{12}|}{\sqrt{zdr+1/zdr+2-4\Re^2\rho_{12}}} \qquad (11)
\end{aligned}
$$

With ρ_2 and imaginary part of decorrelation inside, $ORTT$ presents better classification among the various precipitation types. Note $ORTT$ comes from co-to-cross circular covariance while ρ_4 comes from co-polar circular covariance. Since co-polar power is less than cross-polar power in circular basis, $ORTT$ will be a better estimator than ρ_4 as we will see next.

The ρ_4 in circular basis should be taken as a counterpart of LDR but it is a pure orientation factor. We can see a clear boundary between rain and hail in the scatterplot of ρ_4 vs Z_h. For the rain-hail mixture, it depends on which one dominates. However, it scatters over large range for snow even over 1, which we believe results from artifacts in transformation because of low SNR. Up to this point, we believe the combination of $ORTT$ and ρ_4 will tell us more about the hydrometeor types than LDR and ρ_{co}. However, low value of ρ_{co} implies a mixture of precipitation types and is valuable for classification.

5. Discussion

In this paper, we reviewed the transformation on polarimetric covariance matrix between different polarization basis and explored it to obtain circular set of radar measurables from linear radar measurements. With a real case from STEPS, it shows that the circular measurements could be obtained with confidence.

In theory, the covariance matrix under different polarization bases are equivalent and can be transformed to other bases. The information is expressed in different manner for circular and linear polarization. Specifically, ρ_4 is only dependent on orientation distribution while CDR is independent of that. Both of them along with $ORTT$ are sensitive to propagation effects. On the contrary, all the radar variables in linear polarization are dependent on shape and orientation of hydrometeors in a complicated way, but they do not get much affected by propagation effects. After correction, CDR closely relates to Z_{dr} for raindrops in Raleigh-Gans scattering. LDR relates to orientation distribution but has much ambiguity for classification. $ORTT$ and ρ_4, however, provide additional information.

In practice, however, the cross-polar power return is fairly low and the cross-polar terms are prone to noise and measurement error, especially for low reflectivity precipitation. The transformation demands an excellent cross-polar performance for antenna subsystem to achieve credible estimation of orientation factors. To account for such concern, CSU-CHILL is undergoing another major upgrade to dual-offset Gregorian antenna. Also, a scheme to measure the full covariance matrix in the slant $45^o/135^o$ basis will be implemented. This will enhance the cross-polar power return and also mitigate somewhat the antenna polarization error contamination as compared to the H/V basis.

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