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NON-PARAMETRIC ESTIMATION OF Z–R RELATIONS AND NORMALIZED RAINDROP SIZE DISTRIBUTIONS USING GAUSSIAN MIXTURES

Remko Uijlenhoet[1*], Rafał Wójcik[1], Paul Torfs[1],
Daniel Sempere Torres[2], GyuWon Lee[3], and Isztar Zawadzki[3]

[1]Department of Environmental Sciences, Wageningen University, The Netherlands
[2]Group of Applied Research on Hydrometeorology, UPC, Barcelona, Spain
[3]Marshall Radar Observatory, McGill University, Montreal, Canada

1 INTRODUCTION

Several approaches towards normalizing raindrop size distributions (DSDs) have been proposed in the scientific literature. The aim of normalizing (or "scaling") DSDs is to collapse many individual and highly variable empirical distributions into (ideally) one single "general" (or "intrinsic") distribution, which is typical for a particular type of rain or climatic setting (Sempere Torres et al., 1994, 1998; Lee et al., 2003). This provides a statistically more robust manner to adjust analytical parameterizations to DSDs than the traditional approach where a particular parametric form is fitted to each empirical distribution separately (e.g. Marshall and Palmer, 1948). An additional advantage of normalizing DSDs is that it becomes easier to establish connections between the shape of DSDs and the physical processes producing them (Uijlenhoet et al., 2003a,b). This will ultimately lead to improved rainfall retrieval algorithms for ground-based and space-borne microwave remote sensors (both active and passive). We present a non-parametric adjustment method to estimate radar reflectivity – rain rate relationships and normalized DSDs (and their associated uncertainties) based on Gaussian mixtures (e.g. McLachlan and Peel, 2000; Wand and Jones, 1995, present kernel smoothing approach).

2 MATERIAL AND METHODS

Sempere Torres et al. (1994, 1998) propose a unified framework for parameterizing DSDs, of which many previous parameterizations are special cases. Their formulation takes the form of a scaling law, in which the DSD depends both on the raindrop diameter (D) and on the value of a reference variable, commonly taken to be the rain rate (R). In this general formulation it is no longer necessary to impose an a priori functional form for the DSD. This form follows directly from the data. In addition, the scaling law formalism naturally leads to the ubiquitous power law relationships between rainfall integral variables (such as the radar reflectivity factor Z and the rain rate R). According to the scaling law formalism, DSDs can be parameterized in terms of 1) an intrinsic DSD, the so-called general DSD $g(x)$, where x is a scaled version of D, and 2) the scaling exponents α and β. It can be demonstrated that the values of α and β and the form and dimensions of $g(x)$ depend on the choice of the reference variable (typically R), but do not bear any functional dependence on its value. An unsolved problem in the area of DSD normalization is that of objectively choosing an appropriate functional form for the scaled distribution $g(x)$. Owing to the generality of the normalization approach, imposing one of the classical functional forms for probability densities (such as exponential, gamma, lognormal, etc.) is unnecessarily restrictive. In this paper we investigate the potential of a non-parametric estimation method based on Gaussian mixtures to infer normalized DSDs and related Z–R relations. We apply the methodology to a climatologically representative sample of 534 DSDs collected using the filter paper method by Herman Wessels (who kindly provided the data to us) at the Royal Netherlands Meteorological Institute (KNMI).

3 RESULTS AND DISCUSSION

Figure 1 shows the bivariate Gaussian mixture adjusted to a scatterplot of the Z–R pairs corresponding to the 534 DSDs. The corresponding regression of Z given R and the associated uncertainties are projected on the z=0-plane. The fact

[*]*Corresponding author address:* Dr Remko Uijlenhoet, Wageningen University, Department of Environmental Sciences, Hydrology and Quantitative Water Management Group, Nieuwe Kanaal 11, 6709 PA Wageningen, The Netherlands; e-mail: remko.uijlenhoet@wur.nl

that the regression line is closely linear in log-log space provides an independent check of the power-law behavior of the climatological Z–R relation for Dutch conditions. We note that inference in linear instead of in logarithmic space provides nearly the same regression line (not shown here). Figure 2 shows the corresponding results for the 534 $g(x)$'s normalized using R as the reference variable according to the method proposed by Sempere Torres et al. (1994, 1998). The inferred form of the regression line cannot be represented by any of the classical DSD parameterizations, although the generalized gamma function may provide satisfactory results in this particular case. According to the scaling law formalism, the coefficient of a Z–R relation should correspond to the 6th moment of the associated $g(x)$. How we can implement this constraint in the Gaussian mixture technique is still under investigation.

4 CONCLUSIONS

We have demonstrated the potential of a non-parametric estimation method based on Gaussian mixtures to infer normalized DSDs and related Z–R relations. The same non-parametric estimation method can be employed to infer scaled DSDs obtained using the double-moment normalization approach that we recently proposed as an extension to the single-moment normalization presented here (Lee et al., 2003).

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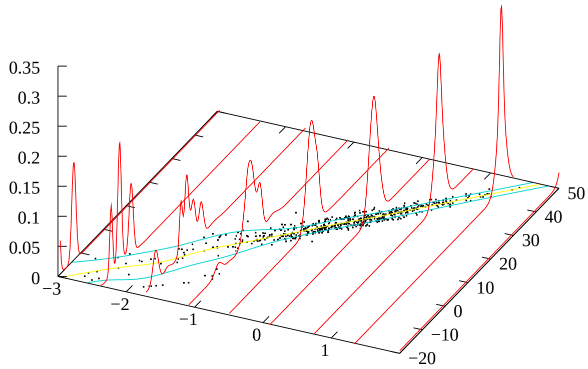

Figure 1: Bivariate Gaussian mixture (z-axis) adjusted to 534 Z (y-axis, dBZ) – R (x-axis, logarithmic scale) pairs. Projected on z=0: Conditional mean (regression) and $\pm 1\sigma$ error bounds.

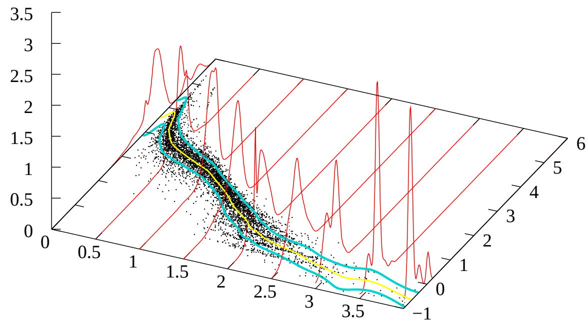

Figure 2: Gaussian mixture (z-axis) adjusted to 534 scaled DSDs (x-axis: scaled size x, linear scale; y-axis: scaled number density $g(x)$, logarithmic scale), including regression and $\pm 1\sigma$ error bounds.

REFERENCES

Lee, G. W., I. Zawadzki, W. Szyrmer, D. Sempere Torres, and R. Uijlenhoet, 2003: A general approach to double-moment normalization of drop size distributions. *J. Appl. Meteor.*, submitted.

Marshall, J. S. and W. M. Palmer, 1948: The distribution of raindrops with size. *J. Meteor.*, **5**, 165–166.

McLachlan, G. and D. Peel, 2000: *Finite mixture models*. John Wiley & Sons Inc., New York.

Sempere Torres, D., J. M. Porrà, and J.-D. Creutin, 1994: A general formulation for raindrop size distribution. *J. Appl. Meteor.*, **33**, 1494–1502.

— 1998: Experimental evidence of a general description of raindrop size distribution properties. *J. Geophys. Res.*, **103D**, 1785–1797.

Uijlenhoet, R., J. A. Smith, and M. Steiner, 2003a: The microphysical structure of extreme precipitation as inferred from ground-based raindrop spectra. *J. Atmos. Sci.*, **60**, 1220–1238.

Uijlenhoet, R., M. Steiner, and J. A. Smith, 2003b: Variability of raindrop size distributions in a squall line and implications for radar rainfall estimation. *J. Hydrometeor.*, **4**, 43–61.

Wand, M. P. and M. C. Jones, 1995: *Kernel smoothing*. Chapman & Hall, London.