

## A METHODOLOGY FOR ATTENUATION CORRECTION BASED ON THE MINIMIZATION OF A COST FUNCTION

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### 1. INTRODUCTION

Attenuation of radar signal due to precipitation is one of the important limitations for quantitative uses of C-band radars. However, most of the operational radars (outside USA) are C-band and, therefore, significantly affected by attenuation.

Although in 1954 Hitschfeld and Bordan derived the analytical expression to invert the attenuation problem (supposing a potential  $Z$ - $k$  relationship), they already showed that this equation is very unstable with errors in radar measurements (in particular with errors in the radar calibration) and in the estimation of the  $Z$ - $k$  parameters.

More recently, the use of attenuated-frequencies for air- or satellite-borne radar has promoted the development of different techniques to correct for attenuation (using single- or double-frequency, single- or double-beam and single- or double-polarized radars), constraining the attenuation equation. In the case of single-frequency methodologies, most of them are based on the Surface Reference Technique (Iguchi and Meneghini, 1994; Marzoug and Amayenc, 1994; Iguchi et al., 2000) that consists on constraining the total Path Integrated Attenuation (PIA) with the surface return. This idea was implemented by Delrieu et al. (1997) in ground-based radars, using mountain returns as constraint.

In this study, a methodology for attenuation correction based on the minimization of a cost function is formulated. The aim of this methodology is to take into account as many sources of information as possible to constrain the attenuation equation.

In the presented formulation, this information comes from measurements at rain gages and the radar returns from mountains.

### 2. ATTENUATION EQUATIONS

Assuming a potential relationship between the specific attenuation,  $k(r)$ , and the (non-attenuated) reflectivity,  $k(r) = \alpha \cdot Z(r)^\beta$ , the attenuation equation can be written as:

$$\begin{aligned} Z_m(r) &= Z(r) \cdot \delta C \cdot A(r) \\ &= Z(r) \cdot \delta C \cdot \exp\left[-0.46 \cdot \int_0^r \alpha \cdot Z(s)^\beta ds\right] \end{aligned} \quad (1)$$

where  $Z_m(r)$  is the attenuated reflectivity at range  $r$  affected by a calibration error,  $\delta C$ ; the term  $A(r)$  represents the attenuation reduction of reflectivity. Expressed in dB,  $A(r)$  becomes the *Path Integrated Attenuation (PIA)*,

$$PIA(r) = -10 \cdot \log[A(r)] = 2 \cdot \int_0^r \alpha \cdot Z(s)^\beta ds \quad (2a)$$

$$PIA(r) = z(r) - (z_m(r) - 10 \cdot \log(\delta C)) \quad (2b)$$

where  $Z(r)$  is in  $\text{mm}^6 \text{m}^{-3}$  and  $z(r)$  in dBZ.

### 3. THE PROPOSED METHOD

The aim of the proposed methodology is to satisfy the attenuation model, but imposing some constraints to avoid the instability problems. The idea is that these constraints may come from different sources (ideally, calculable from radar measurements). In a previous formulation of this methodology, rain gage measurements were incorporated to constrain Hitschfeld and Bordan's equation (Berenguer et al., 2002). In this case, we will also use measured ground echo returns.

The proposed methodology consists on the minimization of the following cost function, which imposes the corrected reflectivity field to be coherent with the attenuation model but limited by the constraints:

$$\begin{aligned} J(\delta C, \alpha, \beta, Z) &= w_1 \cdot \sum_{i=1}^{n_{bins}} \left[ z(r_i) - (z_m(r_i) + PIA(r_i)) \right]^2 \\ &+ w_2 \cdot \sum_{j=1}^{n_{gages}} \left[ z(r_{g,j}) - 10 \cdot \log(a \cdot R_j^b) \right]^2 \quad (3) \\ &+ w_3 \cdot \sum_{k=1}^{n_{gc}} \left[ PIA(r_{gc,k}) - PIA_{gc}(r_{gc,k}) \right]^2 \end{aligned}$$

where,  $w_i$  are the weights given to each term of the cost function;  $z(r_i)$  is the corrected reflectivity field, that is also used to calculate  $PIA(r_i)$  from equation (2a);  $z_m(r_i)$  is the measured (attenuated) reflectivity to be corrected.  $a$ ,  $b$  are the parameters of the  $Z$ - $R$  relationship used to convert the rain-rate measured at the gages,  $R_j$ , into reflectivity. Finally,  $PIA(r_{gc,k})$  and  $PIA_{gc}(r_{gc,k})$  are the  $PIAs$  at ground clutter areas, on the one hand, calculated using equation (2a), and, on the other, estimated from the difference between measured ground echo returns and mean ground echo returns in dry weather conditions (using equation (2b)).

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Therefore, the philosophy of this methodology is quite similar to the idea of Serrar et al. (2000) in the sense that it estimates an “effective” calibration error that enables the attenuation correction.

#### 4. PRELIMINARY DISCUSSION OF THE METHOD

A methodology based on the minimization of a cost function is proposed for the correction of attenuation of radar signal due to precipitation. The proposed function avoids the divergence of the analytical solution and allows more constraints to be easily incorporated.

In this case, the formulation of the cost function includes a first term that imposes that the corrected reflectivity must satisfy the attenuation model and at the same time, it is constrained by rain gage information and mountain returns (second and third terms, respectively).

The use of rainfall rate from rain gages is not optimal because it represents an external constraint (not directly derivable from radar).

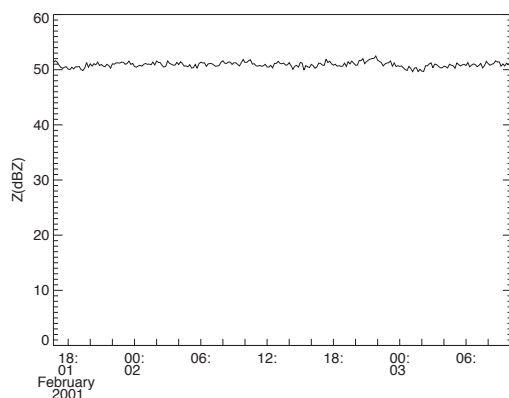


Fig. 1. Evolution of the mean reflectivity for a given mountain as measured by the Barcelona C-band radar of the Spanish INM during a non-rainy period.

It must be stated that using ground echoes as constraint may have associated difficulties: (i) their variability in dry-weather conditions (see Fig. 1), (ii) the fact that mountain returns change from dry to wet conditions or (iii) that ground echoes are not uniformly distributed in the radar scanning area to be corrected.

Finally, an important point for future work is the inclusion of more constraints into the cost function, especially from radar measurements. Some of these constraints could come from the overlapping region of two or more radars or, imposing some kind of spatial continuity of the reflectivity field in contiguous azimuths (Vignal, 1998).

*Acknowledgements:* This study has been carried out in the framework of the EC project VOLTAIRE (EVK2-CT-2002-00155) and of the Spanish CICYT project REN2000-1755-C03-01. Thanks are also due to the INM for providing radar data.

#### References

- Berenguer, M., G. W. Lee, D. Sempere-Torres, and I. Zawadzki, 2002: A variational method for attenuation correction of radar signal. *2nd European Conf. on Radar Meteor.*, 11-16.
- Delrieu, G., S. Caoual, and J. D. Creutin, 1997: Feasibility of using mountain return for the correction of ground-based X-band weather radar data. *J. Atmos. Oceanic Technol.*, **14**, 368-385.
- Hitschfeld, W. F. and J. Bordan, 1954: Errors inherent in the radar measurement of rainfall at attenuating wavelengths. *J. Meteor.*, **11**, 58-67.
- Iguchi, T. and R. Meneghini, 1994: Intercomparison of single-frequency methods for retrieving a vertical rain profile from airborne or spaceborne radar data. *J. Atmos. Ocean. Tech.*, **11**, 1507-1516.
- Iguchi, T., T. Kozu, R. Meneghini, J. Awaka, and K. Okamoto, 2000: Rain-profiling algorithm for the TRMM precipitation radar. *J. Appl. Meteor.*, **39**, 2038-2052.
- Marzoug, M. and P. Amayenc, 1994: A class of single- and dual-frequency algorithms for rain-rate profiling from a spaceborne radar. Part I: Principle and tests from numerical simulations. *J. Atmos. Oceanic Technol.*, **11**, 1480-1506.
- Serrar, S., G. Delrieu, J.-D. Creutin, and R. Uijlenhoet, 2000: Mountain reference technique: Use of mountain returns to calibrate weather radars operating at attenuating wavelengths. *J. Geophys. Res.*, **105**, 2281-2290.
- Vignal, B., 1998: Identification par methode inverse des profils verticaux de reflectivite et des profils radiaux d'attenuation, LCPC. Ph.D. Thesis.