P4B.6 ON SIMULTANEOUS TRANSMISSION AND RECEPTION OF VERTICAL AND HORIZONTAL POLARIZED WEATHER RADAR SIGNALS

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1. Introduction*

Polarization diversity includes the use of any pair of orthogonal polarization states and with few exceptions the same polarization basis was always chosen for transmission and reception in weather radar.

The main reason for polarization-diversity radar transmitting and receiving simultaneously linear vertical (V) and linear horizontal (H) polarization is to eliminate the high power microwave switch, which has been a critical component in dual-linear polarization research radar. Concern is now mainly given to receiver technology, and less attention is paid to the polarization state of the transmitted wave, viz. to the relative phase, ϕ_{HV}, between the V and H components of the transmitted radiation. In general the transmitted wave is elliptical. In particular, if the V and H components of the signal have equal amplitude, 45° slant linear polarization is obtained when $\phi_{HV} = 0°$, and circular polarization when $\phi_{HV} = \pm 90°$.

In this work we examine how the radar echo signals depend on ϕ_{HV}. We assume a radar system with pulse train switching between orthogonal polarization states whose V and H components have equal amplitude and simultaneous reception of V and H radiation. In section 2 we show that the radar echo signal covariances and cross covariances obtained when elliptical polarization states are transmitted, can be expressed in term of the echo signal covariances and cross covariances from transmission of slant linear +/- 45° polarization states. In section 3 we derive one general set of equation for separation of propagation and backscattering effects that can be applied for any elliptical polarization state of the transmitted wave.

2. Radar echo covariances and cross covariances at elliptical polarization

Following Torlaschi and Holt (1998), when linear V/H polarization is used as a basis, the radar equation for a single particle can be represented by

$$\begin{bmatrix} V_1 & V_2 \\ H_1 & H_2 \end{bmatrix} = \frac{C}{r^2}\mathbf{M}\begin{bmatrix} T_{1V} & T_{2V} \\ T_{1H} & T_{2H} \end{bmatrix}, \tag{1}$$

where subscript 1 refers to transmission of an elliptical polarization state and subscript 2 to transmission of the orthogonal state, for example, V_1 denotes the complex value of the signal amplitude for transmission of the elliptical polarization state (T_{1V}, T_{1H}) and reception of vertical polarization; C is a constant dependent on radar parameters; r is the distance from the radar; and matrix **M** relates the transmitter output signals to the received signals. Let us define matrix **A** such that

$$\begin{bmatrix} V_1 & V_2 \\ H_1 & H_2 \end{bmatrix} = \begin{bmatrix} V_+ & V_- \\ H_+ & H_- \end{bmatrix}\mathbf{A}, \tag{2}$$

where subscript +/- designates the components of the received signals when slant linear +/- 45° polarization states are transmitted. For unit power delivered to the antenna, the transmitter output components that give orthogonal elliptical polarizations are $(T_{1V}, T_{1H}) = 1/\sqrt{2}\,(1, e^{j\phi_{HV}})$, and $(T_{2V}, T_{2H}) = 1/\sqrt{2}\,(1, -e^{j\phi_{HV}})$; whilst $(T_{1V}, T_{1H}) = 1/\sqrt{2}\,(1, 1)$, and $(T_{2V}, T_{2H}) = 1/\sqrt{2}\,(1, -1)$ correspond to slant linear +/- 45° polarizations. By substituting for both polarizations the transmitter output components in (1) and then the result in (2), we obtain

$$\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}^{-1}\begin{bmatrix} 1 & 1 \\ e^{j\phi_{HV}} & -e^{j\phi_{HV}} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1+e^{j\phi_{HV}} & 1-e^{j\phi_{HV}} \\ 1-e^{j\phi_{HV}} & 1+e^{j\phi_{HV}} \end{bmatrix}, \tag{3}$$

where $[.]^{-1}$ is the inverse matrix. From (2) and (3) the components of the received signals at elliptical polarization as function of the components at slant linear +/- 45° polarization are

$$V_1 = \frac{1}{2}\left[(V_+ + V_-) + e^{j\phi_{HV}}(V_+ - V_-)\right], \tag{4a}$$

$$V_2 = \frac{1}{2}\left[(V_+ + V_-) - e^{j\phi_{HV}}(V_+ - V_-)\right], \tag{4b}$$

$$H_1 = \frac{1}{2}\left[(H_+ + H_-) + e^{j\phi_{HV}}(H_+ - H_-)\right], \text{ and} \tag{4c}$$

$$H_2 = \frac{1}{2}\left[(H_+ + H_-) - e^{j\phi_{HV}}(H_+ - H_-)\right]. \tag{4d}$$

The coherency matrices and the covariance matrix of the received signals represent the radar echo signals from an ensemble of particles filling a radar resolution volume. The coherency matrices are defined as

$$\mathbf{J}_1 = \begin{bmatrix} \langle |V_1|^2 \rangle & \langle V_1 H_1^* \rangle \\ \langle V_1^* H_1 \rangle & \langle |H_1|^2 \rangle \end{bmatrix} \text{ and} \tag{5a}$$

$$\mathbf{J}_2 = \begin{bmatrix} \langle |V_2|^2 \rangle & \langle V_2 H_2^* \rangle \\ \langle V_2^* H_2 \rangle & \langle |H_2|^2 \rangle \end{bmatrix}, \tag{5b}$$

where, the angle brackets indicate the expected values, and the asterisk denotes the complex conjugate; and the covariance matrix **R** at lag time T_s is

$$\mathbf{R} = \frac{1}{2}\left(\mathbf{R}_a + \mathbf{R}_b^T\right) = \begin{bmatrix} R_{VV} & R_{VH} \\ R_{HV} & R_{HH} \end{bmatrix} e^{j\phi_D} \tag{6a}$$

where the matrix $\mathbf{R}_b^T$ is the transpose of $\mathbf{R}_b$, ϕ_D is the

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Doppler phase due to radial velocity of the scatterers,

$$\mathbf{R}_a = \left\langle \begin{bmatrix} V_1^*(t) \\ H_1^*(t) \end{bmatrix} \begin{bmatrix} V_2(t+T_s) & H_2(t+T_s) \end{bmatrix} \right\rangle, \tag{6b}$$

$$\mathbf{R}_b = \left\langle \begin{bmatrix} V_2^*(t) \\ H_2^*(t) \end{bmatrix} \begin{bmatrix} V_1(t+T_s) & H_1(t+T_s) \end{bmatrix} \right\rangle. \tag{6c}$$

Substituting (4) into (5) and (6) we obtain the radar echo signal covariances and cross covariances at elliptical polarization as function of slant linear +/- 45° polarization radar observables.

3. Separation of propagation and backscattering effects

Assuming a medium where the hydrometeors are axially symmetric and their distribution of canting angle is uniform along the propagation path, Holt (1984) shows that the elements of matrix **M** are equal to

$$M_{VV} = d_V^2\, S_{VV} \cos^2\theta + d_H^2\, S_{HH} \sin^2\theta, \tag{7a}$$

$$M_{VH} = M_{HV} = \left(d_V^2\, S_{VV} - d_H^2\, S_{HH}\right)\sin\theta\cos\theta, \quad \text{and} \tag{7b}$$

$$M_{HH} = d_V^2\, S_{VV} \sin^2\theta + d_H^2\, S_{HH} \cos^2\theta, \tag{7c}$$

where S_{VV}, S_{HH} are the copolar components of the backscattering matrix, θ is the apparent canting angle, and d_V, d_H are complex propagation parameters. From the definitions of the two-way mean attenuation, A, the two-way differential attenuation, Δa, and the two-way differential propagation phase shift, Φ_{DP}, Torlaschi and Holt (1993) has shown that

$$e^{-A} = \left|d_V d_H\right|^2, \tag{8a}$$

$$e^{\Delta a} = \left|\frac{d_V}{d_H}\right|^2, \text{ and} \tag{8b}$$

$$e^{j\Phi_{DP}} = \frac{d_V d_H^*}{d_V^* d_H}. \tag{8c}$$

The substitution of (7) and the transmitter output components for elliptical polarization states into (1) yields the expressions of the components of the received signals and from (5) and (6) we obtain the radar echo signal covariances and cross covariances at elliptical polarization. Following Torlaschi and Gingras (2000), we define from the radar echo signal covariances and cross covariances the following quantities:

$$p = \frac{1}{2}\left[\left\langle|V_1|^2\right\rangle + \left\langle|V_2|^2\right\rangle + \left\langle|H_1|^2\right\rangle + \left\langle|H_2|^2\right\rangle\right] = \frac{|C'|^2}{2r^2} e^{-A}\left\langle e^{\Delta a}|S_{VV}|^2 + e^{-\Delta a}|S_{HH}|^2\right\rangle, \tag{9a}$$

$$q = \mathrm{Re}\{R_{VV} + R_{HH}\} + j\left(\left\langle V_1^* H_1\right\rangle + \left\langle V_2^* H_2\right\rangle\right) = \frac{|C'|^2}{2r^2} e^{-A}\left\langle \rho_2\left(e^{\Delta a}|S_{VV}|^2 - e^{-\Delta a}|S_{HH}|^2\right)\right\rangle e^{j2\bar{\theta}}, \tag{9b}$$

$$u = \mathrm{Re}\{R_{HV} - R_{VH}\}\cos\phi_{HV} + \mathrm{Im}\left(\left\langle V_1^* H_1\right\rangle - \left\langle V_2^* H_2\right\rangle\right)\sin\phi_{HV} = \frac{|C'|^2}{r^2} e^{-A}\left|\left\langle S_{VV} S_{HH}^*\right\rangle\right|\cos\Phi \tag{9c}$$

$$\begin{aligned} v &= \mathrm{Im}\{R_{VH} + R_{HV}\}\cos\phi_{HV} + \mathrm{Re}\left\{\left\langle V_1^* H_1\right\rangle - \left\langle V_2^* H_2\right\rangle\right\}\sin\phi_{HV} + j\,\mathrm{Im}\{R_{VH} - R_{HV}\} \\ &= \frac{|C'|^2}{r^2} e^{-A}\left|\left\langle \rho_2 S_{VV} S_{HH}^*\right\rangle\right|\sin\Phi\; e^{j2\bar{\theta}}, \quad \text{and} \end{aligned} \tag{9d}$$

$$\begin{aligned} w &= \mathrm{Re}\{R_{VV} - R_{HH}\} - \mathrm{Re}\left\{\left\langle V_1^* H_1\right\rangle - \left\langle V_2^* H_2\right\rangle\right\}\cos\phi_{HV} + \mathrm{Im}\{R_{VH} - R_{HV}\}\sin\phi_{HV} \\ &= \frac{|C'|^2}{2r^2} e^{-A}\left[\left\langle \rho_4\left(e^{\Delta a}|S_{VV}|^2 + e^{-\Delta a}|S_{HH}|^2\right)\right\rangle - 2\left|\left\langle \rho_4 S_{VV} S_{HH}^*\right\rangle\right|\cos\Phi\right]\cos 4\bar{\theta}, \end{aligned} \tag{9e}$$

where $\Phi = \Phi_{DP} + \delta$ is the two-way total differential phase shift, $\delta = \arg\langle S_{VV} S_{HH}^*\rangle$ is the backscattering differential phase shift, $\bar{\theta}$ is the mean apparent canting angle throughout the region of precipitation, and ρ_2 and ρ_4 are two parameters representing the degree of common orientation of hydrometeors filling the backscattering volume (McCormick and Hendry 1975; Hendry et al. 1987). The relationship between ρ_2 and ρ_4 for Gaussian-like canting angle distribution is (Torlaschi and Gingras 2000)

$$\rho_2 = 0.323 - 0.170\,\rho_4 + 0.847\sqrt{\rho_4}\,. \tag{10}$$

Note that by setting $\phi_{HV} = 0°$ in (9), we obtain the expressions of p, q, u, v, and w presented in Torlaschi and Gingras (2000) for alternate transmission of 45° slant polarization. Equations (9) are more general than Torlaschi and Gingras' and can be applied to any transmitted couple of elliptical polarizations states whose V and H components have equal amplitude.

It follows from (9) that

$$\bar{\theta} = \frac{1}{2}\min\left[\arg\{q\} + k\pi\right], \qquad k = 0, 1, \tag{11a}$$

$$\rho_4 = \frac{w}{(p - u)\cos 4\bar{\theta}}, \tag{11b}$$

$$\left\langle |S_{VV}|^2 \right\rangle = \frac{r^2}{|C'|} e^{A} e^{-\Delta a}\left(p \pm \frac{|q|}{\rho_2}\right), \tag{11c}$$

$$\left\langle |S_{HH}|^2 \right\rangle = \frac{r^2}{|C'|} e^{A} e^{\Delta a}\left(p \mp \frac{|q|}{\rho_2}\right), \tag{11d}$$

$$\left\langle S_{VV} S_{HH}^* \right\rangle = \frac{r^2}{|C'|^2} e^{A}\left(u + j\frac{|v|}{\rho_2}\right), \text{ and} \tag{11e}$$

$$\Phi = \arg\left(u + j\frac{|v|}{\rho_2}\right). \tag{11f}$$

The intrinsic radar observables can be estimated by: the radar reflectivity at V and H polarization

$$Z_V = \frac{4\lambda^4}{\pi^4 |K_w|^2}\left\langle |S_{VV}|^2 \right\rangle = 10^{\frac{A_V}{10}} \frac{4\lambda^4}{\pi^4 |K_w|^2} \frac{r^2}{|C'|^2}\left(p \pm \frac{|q|}{\rho_2}\right), \tag{12a}$$

$$Z_H = \frac{4\lambda^4}{\pi^4 |K_w|^2}\left\langle |S_{HH}|^2 \right\rangle = 10^{\frac{A_H}{10}} \frac{4\lambda^4}{\pi^4 |K_w|^2} \frac{r^2}{|C'|^2}\left(p \mp \frac{|q|}{\rho_2}\right), \tag{12b}$$

where λ is the radar wavelength, K_w is the refraction factor for water, and $A_{V,H} = A \mp \Delta A/2$ is the total two-way attenuation at V, H polarization, respectively, ΔA is the two-way differential attenuation in decibel units, in the right-hand side of (11c) and (11d) the superior signs are taken when $R_{VV} + R_{HH} \geq 0$, and the inferior signs when $R_{VV} + R_{HH} < 0$; the differential reflectivity

$$Z_{DR} = 10\log\frac{\left\langle |S_{HH}|^2 \right\rangle}{\left\langle |S_{VV}|^2 \right\rangle} = 10\log\frac{\rho_2 p \mp |q|}{\rho_2 p \pm |q|} + \Delta A; \tag{9c}$$

and the copolar correlation coefficient at zero lag time

$$\rho_{HV} = \frac{\left|\left\langle S_{VV} S_{HH}^* \right\rangle\right|}{\sqrt{\left\langle |S_{VV}|^2 \right\rangle \left\langle |S_{HH}|^2 \right\rangle}} = \frac{\left|\rho_2 u + j|v|\right|}{\sqrt{\rho_2^2 p^2 - |q|^2}}. \tag{9d}$$

4. Summary

This paper shows that the radar echo signal covariances and cross covariances from a given couple of orthogonal polarization states on transmit can be expressed in terms of the echo signal co-variances and cross co-variances from transmission of slant linear +/- 45° polarization states. Furthermore, for axially symmetric hydrometeors with canting angle distribution symmetric about the mean canting angle, a general set of equations for separation of propagation and backscattering effects is developed. The mean apparent canting angle, the degree of common orientation of the hydrometeors, and the differential phase shift are then obtained together with the intrinsic value of the reflectivity, the differential reflectivity, and the co-polar correlation coefficient at zero lag time. The analysis of the bias on these radar observables due to the transmission of only one polarization state remain to be examined.

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