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Introduction

Herein we investigate spectral characteristics of copolar and cross-polar signals for a radar that transmits horizontally (H) and vertically (V) polarized waves. Because radial displacements of hydrometeors dominate the physical process that contributes to the spectral shapes at either polarization, the spectra at H and V polarizations are very similar. Observations on NSSL's Research&Development polarimetric WSR-88D confirm this likeness in shape.

Besides radial motion of scatterers, an additional significant mechanism that influences spectra of cross-polar signals is the change in shape and orientation of scatterers. This is because these factors contribute to depolarization. Depolarization occurs if cloud particles are canted and/or the particles are not symmetrical. Changes in the canting angles, i.e., wobbling of raindrops and snowflakes modulate both the copolar and the depolarized signals. But the relative amount of modulation is much larger in the cross-polar signal and that could cause the Doppler spectra to differ. Observations of the copolar and cross-polar spectra presented herein have characteristics that might be attributed to this effect.

Radar data was collected on the NSSL's polarimetric WSR-88D. The radar can operate in two modes: 1) Simultaneous transmission and reception of H and V signals (SHV mode), and 2) transmission of H and reception of H&V, called depolarization mode (Melnikov et al., 2003). In the SHV mode, the system computes reflectivity (Z), Doppler velocity (v), and spectral width (σ_v) measured at H-polarization along with differential reflectivity (Z_{dr}), differential phase (φ_{dp}), and the correlation coefficient (ρ_{hv}). In the depolarization mode, the system calculates Z , V , and σ_v in the H channel along with the φ_h and copolar to cross-polar correlation coefficient ρ_{cx} (see Doviak and Zrnica 1993).

Copolar spectra

Typical spectra of H and V signals in the SHV mode are in Fig. 1; the signals were recorded simultaneously. For signal-to-noise ratios larger than 15 dB, the spectra almost coincide, i.e., the correlation coefficient between H and V signals is very close to one. Using time series voltages $V_h(nT)$ and $V_v(nT)$, where T is the pulse repetition interval and n is an integer, three correlation functions can be calculated

$$\begin{aligned} R_{hh}(nT) &= \overline{V_h(t)V_h^*(t+nT)} = S_h D_{hh}(nT) + N_h \delta_{0n}, \\ R_{vv}(nT) &= \overline{V_v(t)V_v^*(t+nT)} = S_v D_{vv}(nT) + N_v \delta_{0n}, \\ R_{hv}(nT) &= \overline{V_h(t)V_v^*(t+nT)} = (S_h S_v)^{1/2} \rho_{hv} D_{hv}(nT), \end{aligned} \quad (1)$$

where S_h and S_v are the powers of weather signals in the channels, ρ_{hv} is the correlation coefficient between H and V signals at zero lag, D_{xy} are the correlations in sample time due to Doppler spread (x or $y = h$ or v), N_h and N_v are noise in the channels, δ_{0n} is the Kronecker's symbol, and the over-bar denotes time averaging. The correlations D_{xy} can be written as (Doviak and Zrnica 1993)

$$D_{xy}(nT) = \exp \left[j \frac{4\pi nT}{\lambda} v^{(xy)} \right] \rho(nt \sigma_v^{(xy)}),$$

where $v^{(xy)}$ and $\sigma_v^{(xy)}$ are the Doppler velocity and the spectral width computed using the H and V signals, the magnitude of the correlation coefficient ρ depends on spectral width, and λ is the wavelength. High similarity of spectra in the channels means that all D_{xy} correlations are almost equal, i.e. the Doppler velocities and the spectral widths calculated from the H and V signals are same. This fact can be used to calculate two main polarimetric parameters free of noise. It follows from (1), that differential reflectivity Z_{dr} and the correlation coefficient ρ_{hv} can be obtained from correlations that are not biased by noise. For instance, for $n=1$ we have

$$Z_{dr} = S_h / S_v = |R_{hh}(T)| / |R_{vv}(T)|, \quad (2)$$

$$\rho_{hv} = |R_{hv}(T)| / [R_{hh}(T)R_{vv}(T)]^{1/2}. \quad (3)$$

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Expressions (2) and (3) can be used in cloud regions with weak backscattered signals where cloud particles are small and follow well the wind, i.e., where all D_{xy} are same.

Cross-polar spectra

In the depolarization mode, we often observe significant difference between Doppler spectra in H (copolar) and V (cross-polar) channels which can not be explained by the weaker signal in the cross-polar channel. For signal-to-noise ratios more than 15 dB in the cross-polar channel, Doppler spectra of the copolar signal often have more pronounced peaks and are narrower than spectra of the cross-polar signal. Sometimes this difference is over two times. For example the spectral widths (Fig. 2) obtained from Fourier transform and noise subtraction are $\sigma_v^{(hh)} = 1.2 \text{ m s}^{-1}$ and $\sigma_v^{(vv)} = 2.4 \text{ m s}^{-1}$, i.e., the cross-polar spectrum is twice wider than the copolar one.

To study bulk properties of spectral width, we recorded fields of reflectivity, Doppler velocity, and spectral width in the two channels. Our processing system currently calculates the base radar moments, i.e. Z , v , and σ_v in one channel only, either H or V. Therefore we used the depolarization mode recorded sequentially the moment fields from the H and V channels (Fig. 3). The time separation between these fields is 2.3 min. Fig 3c presents the power in the cross-polar channel weighted with range as it is done for reflectivity. Smaller extent of echo in the cross-polar channel is due to weaker signal than in the copolar channel. One can see that values of the spectral width in the cross-polar channel are noticeable larger than in the copolar channel. Note that due to advection and evolution data in Figs 3b and 3d represent somewhat different environments. But studying many sequential pairs as in Fig. 3 we have never observed an opposite relation, i.e., that bulk spectral widths in the copolar channel are larger than in the cross-polar channel. Thus the evidence from radar data is compelling to conclude that spectral widths in the cross-polar channel are generally wider than in the copolar channel.

Several contributions might be responsible for the difference of the spectral widths: wobbling of raindrops and snowflakes, raindrop vibrations, effects of antenna sidelobes, system noise, and/or multiple scattering of electromagnetic radiation in clouds. Next we address these potential contributors.

Wobbling of cloud particles. A cloud particle that has non zero canting angle causes depolarization of backscattered radiation. Wobbling cloud particles change the canting angles which then affects the spectrum of the depolarized signal. The contribution by wobbling and precession to the spectral width squared in the cross-polar channel is given by the following expression, $\lambda^2 (F_w^2 + F_p^2)/4$, where F_w is the wobbling frequency and F_p is the frequency of precession weighed with the depolarized backscattered power. Precession is considered to be around the vertical axis. If we assume $F_w = 10 \text{ Hz}$ (probably on a high side for wobbling), we get $\lambda F_w / 2 = 0.5 \text{ m s}^{-1}$ which is very small. The same holds for precession. Thus we conclude that wobbling cannot explain the difference of the observed spectral widths.

Oscillations of the raindrops. Effects of the raindrop oscillations on the scattering of copolar signal have been studied by Zrnica and Doviak (1989). The oscillations contribute also to fluctuations of cross-polar signal if the droplets have non-zero canting angle. Large raindrops oscillate with frequencies of 30 to 40 Hz (Musgrove and Brook 1975; Beard and Jameson 1983) and the frequency of oscillations of 1 mm raindrops reaches 100 Hz. The frequency f of the dominant mode can be calculated using the equation $f = (24.5/a)^{3/2}$, where a is the radius of a raindrop (Landau and Lifshits 1959). It's worth noting that large raindrops may oscillate with harmonics different from the dominant mode, creating apparent changes to the canting angle.

For oscillations with the dominant mode, the variance of a Doppler spectrum caused by raindrop oscillation is proportional to the frequency of oscillations squared weighted with deviations of scattering coefficients for the cross-polar signal. Assuming that the relative deviations of principal axes of a raindrop is of order of 0.1 (Beard and Johnson 1983), the corresponding contribution to the spectral width can be of 1 to 3 m s^{-1} . That is the raindrop oscillations could contribute significantly to the widening of the cross-polar spectra.

Multiple scattering. Multiple scattering in clouds contributes to the depolarized backscattered wave. For double scattering, we estimated a ratio of powers scattered in the copolar direction, P_{co} (from single scattering), and cross-polar direction, P_{cross} (from double scattering). For this ratio, we estimated

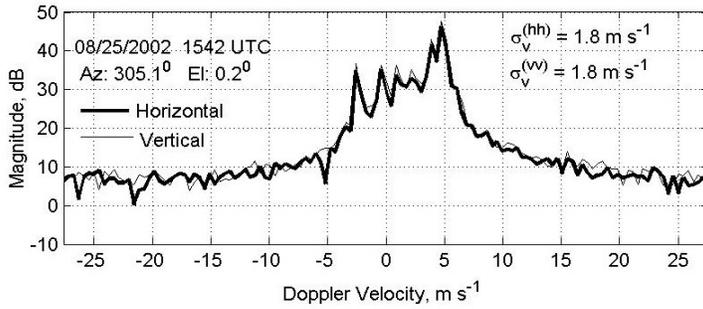


Fig. 1. Doppler spectra of the horizontally and vertically polarized signals. The mode is SHV, distance is 56 km, spectral window is rectangular.

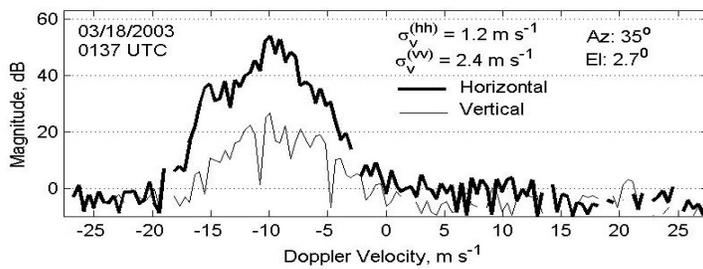


Fig. 2. Doppler spectra of the horizontally (copolar) and vertically (cross-polar) polarized signals in the depolarization mode. Distance is 57 km, spectral window is hamming.

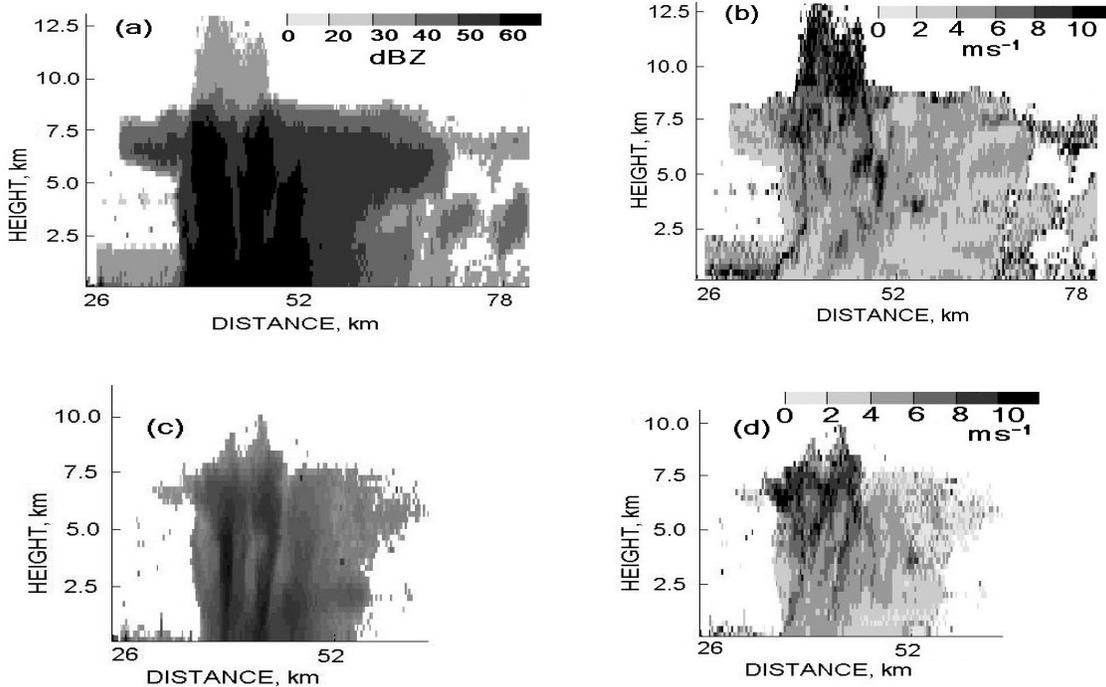


Fig. 3. Vertical cross-sections of reflectivity (a) and spectral width (b) in the co-polar channel on 03/18/2003 0133 UTC, azimuth is 35.1° . Fields of the powers (c) recorded as reflectivity and spectral width (d) in the cross-polar channel recorded at 0130 UT at same azimuth.

$10 \log(P_{\text{cross}}/P_{\text{co}}) \leq Z - 80$, where Z is measured in dBZ and the radar resolution volume is approximated with a sphere of 1 km diameter. For reflectivity of 60 dBZ or more, the contribution of multiple scattering could be of the order of depolarized signal. Because depolarized signal from multiple scattering have large spread of Doppler shifts this signal could have wider spectra than copolar signal.

Non meteorological factors.

Low signal-to-noise ratios, SNR, can bias measured spectral width. But for SNR more than 15 dB, as in Fig. 2 and in the core part of echo in Fig. 3, this bias is negligible.

Antenna sidelobes can affect spectral width measurements. This effect is noticeable when the antenna directed to the side of a reflectivity core and sufficient signal comes via sidelobes that illuminate the core. Data in Fig. 2 were recorded in a reflectivity core where influence of the sidelobes is negligible.

Discussions

In the simultaneous transmission and reception mode, spectra in the H and V channels are very similar and the signals are highly correlated. Hence correlations at lags other than zero can be used to eliminate noise bias in differential reflectivity and copolar to cross-polar correlation coefficient.

Radar observations in thunderstorms show that the spectra of the cross-polar signal are often wider than the spectra of the co-polar signal. Several processes may contribute to the spectral widening in the cross-polar channel. Among these we have examined wobbling of the scatterers, drops' oscillations, multiple scattering, antenna sidelobes, and receiver noise. Wobbling has little influence on the spectral difference. Further we identify and thus exclude from analysis signals with possible contaminations through antenna sidelobes and poor signal-to-noise ratios. Our calculations show that either one or both, oscillations of drops and multiple scattering can explain observed wide spectra in the cross-polar channel. Although we have no independent means to verify which mechanism causes this broadening, by using a simple scattering model we can exclude multiple scattering as a contributor in some regions of the storm. We are thus left with the question: are drop's oscillations responsible and can these be identified in the spectra of the cross-polar signals?

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