1. Introduction

Multiparameter radar enables detection and estimation of the types of precipitation by exploring the relations between polarimetric measurables and hydrometeor characteristics such as size, shape, orientation and concentration of the particles. In linear polarization, the polarimetric measurables used for classification purpose include reflectivity factor ($Z_V$), differential reflectivity ($Z_D$), specific differential phase ($K_{dp}$), linear depolarization ratio ($LDR$) and co-polar correlation coefficient ($\rho_{cc}$). By combined rules based on these variables, the knowledge on precipitation types can be obtained from radar measurements (Liu and Chandrasekar 2000). However, all the physical factors of particles jointly affect the linear polarimetric measurables so that there exists ambiguity in classification (Straka et al. 2000). For example, LDR depends on mean canting angle as well as shape irregularity and canting dispersion; $\rho_{cc}$ shows the degree of decorrelation which results from the factors such as non-zero backscatter differential phase shift, shape irregularity, mixture of different precipitation types, etc.

Compared to linear polarization basis, the circular polarimetric measurables can separate some physical factors. Circular depolarization ratio ($CDR$) is dominated by particle shape and independent of particle orientation while the estimator $\rho_t$ ($\rho_t = \mathbb{E}[\cos 4\beta]$ where $\mathbb{E}$ stands for expectation over the pdf of the canting angle $\beta$) is only related to the latter. With the availability of the full covariance matrix, it is feasible to implement basis transformation. We can take advantage of circular basis transformation to separate the orientation and shape factors.

However, it is well known that propagation effects will corrupt the circular radar measurements while the linear measurements are much less sensitive to such effects (Jameson and Davé 1988, Torlaschi and Holt 1993). In this paper, propagation effects will be corrected in linear measurement and basis transformation will be employed to retrieve hydrometeor properties on orientation as well as on shape for improving precipitation classification.

2. Polarimetric Covariance Matrix and Basis Transformation

In linear polarization basis, the scattering matrix can be expressed as

$$ S = \begin{bmatrix} S_{hh} & S_{hv} \\ S_{vh} & S_{vv} \end{bmatrix} $$  \hfill (1)

To study randomly distributed medium, we employ the second order moments and get the covariance matrix in linear basis as

$$ \Sigma_l = \begin{bmatrix} \langle |S_{hh}|^2 \rangle & \sqrt{2} \langle S_{hh} S_{hv}^* \rangle & \langle S_{hh} S_{vv}^* \rangle \\ \sqrt{2} \langle S_{hh}^* S_{hv} \rangle & 2\langle |S_{hv}|^2 \rangle & \sqrt{2} \langle S_{hv}^* S_{vv} \rangle \\ \langle S_{hh}^* S_{vv} \rangle & \sqrt{2} \langle S_{vv}^* S_{hv} \rangle & \langle |S_{vv}|^2 \rangle \end{bmatrix} $$  \hfill (2)

The linear dual-polarization radar such as CSU-CHILL is able to measure this full 3X3 covariance matrix. The covariance matrix in circular basis ($\Sigma_c$) can be obtained from the transformation of $\Sigma_l$ in Eq. (2) as,

$$ \Sigma_c = T\Sigma_l T^H $$

where $T$ is defined as,

$$ T = \frac{1}{2} \begin{bmatrix} 1 & j\sqrt{2} & -1 \\ \sqrt{2} & 0 & \sqrt{2} \\ 1 & -j\sqrt{2} & -1 \end{bmatrix} $$  \hfill (4)

To study the effect of particle orientation, the hydrometeor is regarded as possessing an axis of symmetry and its orientation can be described by $\psi$, the angle between incidence direction and the symmetry axis, and $\beta$, the angle between horizontal polarization and the projection of the symmetry axis in the polarization plane. Applying the concept of basis transformation, the covariance transformation matrix incurred by canting angle in linear basis is:

$$ T_l(\beta) = \begin{bmatrix} \cos^2 \beta & \sin 2\beta & \sin^2 \beta \\ -\sin 2\beta & \cos^2 \beta & \sin \beta \cos \beta \\ \sin^2 \beta & -\sin \beta \cos \beta & \cos^2 \beta \end{bmatrix} $$  \hfill (5)

Similarly, in circular basis the covariance transformation matrix is

$$ T_c(\beta) = \frac{1}{2} \begin{bmatrix} e^{-j2\beta} & j\sqrt{2}e^{-j2\beta} & -e^{-j2\beta} \\ e^{j2\beta} & 0 & \sqrt{2} \\ -j\sqrt{2}e^{j2\beta} & e^{j2\beta} & -e^{j2\beta} \end{bmatrix} $$  \hfill (6)

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In order to correct the propagation effects, generally the net mean canting angle along the propagation path is assumed to be zero (in the absence of electrification). Once such assumption holds, the propagation matrix in linear polarization is diagonal

$$ P_l = \begin{bmatrix} e^{\lambda_1 r} & e^{\lambda_2 r} \\ \end{bmatrix} $$  \hspace{1cm} (7)

So is the corresponding covariance transformation matrix imposed by propagation effects:

$$ T_l(P) = e^{2\lambda r} \begin{bmatrix} 1 & e^{u} \\ e^{-u} & 1 \\ \end{bmatrix} $$  \hspace{1cm} (8)

where

$$ u = (\lambda_1 - \lambda_2)r = \frac{r}{2}(\frac{A_{dp}}{8.686} + jK_{dp}) $$

However, the propagation matrix in circular basis is not diagonal and its covariance transformation matrix is rather complicated as shown in the symmetric matrix:

$$ T_c(P) = e^{(\lambda_1 + \lambda_2)r} \begin{bmatrix} \cosh^2 \frac{u}{2} & \sqrt{2} \cosh \frac{u}{2} \sinh \frac{u}{2} \\ \sqrt{2} \cosh \frac{u}{2} \sinh \frac{u}{2} & \cosh^2 \frac{u}{2} \sinh^2 \frac{u}{2} \\ \end{bmatrix} $$  \hspace{1cm} (9)

Eq. (6) implies that it is convenient to retrieve the information on canting angle from circular covariance matrix. On the contrary, Eq. (8) implies that it is feasible to correct the propagation effect in linear polarization basis. Note that for correcting propagation effects the same assumption is considered here as in other related literature (Jameson and Davé 1988, Torlaschi and Holt 1993).

3. Data Analysis

In the Severe Thunderstorm Electrification and Precipitation Study (STEPS, 2000) project, a strong convective storm developed in the dual-doppler coverage area of CSU-CHILL and S-Pol radars from 2143UTC to 2245UTC on 11 June 2000. At 2225UTC, the storm was developing toward the baseline of the two radars and CSU-CHILL took a RHI scan along 338° azimuth angle. Fig. (1) illustrates a ray of data at low elevation with conventional linear radar measurables. At 30 km along range, the reflectivity reaches 60 dB and $Z_{dr}$ shows a large reading around 4 dB as well as rapidly increasing $\Phi_{dp}$. There is no apparent increase in $\rho_{zh}$, which means the net mean canting angle along the propagation path is close zero according to Ryzhkov (2001). Furthermore, $LDR$ suggests the canting angle is distributed with a finite width.

Before applying polarization transformation, the constructed covariance matrix in linear polarization basis must be calibrated carefully, especially for $Z_{dr}$, $LDR$ and the co-to-cross covariance phases. Hubbert et al. (2003) addressed this issue and gave a detailed methodology, which is utilized here to first construct the calibrated covariance matrix.

The antenna may also introduce error in polarization states and result in incorrect interpretation after polarization transformation. Assuming the precipitation is homogenous, the antenna-induced polarization error can be represented by another transform matrix and for CSU-CHILL it was estimated by Hubbert and Bringer (2003b).

What is left is the intrinsic covariance matrix with the propagation part intact. With the assumption of zero net mean canting angle, the transformation matrix is simply a diagonal matrix shown in Eq. (8) with two parameters: $\Phi_{dp}$ and $A_{dp}$. The estimation on $\Phi_{dp}$ is obtained by adaptively filtering the measured differential phase along the propagation path (Hubbert and Bringer 1995), and the estimation of $A_{dp}$ is based on the $\Phi_{dp}$ constraint algorithm described by Bringer et al. (2001). For this single RHI sweep, a linear relation between $A_{dp}$ and $K_{dp}$ is assumed and the averaged coefficient is applied to the whole sweep.

Once these steps are done, Eq. (3) is implemented to transform the covariance matrix from linear basis to circular basis and the radar measurables in circular po-
larization are retrieved. Fig. (2) shows both the circular 
measurables that are retrieved before and after the prop-
agation correction. Note the dramatic distortion caused 
by propagation effects on CDR, ORTT and ρ4. Mean 
canting angle is also estimated from the circular vari-
cance matrix for every resolution bin and its reading shows 
close zero.

4. Improvement on Precipitation Classification

In circular basis, CDR is independent of hydrometeor 
orientation and ρ4 is only dependent on the orientation. 
Hydrometeor classification procedure may be able to take 
advantage of such property. To study this possibility, 
a variety of storm types are selected for heavy rain (as 
analyzed above), rain hail mixture (2105UTC 29 Aug. 
2002), hail (2115UTC 4 Jun. 2001) and snow (2139UTC 
17 Mar. 2003). The scatterplots versus reflectivity are 
given in Fig. (3).

Zdr is able to present excellent separation among the 
precipitation types in the sense of mean shape. How-
ever, LDR looks similar for heavy rain and rain-hail mix-
ture. Even for rain and hail, there is still some overlapping 
regions in the scatterplot. In general, LDR depends 
on particle shape, its orientation and the decorrelation. 
Many factors could contribute to decorrelation, which 
makes it hard to interpret the reading of ρco. Much 
overlap can also be found except that part of rain-hail mixture 
leads to fairly low ρco (below 0.95).

As for CDR comparing to Zdr, even though it is in-
dependent of orientation distribution, it introduces more

Figure 2: Same as Fig. (1) except for the transformed 
circular polarimetric measurements. The variables 
before propagation correction are plotted in dash lines, while 
those after correction are plotted in solid lines.

Figure 3: Scatterplots of (a) Zdr, (b) LDR, (c) ρco, (d) 
CDR, (e) ORTT, (f) ρ4 vs Zdr (in the sequence of left to 
right, top to bottom), where marker ‘x’ represents data 
for the rain case, marker ‘o’ for the hail case, marker ‘+’ 
for snow and ‘Δ’ for rain-hail mixture.

ambiguity than Zdr. Al-Jumily et al. (1991) illustrated 
the ambiguity of distinguishing between rain and hail by 
CDR with real circular polarization radar measurement. 
Here we will investigate the reason as follows. From 
Eq. (6) we can express propagation-corrected CDR by 
the variables in the principle plane as:

\[ CDR = 10 \log_{10} \left( \frac{(|S_{11} - S_{22}|^2)}{|S_{11} + S_{22}|^2} \right) \]

\[ = 10 \log_{10} \left( \frac{\sqrt{zdr} + 1/\sqrt{zdr} - 2\Re p_{12}}{\sqrt{zdr} + 1/\sqrt{zdr} + 2\Re p_{12}} \right) \]

where zdr refers to the differential reflectivity in principle 
plane in linear scale and p_{12} refers to correlation co-
efficient between S_{11} and S_{22}. Therefore, CDR will fluctu-
ate with decorrelation besides mean shape. Even with-
out decorrelation, CDR will give same value for both
negative $Z_{dr}$ and positive $Z_{dr}$ as shown here for hail ($Z_{dr}$ around -0.5 dB) and snow ($Z_{dr}$ around 0.5 dB), i.e., CDR cannot distinguish between oblate and prolate shapes at low elevation angles.

Similarly, ORTT could be expressed as:

$$\text{ORTT} = \langle \cos(2\beta') \rangle \frac{|(S_{11} + S_{22})^2(S_{11} - S_{22})|}{\sqrt{|(S_{11} + S_{22})^2|(S_{11} - S_{22})^2|}}$$

$$= \rho_2 \frac{\sqrt{2dr} - 1/\sqrt{2dr} + j23\rho_{12}}{\sqrt{2dr} + 1/\sqrt{2dr} + 2 - 4\rho^2_{12}}$$

With $\rho_2$ and imaginary part of decorrelation inside, ORTT presents better classification among the various precipitation types. Note ORTT comes from co-to-cross circular covariance while $\rho_1$ comes from co-polar circular covariance. Since co-polar power is less than cross-polar power in circular basis, ORTT will be a better estimator than $\rho_1$ as we will see next.

The $\rho_1$ in circular basis should be taken as a counterpart of LDR but it is a pure orientation factor. We can see a clear boundary between rain and hail in the scatterplot of $\rho_1$ vs $Z_{dr}$, For the rain-hail mixture, it depends on which one dominates. However, it scatters over large range for snow even over 1, which we believe results from artifacts in transformation because of low SNR. Up to this point, we believe the combination of ORTT and $\rho_1$ will tell us more about the hydrometeor types than LDR and $\rho_{co}$. However, low value of $\rho_{co}$ implies a mixture of precipitation types and is valuable for classification.

5. Discussion

In this paper, we reviewed the transformation on polarimetric covariance matrix between different polarization bases and explored it to obtain circular set of radar measurable from linear radar measurements. With a real case from STEPS, it shows that the circular measurements could be obtained with confidence.

In theory, the covariance matrix under different polarization bases are equivalent and can be transformed to other bases. This information is expressed in different manner for circular and linear polarization. Specifically, $\rho_1$ is only dependent on orientation distribution while CDR is independent of that. Both of them along with ORTT are sensitive to propagation effects. On the contrary, all the radar variables in linear polarization are dependent on shape and orientation of hydrometeors in a complicated way, but they do not get much affected by propagation effects. After correction, CDR closely relates to $Z_{dr}$ for raindrops in Raleigh-Gans scattering. LDR relates to orientation distribution but has much ambiguity for classification. ORTT and $\rho_1$, however, provide additional information.

In practice, however, the cross-polar power return is fairly low and the cross-polar terms are prone to noise and measurement error, especially for low reflectivity precipitation. The transformation demands an excellent cross-polar performance for antenna subsystem to achieve credible estimation of orientation factors. To account for such concern, CSU-CHILL is undergoing another major upgrade to dual-offset Gregorian antenna. Also, a scheme to measure the full covariance matrix in the slant $45^\circ/135^\circ$ basis will be implemented. This will enhance the cross-polar power return and also mitigate somewhat the antenna polarization error contamination as compared to the H/V basis.

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References


