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A GENERALIZATION OF NORMALIZED N_0 AND ITS RELEVANCE TO RADAR RAINFALL REMOTE SENSING

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1. INTRODUCTION

Raindrop size distribution (DSD) is formed as complicated microphysical processes in rain, and important for cloud microphysics and radar rainfall remote sensing. The modeling of DSD has been a long-year interest in radar meteorologists. In recent years, the "normalization" of DSD has been studied extensively to simplify the problem of variability of DSD and relations between integral rain parameters (IRPs) (*e.g. Dou et al*., 1999; Testud *et al*., 2001; Illingworth and Blackman, 2002). The normalization is made with the two quantities; normalized N_0 (N_0^*) and either median volume diameter (D_0) or mean volume diameter (D_m), N_0^* being given as a ratio of the 3rd moment of DSD (M_3) to D_0^4 or D_m^4 with a proper scaling. The fundamental concept is that the liquid water content (LWC, proportional to M_3) is the central to the normalization because it has a clear physical meaning and is also important for rainfall remote sensing. We can notice, however, that the weighting function to obtain a "mean" diameter need not be the mass of a drop. Other weighting can be used as well. The parameter N_0^* is modified accordingly. With such generalization, we could get more insight into properties of DSD theoretically and experimentally. In this paper, a concept of the generalization of normalized N_0 is described and relevance to radar rainfall remote sensing is discussed.

2. Definition of generalized N_0*

Let us assume that we have a DSD, $N(D)$, which may be fitted with a functional model (*e.g.,* a gamma model). Regardless of the selection of the model, we have xth moment M_x (x is in general non-negative integer but non-negative, non-integer number can be used as well).

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Assuming the gamma model, $N(D)$ is expressed as:

$$N(D) = N_0 D^m e^{-LD} = M_0 \frac{L^{m+1}}{G(m+1)} D^m e^{-LD} \qquad (1)$$

The moment M_x is given by

$$M_x = N_0 \frac{G(m+x+1)}{L^{m+x+1}} = M_0 \frac{G(m+x+1)}{L^x G(m+1)} \qquad (2)$$

If we fix m, the scaling parameter L can be obtained from the two different DSD moments, M_x and M_y as follows:

$$L_{xy} = \left(\frac{M_x}{M_y} \frac{G(m+y+1)}{G(m+x+1)} \right)^{\frac{1}{y-x}} \qquad (3)$$

where L_{xy} indicates the L derived from M_x and M_y. If $y - x = 1$, M_y/M_x can be regarded as "M_x-weighted" mean drop diameter, D_{mx}, and the ratio of the gamma function in Eq.3 ($G(m+y+1)/G(m+x+1)$) becomes $m+x+1$. Thus, in the case of $y - x = 1$, we have the expression:

$$L_x = \frac{m+x+1}{D_{mx}} \qquad (4)$$

where L_x indicates the L derived from M_x and M_{x+1}. When $x = 3$, it is the same as D_m used in Testud *et al.* (2001). We focus our attention to the cases where $y - x = 1$ to simplify the discussion.

We next try to derive the expression of DSD with D_{mx} and corresponding "generalized N_0*". Noting that the relation between N_0 and M_x (Eq.2), $N(D)$ is expressed as:

$$N(D) = N_0 D^m e^{-LD} = M_x \frac{L^{m+x+1}}{G(m+x+1)} D^m e^{-LD} \qquad (5)$$

Also M_x is related to D_{mx} and N_0 (or N_{00} in the case of $m = 0$, where N_{00} stands for N_0 in the exponential DSD model).

$$M_x = N_0 \frac{G(m+x+1)D_{mx}^{m+x+1}}{(m+x+1)^{m+x+1}} = N_{00}\frac{G(x+1)D_{mx}^{x+1}}{(x+1)^{x+1}} \qquad (6)$$

Thus it is reasonable to define a "generalized N_0*", N_{0x}* as

$$N_{0x}^{*} \equiv \frac{(x+1)^{x+1}}{G(x+1)} M_x D_{mx}^{-(x+1)} \qquad (7)$$

The N_0* defined in Testud *et al*. (2001) is the N_{0x}* with $x = 3$. (Note that it is basically defined only with LWC and D_m.) Using the parameters D_{mx} and N_{0x}*, and letting $x_1 = x + 1$, $N(D)$ is expressed as

$$N(D) = N_{0x}^{*} F_{mx}(D/D_{mx}),$$
$$F_{mx}(X) \equiv \frac{G(x_1)}{x_1^{x_1}} \frac{(m+x_1)^{m+x_1}}{G(m+x_1)} X^m e^{-(m+x_1)X} \qquad (8)$$

This is consistent with Eqs.16 and 17 in Testud *et al*. (2001). In this "generalized" version, N_{0x}* is recognized as the intercept parameter of the exponential-fitted DSD of interest having the same xth moment M_x and M_x-weighted mean diameter D_{mx}.

3. Moment relations from disdrometer data

It is anticipated that the characteristics of normalized DSD thus obtained, and relations between normalized IRPs depend on the value of x. In Fig. 1, scattergrams between original R and Z, and normalized R and Z (R/N_{0x}* and Z/N_{0x}*) are plotted for $x = 0$ to 5 using the disdrometer data with 3-min integration, having drop counts > 200 and R > 1 mm/h, measured at Gadanki, south India, between September 1997 to December 1999, where seasonal variation of DSD properties is very significant (Kozu *et al.,* 2001). It is clear that as x approaches 3 or larger, the effect of the normalization to reduce the scatter is clear. The best correlation is obtained when x is around 4, although $x = 3$ also gives an excellent fit. We should note that the effect of normalization depends on the IRPs of interest and $x = 4$ would not be the "universal" optimum value.

4. Normalized M_i-M_j relation: Gamma DSD

Using Eq.7 and eliminating D_{mx}, we have the relation between DSD moments M_i and M_j normalized with N_{0x}*, *i.e.* $M_{ix}^* \equiv M_i/N_{0x}^*$ and $M_{jx}^* \equiv M_j/N_{0x}^*$:

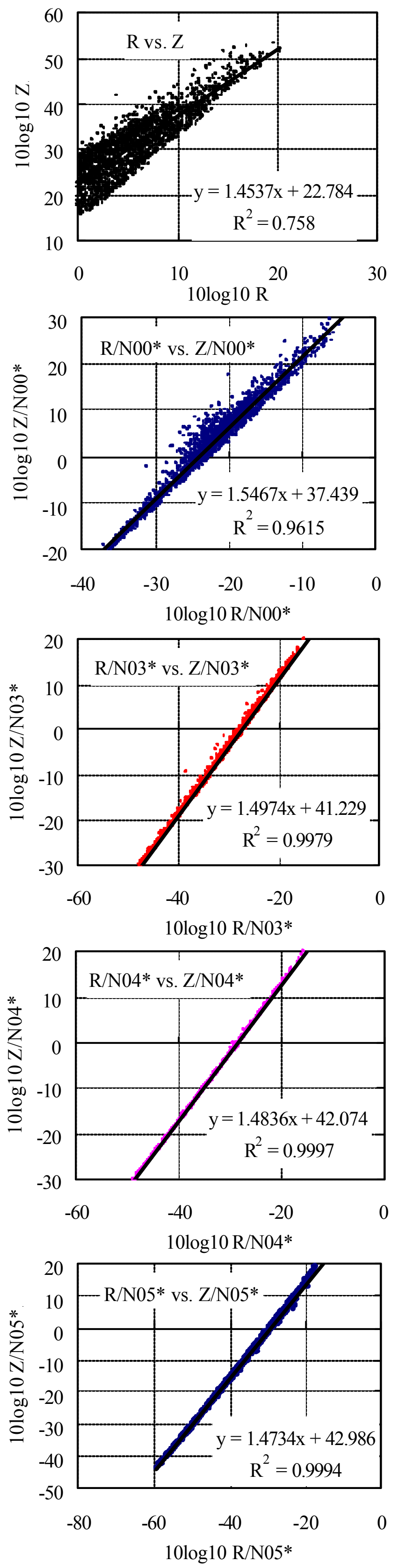

Fig.1. Scattergrams of R and Z, and normalized R and Z with $x = 0, 3, 4,$ and 5.

$$\left(M_{ix}^*\right)^{1/i_1} = C_m(m,x,i,j)\,C_x(x,i,j)\left(M_{jx}^*\right)^{1/j_1} \qquad (9)$$

where $i_1 = i+1, j_1 = j+1$. Letting $x_1 = x+1$,

$$C_m(m,x,i,j) = \frac{G(m+i_1)^{1/i_1}\,G(m+x_1)^{(i-j)/(i_1 j_1)}}{G(m+j_1)^{1/j_1}\,(m+x_1)^{x_1(i-j)/(i_1 j_1)}} \qquad (10)$$

$$C_x(x,i,j) = \frac{x_1^{\,x_1(i-j)/(i_1 j_1)}}{G(x_1)^{(i-j)/(i_1 j_1)}} \qquad (11)$$

Given a set of i, j and x, the normalized moment relation is expressed with a simple power law. The coefficient is the product of two parts; the first one Cm is a function of m, and the second one Cx is constant. Fig.2 shows the dependence of Cm on m from 0 to 8 for the normalized M_6-$M_{3.67}$ relation (almost the same as that for the normalized Z-R relation). It is found that Cm is nearly constant (independent of m) when $x = 4.2$, which is consistent with the finding from the Gadanki disdrometer data.

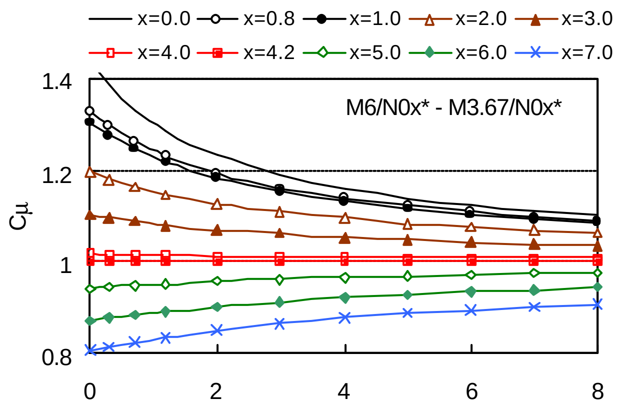

Fig.2. Dependence of Cm on m for normalized M_6-$M_{3.67}$ relation.

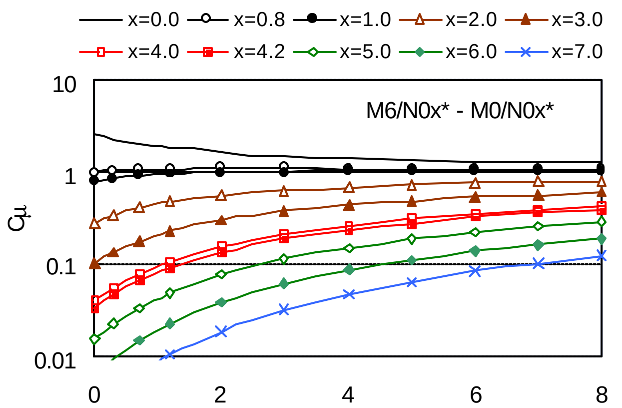

Fig.3. Same as Fig.2 but for M_6-M_0 relation.

As we anticipated above, the "optimum" x in the sense of minimizing the m dependence of Cm is dependent on the values of i and j; when $i = 3$ and $j = 6$, $x = 3.7$ is optimum. We can find an optimum x for any combination of i and j. What does this imply?

If we find the "best" x (x_o), the coefficient CmC_x in Eq.9 becomes independent of m and we can use $m = 0$ instead of general values of m:

$$C_m(m,x_o,i,j)C_x(x_o,i,j) = \frac{G(i_1)^{1/i_1}}{G(j_1)^{1/j_1}} \qquad (12)$$

Eq.12 (and Eq.9 with x_o) implies that the relation between DSD moments normalized by N_{0xo}* has the coefficient independent of m. Finding the value of N_{0xo}* for a DSD (or a set of DSDs) that is determined from M_{xo} and M_{xo+1} is equivalent to defining the M_i-M_j relation. It is noted that the exponent is not necessarily i_1/j_1 since N_{0xo}* is generally a variable.

The reason of the stability of N_{03}*-normalized moment relations originally recognized (Dou *et al.*, 1999; Testud *et al.*, 2001) should be that $x = 3$ approximately satisfies the above optimum condition.

5. Validity of 2 or 3-parameter expression of DSD

What we found from the consideration of moment relations normalized with N_{0x}* and its "optimum" value can be summarized as follows:

(i) Although we have derived N_{0x}* assuming the gamma DSD model, this assumption is not actually necessary because N_{0x}* can be defined from M_x, M_{x+1} and a constant $x_1^{x_1}/G(x_1)$. When x is around 4, we may relate N_{0x}* with physically meaningful rain quantities, *i.e.* rain attenuation coefficients, approximately proportional to higher order moments (Kozu, 1991). The finding from the disdrometer data analysis indicates that DSD can be expressed as a set of three parameters (*i.e.* N_{0x}*, D_{mx} and M_j, or equivalently M_x, M_{x+1} and M_j) for a limited drop diameter domain (significant to higher order moments).

(ii) DSD can even be expressed as two parameters since $x = 5$ also gives an excellent correlation between normalized R and Z in which $x{+}1 = j = 6$.

(iii) With having the three DSD parameters above, we can apply any DSD models. In the case of the gamma model, the variability of the relation between normalized moments appears to be expressed as the variation of the shape parameter m. It is noted that the excellent correlations between normalized Z and R experimentally obtained (Fig.1) are quite consistent with the properties of power law relations assuming the gamma model (Eqs.9-12).

A question is to what extent we can apply the 2 or 3 parameter expression of DSD. A way to examine this is to check the variability of Cm in the relation between lower- and higher-order DSD moments (*e.g.* M_6-M_0), which is shown in Fig.3. We can clearly see that Cm is heavily dependent on m for most x values. When $x = 0.8$ to 1, however, Cm is almost constant over the m values considered. We know that the correlation between M_6 and M_0 is very small in general. How about the correlation between M_0 and M_6 normalized with N_{01}*? A test result using the same disdrometer data is shown in Fig.4 along with the gamma model relations (Eq.9 with i=6, j=0, x=1, m=4). It seems that the normalized M_6 and M_0 are fairly well correlated and consistent with the gamma model; however, we need to be careful about this result because the Joss-disdrometer may be inaccurate to measure M_0, in particular when m is small. The scatter of the measured points from the gamma model line indicates the variation of m or departure of actual DSDs from the gamma model. It is noted that the goodness of the correlation is very sensitive to x; it becomes much worse when the normalization is made with, *e.g.* $x = 0$ or 2.

6. Concluding remarks

The expression of DSD with the parameters (N_{0x}*, D_{mx}, m), through the generalization of N_0* by "generalizing" the weight to obtain mean diameter from D^3 to D^x, is an extension of the original version (N_0*, D_m, m) (Testud *et al*., 2001). In contrast with the original version, N_{0x}* and D_{mx} may not have clear physical meaning. Nevertheless, it can be defined without assuming a specific DSD model and may keep some physical meaning when $x = 4$ to 5, considering that rain attenuation coefficients are approximately proportional to higher order DSD moments. Using an "optimum" x, which is dependent on the order of the DSD moments of interest, the correlation between the normalized moments can become nearly perfect. This finding again confirms the well-known fact of the validity of expressing DSD with 3 parameters, or even with 2 parameters within the limitation of "DSD expression for relating higher order moments". The normalization of DSD with this scheme appears to be useful because it can express the DSD by 3 terms separately; magnitude, scale, and shape, with a proper weighing that can be adjusted depending on the DSD moments (or IRPs) of interest. With the optimization of x, the variability of relations between two different DSD moments can totally be included in the variability of N_{0x}*, which may be useful to simplify radar rainfall remote sensing problems in which many unknown rain parameters are generally involved. More study is needed for the physical meaning of this optimization and how it is reflected into the normalized shape of DSD.

Acknowledgments

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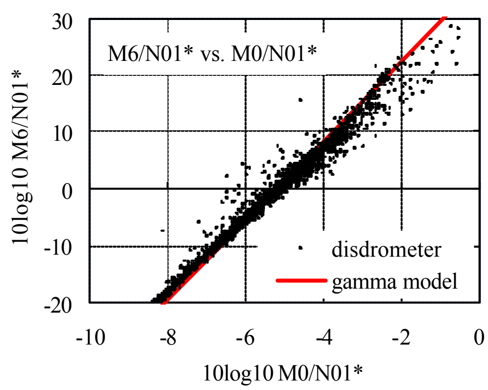

Fig.4. Scattergrams of normalized M_6 and M_0 with $x = 1$, and corresponding gamma model relation with m=4.