DETECTION OF ROTATION AND BOUNDARIES USING TWO-DIMENSIONAL, LOCAL, LINEAR LEAST SQUARES ESTIMATES OF VELOCITY DERIVATIVES

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1. Introduction

Traditional methods of calculating rotational shear from Doppler radial velocity data can give results that vary widely from the true value of shear for a vortex. Some factors that must be considered include noisy data and the offset of sample volumes from the center of rotation. This work illustrates preliminary results from a two-dimensional, local, linear least squares (LLSD) method to minimize the large variances in rotational and divergent shear calculations.

Besides creating greater confidence in the value of intensity of meteorological features that are sampled, the LLSD method for calculating shear values has several other advantages. The LLSD removes many of the radar dependencies involved in the detection of rotation and radial divergence (or radial convergence) signatures. Thus, these derivatives of the radial velocity field may be viewed in three-dimensional space or used as input to multi-sensor meteorological applications that are not single-radar based. Additionally, fields of these radial estimates of rotation and divergence have specific signatures when boundaries or circulations are sampled. This manuscript describes how the derivatives are calculated as well as how the rotational LLSD compares with the less-robust (but operationally used) "peak-to-peak" estimates of azimuthal shear. The accompanying poster presentation will describe the divergent LLSD and examples of data mining techniques that use LLSD for boundary and rotation detection.

2. Local, Linear, Least Squares Derivatives

Elmore et al. (1994) describe a method for estimating divergent shear from single Doppler radar data for use in calculating headwind loss estimates for aircraft that encounter microbursts. The rotation portion of the derivative was also derived by Elmore et al., but not utilized for microburst detection. The LLSD technique was implemented in NSSL's Damaging Downburst Prediction and Detection Algorithm (Smith et al. 2003) for detecting low-level outflows and midlevel convergence and rotation in storm cells. Mitchell and Elmore (1998) first explored the uses of the LLSD for identifying regions of high shear in mesocyclones and tornadic vortex signatures.

Elmore et al. (1994) show that the estimates of radial divergence (u_r) and rotational shear (u_s) can be calculated on a local neighborhood surrounding each range gate, where:

$$u_r = \frac{\sum i V_{ij} w_{ij}}{\Delta r \sum i^2 w_{ij}}, \text{ and}$$
$$u_s = \frac{\sum s_{ij} V_{ij} w_{ij}}{\sum (\Delta s_{ij})^2 w_{ij}}$$

Here, V_{ij} is the radial velocity, Δr is the pulse volume width, s_{ij} is the azimuthal distance from the center of the kernel to the point (i,j), and w_{ij} is a uniform weight function. Because u_r and u_s are derived from only the radial component of the wind, they are approximations of one half the horizontal divergence and vertical vorticity ("half vorticity", hereafter), respectively, assuming a symmetric wind field.

In order to make LLSD calculations on a field of radial velocities, the data are passed through a 3x3 median filter to reduce noise. Then, because the LLSD calculations require a complete kernel of data in order to produce a result, missing radial velocities are filled in with the median of the four adjacent range gates. Finally, the fields of u_r and u_s are calculated.



The size of the kernel that is used in the calculation is variable, and is described below.

3. LLSD of Rotational Shear

LLSD of rotational, or azimuthal, shears are calculated for simulated circulation signatures of different sizes and at different ranges from a hypothetical radar in order to compare with traditional methods of estimating the strength of circulations. We use a Rankine combined vortex model to generate simulated circulation signatures in the Doppler radial velocity field (Wood and Brown 1997). We superimpose 2 ms⁻¹ uniform noise on the radial velocity field to test the robustness of the LLSD calculations.

We compare the LLSD values to the more traditional "peak-to-peak" azimuthal shear calculation, given by

$$u_{as} = \frac{V_{\max} - V_{\min}}{d}$$

where V_{max} and V_{min} are the maximum outbound and minimum inbound radial velocities (on opposite sides of a circulation), respectively, and *d* is the distance between those two peaks. For the LLSD calculations, we choose three different kernel sizes that are each 3 range gates deep and approximately 2500 m, 5000 m, or 8000 m



wide. Thus the number of radials used in the calculation varies with range from the radar, although a minimum of three radials of data are required for a complete calculation. Kernels that use a fixed number of radials at all ranges usually only provide good shear estimates in a small percentage of those ranges.

To test the variability of the three LLSD kernels, we generate synthetic radial velocity signatures of vortices at ranges every 5 km from 20 km to 200 km. Because radar data suffer from many imperfections, including noise and sampling issues that can affect azimuthal shear values (Wood and Brown 2000), 1000 vortices of the same size and strength are generated at each range, each with different noise patterns and azimuthal offsets to the center of the simulated vortex. This allows for calculation of mean azimuthal shear values and 95% confidence intervals for the three LLSD kernels and u_{as} . We test these calculations on simulated vortices of three different diameters: 2.5 km, 5 km, and 8 km.

Figure 1 shows the 2500m LLSD kernel and peak-to-peak azimuthal shear estimates for a 5 km diameter vortex with half vorticity of 0.01 s^{-1} . In this case, the mean LLSD value is within about 20% of the true value out to about 140 km, with a much smaller variance than that of the peak-to-peak azimuthal shear calculations.



Figure 3: The distribution of range positional errors for the 2500 m LLSD kernel (top) and the peak-to-peak azimuthal shear estimate (bottom) for a 5 km diameter vortex. The center grey line is the median, the box is the interquartile range (IQR), the whiskers are the lesser of 1.5x(IQR) or the data range, and the single dots are outliers.

These values drop with range because of the geometry of the radar beam – circulations are not well sampled at long ranges. For a larger-scale 8 km diameter vortex (Fig. 2) sampled with the 2500 m kernel, the mean LLSD values are within 5% of the true value out to about 150 km. For brevity, results from the 5000 m and 8000 m kernel are not shown. However, these larger



kernels tended to underestimate the strength of smaller vortices compared to the 2500 m kernel.

Because we use synthetic radar data, the true location of the center of the circulation is known. Range and azimuthal position errors were calculated for both the LLSD and peak-topeak methods. For azimuthal shear, the center of circulation was considered to be halfway between velocity absolute maxima on each side of the circulation. The NSSL Mesocyclone Detection Algorithm (Stumpf et al 1998) uses this method to determine the center of a circulation. For the LLSD rotation, the center of circulation was considered to be at the LLSD rotation maximum.



The errors in range (Fig. 3) for both methods are quite similar, although the variance is smaller for the LLSD estimate. However, the azimuthal distance errors (Fig. 4) for the peakto-peak method are significantly larger than the LLSD. Additionally, the distribution of the peakto-peak location estimates is not Gaussian. This is illustrated in Fig. 5. While the LLSD position estimates are clustered around the center of the diagram, there are three distinct groupings for the peak-to-peak data. Because the peak-topeak method only uses two data points in its calculations, it is highly susceptible to errors caused by the radial offset from the center of the circulation and noise.

4. Conclusion

The local, linear, least squares approach to calculating radial velocity derivatives is a vast improvement over the frequently-used but simplistic and grossly inaccurate method of calculating shear from two data points. The LLSD provides relatively smooth fields that may be used in other applications to identify features such as boundaries and vortices, as well as to accurately assess their strength and position.

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