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A 3DVAR METHOD FOR DOPPLER RADAR WIND ANALYSIS WITH RECURSIVE FILTER

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1. INTRODUCTION

Dual-Doppler wind analysis methods have greatly aided our understanding of weather phenomena ranging from mesoscale convective complexes to clear air boundary layers (e.g., Doviak et al. 1976; Brandes 1977; Ray et al. 1981; Testud and Chong 1983; Chong et al. 1983a,b; Ziegler et al. 1983; Parsons and Kropfli 1990; Atkins et al. 1995; Dowell and Bluestein 1997; Sun and Crook 1997,1998; Shapiro and Mewes 1999). However, most of these traditional methods still suffer from notable deficiencies, including the setting of often-arbitrary vertical velocity boundary conditions, spatial interpolation and discretization errors, uncertainties in radial wind estimates, and the non-simultaneous nature of the measurements. The most pronounced difficulties among the above concern the vertical velocity boundary conditions and interpolation procedure, especially in data voids.

In order to address the vertical velocity boundary problem, Gao et al. (1999) proposed a variational method that performs analysis in a Cartesian coordinate and permits flexible use of radar data in combination with other information (e.g., soundings, or a vertical profile obtained with the VAD method). Furthermore, it allows for the use of mass continuity and smoothness constraints by incorporating them into a cost function. In particular, by applying the anelastic mass conservation equation as a weak constraint, the severe accumulation of error in the vertical velocity can be reduced because the explicit integration of the anelastic continuity equation is avoided. The method performs well in both idealized OSSE and real data cases. However, there exist some difficulties in specifying the optimal weighting for each constraint. In the previous study, weights were selected based on past experience and repeated tuning experiments. Finding the optimal weighting for each constraint may require many numerical experiments.

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In the present study, a new technique, based on a standard 3D variational (3DVAR) approach, is proposed. In it, a background error covariance matrix, though simple, is modeled by a recursive filter, and the square root of the matrix is used for preconditioning. Using the recursive filter is a simple and efficient way to spread the effect of each radar observation to the analyzed grid points (Wu et al. 2002). Recent developments with spatially recursive filters (Purser et al. 2002) enable the construction of a variational analysis in physical space, which allows more degrees of freedom in defining the error statistics adaptively. The eventual goal is to have an analysis system with inhomogeneous and generally anisotropic three-dimensional background error covariance. Compared to the smoothness constraint used in Gao et al (1999), the recursive filter is more effective in achieving the desired spreading of observations to nearby grid points, and theory (Purser et al. 2002) provides guidance for specifying filter coefficients.

2. DESCRIPTION OF VARIATIONAL METHODOLOGY

a) Formulation of cost function

The basic cost function *J*, may be written as the sum of two quadratic terms:

$$J(x) = \frac{1}{2}(x - x^b)^T B^{-1}(x - x^b) + \frac{1}{2}(H(x) - y^o)^T R^{-1}(H(x) - y^o) + J_c. \qquad (1)$$

The first term measures the departure of the analysis vector, x from the background x^b, which is weighted by the inverse of the background error covariance matrix B^{-1}; the second term measures the departure of the projection of analysis to the observation space, $H(x)$, from the observations themselves (y^o), which is weighted by the inverse of the combined observation and observation-operator error covariance matrix, R^{-1}. J_c may include penalty terms added to the system and may play important roles to correlate certain analysis variables.

The analysis seeks to find a model state x^a for which J is at a minimum. At the minimum, the derivative of J vanishes. The Hessian of $J(x)$ is:

$$\nabla^2 J(x) = B^{-1} + H^T R^{-1} H + \nabla^2 J_c. \qquad (2)$$

If $\nabla^2 J(x)$ is positive definite, then there exists a unique x^a that minimizes the cost function $J(x)$.

By defining $v = \sqrt{B^{-1}}(x - x^b) = C^{-1}\delta x$, (3)

and letting,

$$H(x) = H(x^b + \delta x) \simeq H(x^b) + \mathbf{H}\,\delta x, \quad (4)$$

we obtain a new representation of the incremental cost function:

$$J_{inc} = \frac{1}{2}v^T v + \frac{1}{2}(\mathbf{H}Cv - d)^T R^{-1}(\mathbf{H}Cv - d) + J_{cinc}. \quad (5)$$

The Hessian of J_{inc} is

$$\nabla^2 J_{inc}(x) = \mathbf{I} + C^T \mathbf{H}^T R^{-1}\mathbf{H}C + \nabla^2 J_{cinc}, \quad (6)$$

where **I** stands for the identity matrix. Comparing (6) with (2), we see that the smallest eigenvalue of the Hessian matrix from (6) will be at least larger than unity, so that the condition number will not become infinite (Courtier, 1997). This new Hessian matrix is much better conditioned than the Hessian matrix of original problem (1).

The matrix C defined in (5) is realized as,

$$C = DF, \quad (7)$$

where D is a diagonal matrix of the standard deviation of the background error. For simplicity, we assume that D has diagonal elements specified by the error estimation of numerical experimentation. F is a recursive filter (Hayden and Purser 1995, Lorenc 1992) defined by

$$\begin{aligned} Y_i &= \alpha Y_{i-1} + (1-\alpha)X_i \qquad for \;\; i = 1,\cdots,n \\ Z_i &= \alpha Z_{i+1} + (1-\alpha)Y_i \qquad for \;\; i = n,\cdots,1 \end{aligned} \quad (8)$$

where X_i is the initial value at grid point i, Y_i is the value after filtering for i=1 to n, Z_i is the initial value after one pass of the filter in each direction and α is the filter coefficient. This is a first-order recursive filter, applied in both directions, to ensure zero phase change. Multi-pass filters are built up by repeated application of (8). This filter is applied in all three directions.

In our variational dual-Doppler wind analysis, the first term, J_B, in the cost function measures how close the variational analysis $x = (u, v, w)$ fits the background fields $x_b = (u_b, v_b, w_b)$. The background may be provided by a previous model forecast, a nearby sounding, or a wind profile from another Doppler radar analysis program, such as the Velocity-Azimuth-Display (VAD) method (Browning and Wexler, 1968). J_O is the difference between the analyzed radial velocity, which can be approximated (for distance between radar site and data point r less than 100 km) as,

$$V_r = \frac{(X - X_0)u + (Y - Y_0)v + (Z - Z_0)w}{r}, \quad (9)$$

and the observed radial velocity $y_0 = V_{rob}$. The forward operator, $H(x)$, in this case is represented by equation (13) and a linear interpolation operator that maps V_r from the grid (Cartesian coordinates) to observation points (spherical coordinates); and u, v, and w are wind components in Cartesian coordinates (X, Y, Z). (X_0, Y_0, Z_0) is the radar location.

The third term, J_c, can be expressed as

$$J_C = \frac{1}{2}\lambda_c D^2, \quad (10)$$

which imposes a weak anelastic mass continuity constraint on the analyzed wind field, where

$$D = \frac{\partial \bar{\rho} u}{\partial x} + \frac{\partial \bar{\rho} v}{\partial y} + \frac{\partial \bar{\rho} w}{\partial z}, \quad (11)$$

and $\bar{\rho}$ is the mean air density at a given horizontal level. The weighting coefficient, λ_C, controls the relative importance of this penalty term in the cost function and can be specified based on radar observation (Gao et al. 2003).

3. EXPERIMENT RESULTS

To demonstrate the effectiveness of the described variational method for real data, we will apply it to the 17 May 1981 Arcadia, Oklahoma supercell storm (Dowell and Bluestein, 1997). Twelve coordinated dual-Doppler scans were obtained from the Norman and Cimarron, Oklahoma S-band Doppler radars over a one-hour period spanning the pre-tornadic phase of the storm. The analysis is performed in Cartesian coordinates with 83×83×37 grid points. The grid spacing in the horizontal is Δx = Δy = 1000 m, and in the vertical is Δz = 500 m. The standard deviation of errors for the radar radial velocity is set to 1 ms^{-1}, and the standard deviation of background errors (only a single sounding is used in this analysis) is taken to be 10 ms^{-1}. The analysis domain and the relative positions of the two radars are shown in Fig. 1.

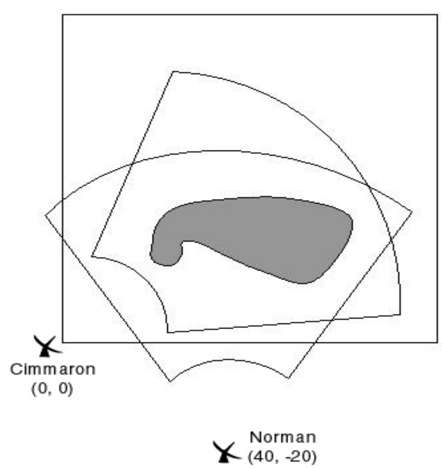

Figure 1. Locations of the Cimarron (0, 0) and Norman (40.0, -20.0) radars that observed the May 17th, 1981 Arcadia storm (shading) and the analysis domain (box).

The results of our analysis for 1641 CST on 17 May are shown in Fig. 2. At low levels (Fig. 2a), a cold outflow originates from rear flank downdrafts that exhibit two maximum centers flanking the occlusion point of the gust front. To the south and east of this region is associated surface convergence and positive vertical velocity. The reflectivity field shows a hook-echo pattern that is roughly consistent with the retrieved flow. Such a flow structure is typical of a tornadic supercell storm with strong low-level rotation (e. g., Lemon and Doswell 1979).

In a vertical slice through line A-B, a very narrow and strong downdraft is located at the center of supercell between the ground and 4.5 km altitude and is surrounded by a ring of updraft (Fig. 2b). This phenomenon agrees with Klemp's theoretical illustration (Fig. 3 in Klemp 1987). Because this analysis is valid 25 minutes before the tornado occurred, the large horizontal shear caused by the narrow downdraft and surrounding updraft could be the source of development of the subsequent tornadic vortex. At 12-km height, a maximum vertical velocity is retrieved and is roughly collocated with the center of maximum reflectivity. These features suggest that both horizontal and vertical flows are kinematically consistent. They qualitatively agree with those analyzed in Dowell and Bluestein (1997) and Gao et al. (1999).

4. SUMMARY

In traditional dual-Doppler analysis, the need for explicitly integrating the mass continuity equation, as well as including 'hole-filling' procedures, increases the solution sensitivity to boundary condition uncertainties. In addition, the separate interpolation from radar observation data points to analysis grid points in traditional methods can introduce errors. In this paper, we developed and tested a variational analysis scheme that is capable of retrieving and analyzing three-dimensional winds from dual-Doppler observations of convective storms. With our method, the horizontal and vertical wind components are analyzed together by adding to the cost function a weak constraint of anelastic mass continuity. The use of a weak instead of strong constraint also leads to procedural simplicity in that the explicit solution of an elliptic equation that would arise from the use of a strong constraint is avoided. The latter tends to be sensitive to the specification of boundary conditions. This finding also agrees with Gao et al. (1999). The application of the method to a supercell tornadic storm case is very promising.

It is our plan to generalize our variational analysis procedure to include additional data sources, and to introduce dynamic constraints in the cost function so that thermodynamic fields are retrieved simultaneously with the winds. This procedure is expected to further improve the wind analysis.

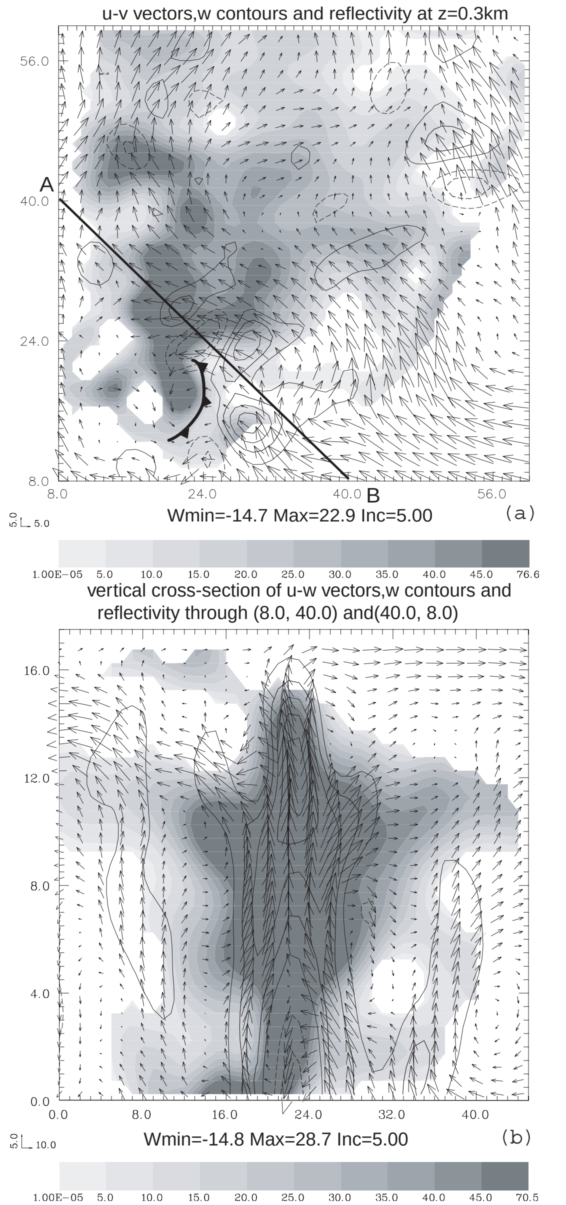

Figure 2. Wind vectors, vertical velocity (contours) analyzed from data sampled by two Doppler radars (located at Norman and Cimarron of Oklahoma) using the variational method for Arcadia, OK at 16:34 CST, 17 May 1981 tornadic storm. Also shown as shaded contours of the reflectivity field. a) Horizontal cross-section at z = 0.3km; b) Vertical cross-section through A-B line in panel a). Rear flank gust front at this level is indicated by the cold front symbol in a). Radar observations are only available where there is reflectivity shading.

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References

Atkins, N. T., T. M. Weckwerth, and R. M. Wakimoto, 1995: Observations of the sea-breeze front during CaPE. Part II: Dual Doppler and aircraft analysis. *Mon. Wea. Rev.*, **123**, 944-969.

Brandes, E. A., 1977: Flow in severe thunderstorms observed by dual-Doppler radar. *Mon. Wea. Rev.*, **105**, 113-120.

Browning, K. A., and R. Wexler, 1968: A determination of kinematic properties of a wind field using Doppler radar, *J. Appl. Meter*., **7**, 105-113.

Chong, M., J. Testud, and F. Roux, 1983a: Three-dimensional wind field analysis from dual-Doppler radar data. Part II: Minimizing the error due to temporal variation. *J. Climate Appl. Meter*., **22**, 1216-1226.

___, ___, and ___, 1983b: Three-dimensional wind field analysis from dual-Doppler radar data. Part III: Minimizing the error due to temporal variation. *J. Climate Appl. Meter*., **22**, 1227-1241.

Courtier, P., 1997: Dual formulation of four dimensional variational assmilation. *Q. J. R. Meteorol. Soc*., **123**, 2449-2461.

Doviak, R. J., P. S. Ray, R. G. Strauch and L. J. Miller, 1976: Error estimation in wind fields derived from dual-Doppler radar measurement. *J. Appl. Meteor*., **8**, 249-253.

Dowell, D. C., and H. B. Bluestein, 1997: The Arcadia, Oklahoma, storm of 17 May 1981: Analysis of a supercell during tornadogensis. *Mon. Wea. Rev*., **125**, 2562-2582.

Gao J., M. Xue, A. Shapiro, and K. K. Droegemeier, 1999: A variational analysis for the retrieval of three-Dimensional mesoscale wind fields from two Doppler radars, *Mon. Wea. Rev.,* **127**, 2128-2142.

Gao J., M. Xue, K. Brewster, and K. K. Droegemeier, 2003: A Three-dimensional Variational Data Analysis Method with Recursive Filter for Doppler Radars. (submitted).

Hayden, C. M. and R. J. Pursur, 1995: Recursive filter for objective analysis of meteorological fields: applications to NESDIS operational processing*, J. of Appl. Meteor*., **34**, 3-15.

Klemp, J. B., 1987: Dynamics of tornadic thunderstorms. *Ann. Rev. of Fluid Mech.*, **19**, 1-33.

Lemon, L. R., and I. C. A. Doswell III, 1979: Severe thunderstorm evolution and mesocyclone structure as related to tornadogenesis. *Mon. Wea. Rev.*, **107**, 1184-1197.

Lorenc, A. C., 1992: Iterative analysis using covariance functions and filters, *Quart. J. of Royal Meteor. Soc*., **118,** 569-591.

Purser, J., W-S Wu, D. Parrish and N. M. Roberts, 2002: Numerical aspects of the application of recursive filter to variational statistical analysis. Part I: spatially homogeneous and isotropic Gaussian covariances. *Mon. Wea. Rev*., (accepted).

Parsons, D. B., and R. A. Kropfli, 1990: Dynamics and fine structure of a microburst. *J. Atmos. Sci.*, **47**, 1674-1692.

Ray., P. S., B. C. Johnson, K. W. Johnson, J. S. Bradberry, J. J. Stephens, K. K. Wagner, R. B. Wilhelmson, and J. B. Klemp, 1981: The morphology of several tornadic storms on 20 May, 1977, *J. Atmos. Sci*., **38**, 1643-1663.

Shapiro A., and J. J. Mewes, 1998: New formulations of dual-Doppler wind radar. *J. Atmos. Oceanic Technol.* (in review).

Sun, J., and N. A. Crook, 1997: Dynamical and microphysical retrieval from Doppler radar observations using a cloud model and its adjoint. Part I: Model development and simulated data experiments. *J. Atmos. Sci*., **54**, 1642-1661.

Sun, J., and N. A. Crook, 1998: Dynamical and microphysical retrieval from Doppler radar observations using a cloud model and its adjoint. Part II: Retrieval experiments of an observed Florida convective storm. *J. Atmos. Sci*., **54**, 1642-1661.

Testud, J. D., and M. Chong, 1983: Three-dimensional wind field analysis from dual-Doppler radar data. Part I: Filtering, interpolation and differentiating the raw data. *J. Climate Appl. Meteor*., **22**, 1204-1215.

Wu, W. –S., R. J. Purser, and D. Parrish, 2002: Three-dimensional variational analysis with spatially inhomogeneous covariances, *Mon. Wea. Rev*., **130**, 2905-2916.

Ziegler, C., P. S. Ray, and N. C. Knight, 1983: Hail growth in an Oklahoma multi-cell storm*. J. Atmos. Sci*., **40**, 1768-1791.