14B.1 THE HURRICANE VOLUME VELOCITY PROCESSING METHOD

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1. INTRODUCTION

The seemingly intractable problem of estimating a 2-dimensional, horizontal wind vector from a field of single-Doppler velocity components has been met with much success in the past. The solution is found by applying simplifying assumptions about the particular phenomena being studied. The Velocity Azimuth Display (VAD - Browning and Wexler 1968) and the Volume Velocity Processing (VVP - Waldteufel and Corbin 1979) methods utilize the assumption of linearity; that is, the wind field is estimated from a first-order Taylor series expansion of its components. The VAD method analyzes the Doppler velocity taken around 360° azimuth at a fixed range and altitude from the radar. The VVP method is an extension of the VAD method that includes an analysis of full-volume Doppler velocity data. By explicitly accounting for the vertical variations in the wind field within its framework, VVP provides estimates of many of the kinematic properties of the wind field that are not extractible in the VAD scheme. However, there is no particular reason to expect a given wind field to vary in a linear fashion. Caya and Zawadzki (1992) showed how an existing nonlinear wind field biases the interpretation of the assumed linear VAD coefficients, and proposed a polynomial fit of the VAD coefficients in range and altitude to circumvent the biases and estimate the degree on nonlinearity.

Donaldson and Harris (1989) developed a Doppler velocity model for *nonlinear* wind fields that included combinations of axisymmetric curvature, diffluence and shear that are likely found in hurricanes. The VAD method was then applied to their model for varying radar range, r, and distance from the radar to the hurricane's circulations center, R. They showed that the ratios of the divergence and deformations to the wind speed in cyclonic flow can be estimated from the VAD method with better than 95% accuracy for $r/R < 0.6$. Building upon these results, Donaldson (1991) developed two useful diagnostic indices that measure the degree that a hurricane conforms to a Rankine vortex. Harasti and List (1995) extended this work by deriving explicate expressions for the axisymmetric wind components (wavenumber zero) and their parameters in terms of the VAD coefficients. Harasti and List (2001) improved upon this methodology further in their development of the Hurricane-customized Extension of the VAD (HEVAD) method. The HEVAD method provides estimates of the wavenumber zero component of the horizontal wind field up to 3 km altitude, and it provides vertical profiles of the mean environmental wind at the radar-origin of coordinates.

Lee et al. (1999) developed the Ground-Based Velocity Track Display (GBVTD) method to estimate the horizontal winds of hurricanes relative to the mean environmental wind vector $\mathbf{V_m} = (u_m, v_m)$. For GBVTD, $\mathbf{V_m}$ represents an average over a domain, centered on the circulation center. Harmonic analyses of the Doppler velocity data are performed on rings concentric with the position of the circulation center, rather than on VAD rings concentric with the radar's position. This permits the retrieval of various wavenumber components of the tangential and radial wind (i.e., the asymmetries), including one component of $\mathbf{V_m}$. Extensive tests of GBVTD were performed by Harasti et al. (2003) on Hurricane Bret (1999) with ground-truth provided by a triple-Doppler wind analysis, courtesy of Dodge et al. (2002). The main conclusion was that, given an accurate estimate of the position of the circulation center, GBVTD is able to retrieve the earth-relative horizontal wind field to within 2 ms^{-1} provided that a complete estimate of $\mathbf{V_m}$ is available from an independent source.

Harasti (2002) applied HEVAD to the same case study of Hurricane Bret and argued that the results revealed a strong anti-cyclonic shear in $\mathbf{V_m}$ across the ~200 km horizontal distance between two Weather Surveillance Radar-1988 Doppler (WSR-88D) radars that observed Bret. Upon close comparisons and exchanges of the results between Dodge et al. (2002), Harasti (2002), and Harasti et al. (2003) it was concluded that it was likely that HEVAD was overestimating the anti-cyclonic shear in $\mathbf{V_m}$. This conclusion had diminished the hopes that HEVAD could be used to provide GBVTD with an independent estimate of $\mathbf{V_m}$. The author subsequently developed an alternative method to estimate $\mathbf{V_m}$, which is the main topic of this paper. Applications to the case study of Hurricane Bret are also presented.

2. THE HURRICANE VOLUME VELOCITY PROCESSING (HVVP) METHOD

Just as VVP is the next logical extension of VAD, HVVP applies the nonlinear aspects of HEVAD to an extension of VVP. The HVVP method begins from the modified VVP method presented in Koscielny et al. (1982). Additional Taylor series terms are added to the equations that account for the quadratic variations of the Cartesian wind components, u, v and w. The observed Doppler velocity (V_D) is assumed equal to the sum of the estimated Doppler velocity and the measurement error ε. The estimated Doppler velocity is expressed as the product of two vectors, **P**, the predictors, and **K**,

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the parameters. The system of equations to be solved via a least squares technique is

$$
\left.
\begin{array}{ll}
V_D = \mathbf{P_m^T K_m} + \varepsilon, & \\
\text{where} & \\
P_1 = \cos\theta_e \sin\phi, & K_1 = u_0, \\
P_2 = r\cos^2\theta_e \sin^2\phi, & K_2 = u_x, \\
P_3 = \cos\theta_e \sin\phi\left(z - z_0\right), & K_3 = u_z, \\
P_4 = \cos\theta_e \cos\phi, & K_4 = v_0, \\
P_5 = r\cos^2\theta_e \cos^2\phi, & K_5 = v_y, \\
P_6 = \cos\theta_e \cos\phi\left(z - z_0\right), & K_6 = v_z, \\
P_7 = r\cos^2\theta_e \sin\phi\cos\phi, & K_7 = u_y + v_x, \\
P_8 = r^2\cos^3\theta_e \sin^3\phi, & K_8 = u_{xx}/2, \\
P_9 = r^2\cos^3\theta_e \sin\phi\cos^2\phi, & K_9 = v_{xy} + u_{yy}/2, \\
P_{10} = r^2\cos^3\theta_e \cos^3\phi, & K_{10} = v_{yy}/2, \\
P_{11} = r^2\cos^3\theta_e \cos\phi\sin^2\phi, & K_{11} = u_{xy} + v_{xx}/2, \\
P_{12} = r\cos^2\theta_e \sin^2\phi\left(z - z_0\right), & K_{12} = u_{xz}, \\
P_{13} = r\cos^2\theta_e \cos^2\phi\left(z - z_0\right), & K_{13} = v_{yz}, \\
P_{14} = r\cos^2\theta_e \sin\phi\cos\phi\left(z - z_0\right), & K_{14} = u_{yz} + v_{xz}, \\
P_{15} = \cos\theta_e \sin\phi\left(z - z_0\right)^2, & K_{15} = u_{zz}, \\
P_{16} = \cos\theta_e \cos\phi\left(z - z_0\right)^2 \ \text{and} & K_{16} = v_{zz}.
\end{array}
\right\} \quad (1)
$$

HVVP uses a spherical system of coordinates (r, ϕ, θ_e), where the elevation angle of the radar beam and the altitude at each V_D datum are θ_e and z, respectively, and z_0 is the altitude of the analysis. As Koscielny et al. (1982) explain, the parameters chosen for estimation in (1) can be loosely separated into three groups. First, there are the parameters of interest, namely, K_1 through to K_7. Next there are the parameters necessary for model accuracy that are not specifically of interest in the current application but which must be included so that unbiased estimates of the parameters of interest can be made. For HVVP, these parameters are K_8 through to K_{16}, which represent terms absent from the linear model employed by VVP. Harasti and List (2001) showed that these parameters account for a large proportion of the nonlinearity present in the wind field of a hurricane. Last, there are nuisance parameters excluded from (1) that are not needed for model accuracy. The nuisance parameters for HVVP are the Taylor series terms related to the vertical component of the motion (vertical wind plus fall speed). If the highest elevation angle of the scanning radar is operationally limited, such as the WSR-88D, one must limit the elevation angle used in the HVVP analysis so that the bias due to the omission of the nuisance parameters in the model is kept to a minimum. For the case of HVVP, the empirically found limit is $\theta_e < 3°$, which is consistent with the results of Koscielny et al. (1982). However, if the full span $0° < \theta_e < 90°$ is used in the volume scan then the nuisance parameters can be estimated accurately and need not be removed from the model.

The procedure for HVVP is as follows. The spherical coordinate system of the radar is rotated ϕ_C degrees clockwise from true north, where ϕ_C is the azimuth angle of the hurricane's circulation center. The methods described in Harasti et al. (2003) may be used to estimate the radar-polar coordinates of the circulation center, (R, ϕ_C). The next step is to divide the radar volume scan up into layers of 500 m thickness starting from a value of z_0 = 0.5 km, and extending upward in 100 m increments to the limit that the data permits. Equation (1) is then solved by least squares, yielding vertical profiles of the parameter vector **K** at the radar-origin of coordinates.

3. INTERPRETATION OF THE HVVP PARAMETERS

The rotated coordinate system used by HVVP (also used by HEVAD and Donaldson (1991)) is such that the Cartesian x and y axes at the radar site are parallel to the tangential and radial wind components of the hurricane, V_t and V_r, respectively. Fig. 1 shows the HVVP coordinate system as seen through the x - y plane at any altitude z, with the radar located at **O**. β is the azimuth angle measured from the circulation center at **C**, θ is the elevation angle of the surveillance scan ($\theta_e = \theta + \theta_c$, where θ_c is the angle subtended by the verticals at the radar and data points), and ζ is the radial distance from **C** to the Doppler velocity measurement point.

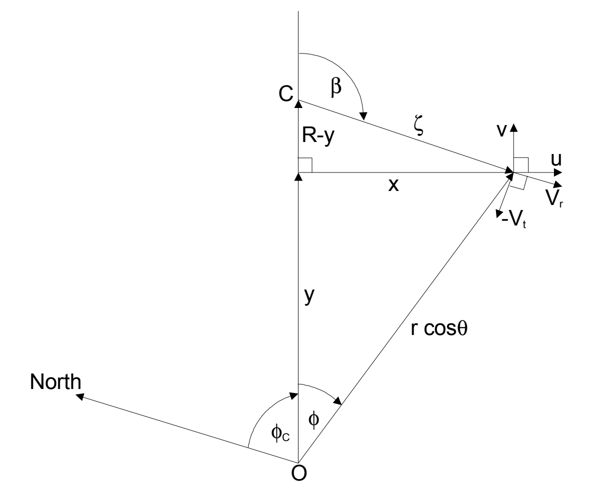

Fig. 1. HVVP geometry through the x - y plane at altitude z.

In order to extract $\mathbf{V_m}$ from the parameter vector **K**, it is necessary to partition u into $u = u_0 - u_m$ and v into $v = v_0 - v_m$. In this way, u and v represent the horizontal components of the wind intrinsic to the hurricane itself. In order to proceed further, it is then necessary to assume a relationship between u, v, V_t and V_r. The current approach utilizes an analytic model for V_t and V_r that includes asymmetries in the β-azimuthal direction superimposed on a modified Rankine vortex:

$$
\left.\begin{aligned}
&V_t(\zeta,z)=V_t(R,z)\ \ [R/\zeta]^{X_t},\\
&V_r(\zeta,z)=V_r(R,z)\ \ [R/\zeta]^{X_r},\\
&\text{where}\\
&V_t(R,z)=\left[1+\sum_n \mu_n\cos n(\beta-\delta_n)\right]V_{t0}(R,z)\\
&\text{and}\\
&V_r(R,z)=\left[1+\sum_n \lambda_n\cos n(\beta-\sigma_n)\right]V_{r0}(R,z).
\end{aligned}\right\}\quad (2)
$$

X_t and X_r are constant exponents that are calculated explicitly in the region $\zeta \geq \zeta_m$, where ζ_m is the radius of maximum wind, which is estimated by the methods described in Harasti et al. (2003). V_{t0} and V_{r0} are the wavenumber zero components of V_t and V_r along the vertical profile (R,z). The asymmetries are expressed as general sinusoidal perturbations with magnitude parameters μ_n and λ_n, and phase parameters δ_n and σ_n. It is expected that $\mu_n < 1$, however, λ_n can be larger than one, particularly when V_r changes sign from one side of the hurricane to the other (e.g. Marks et al. 1992). In heuristic terms, the asymmetries shown in (2) represent the downwind shear of V_t and the crosswind shear of V_r. It is important to note that HVVP only estimates the vertical profiles of $V_t(R,z)$ and $V_r(R,z)$; it does not have the ability to estimate the individual parameters μ_n, λ_n, δ_n and σ_n. These parameters are included in (2) in order to characterize the potential biases in the expressions that follow. It is noteworthy to mention that given an assumed ratio λ_n / μ_n, δ_n and σ_n may be deduced from the GBVTD method, and this information may be useful to HVVP, as explained in the following.

The horizontal wind components are related according to

$$u(\zeta,z)=V_r(\zeta,z)\sin\beta-V_t(\zeta,z)\cos\beta \qquad (3)$$

and

$$v(\zeta,z)=V_r(\zeta,z)\cos\beta+V_t(\zeta,z)\sin\beta\ . \qquad (4)$$

The equations shown in (2) are inserted into (3) and (4), then the parameters K_1 through to K_7 are evaluated at (x, y) = (0,0) with the assistance of the basic trigonometric relations between (ζ, β) and (x, y) that are straightforwardly inferred from Fig. 1. This procedure leads to the expressions

$$V_t(R,z)=R\,K_7\big/\left[X_t+1\right], \qquad (5)$$

$$V_r(R,z)=R\,K_2\,, \qquad (6)$$

$$X_r=-K_5/K_2\,, \qquad (7)$$

$$u_m(z)=u_0-V_t(R,z), \qquad (8)$$

and

$$v_m(z)=v_0+V_r(R,z)\,. \qquad (9)$$

Equations (5)-(9) are unbiased estimates provided that (1) and (2) are adequate models for V_D and the hurricane's wind field, respectively. In addition, (5) is unbiased provided that the contribution to K_7 (the shearing deformation) from the crosswind shear of V_r is negligible, or if this contribution is offset by a bias in the estimate of X_t. Similarly, (6) and (7) are unbiased if the contribution to K_2 (the radial confluence) from the downwind wind shear of V_t is negligible (this potential bias to (6) was also noted by Donaldson 1991).

4. RESULTS AND DISCUSSION

As briefly mentioned above, Hurricane Bret was observed by two WSR-88D radars. These radars are located at Corpus Christi (KCRP) and Brownsville (KBRO), along the Gulf of Mexico coast of Texas. Fig. 2 shows a reflectivity map of Bret as seen by KCRP at 23:42:54 UTC August 22 1999 just as it made landfall. KCRP was located in a region of convective precipitation whereas KBRO was located in a region of shallow, stratiform precipitation (below KCRP's radar beam). As discussed in Harasti (2002), the polar coordinates of Bret's circulation center relative to KBRO and KCRP were determined to be (R, ϕ_C) = (104.97 km, 4.02°) and (R, ϕ_C) = (102.04 km, 171.67°), respectively.

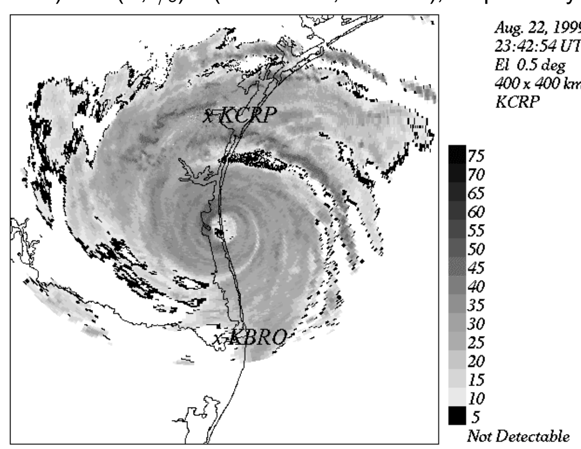

Fig. 2. Reflectivity (dBZ) map of Hurricane Bret derived from the KCRP surveillance scan taken at an elevation angle of 0.5° on 23:42:54 UTC August 22 1999. The locations of KCRP and KBRO are shown ("x" labels).

Equations (5)-(9) do not represent a closed system of equations; a method to estimate X_t is currently under development. The following equations are being accessed for their accuracy in the context of HVVP applied to this case study of Hurricane Bret:

$$X_t=\frac{4Rb_3}{rb_2\cos\theta_e}-3,\qquad V_r>0, \qquad (10)$$

and

$$X_t=\begin{cases}\dfrac{X_r}{2}, & X_r>0,\ V_r<0\\ 1-X_r, & X_r<0,\ V_r<0\end{cases}. \qquad (11)$$

Equation (10) is the expression for X_t used by the HEVAD method, where b_2 and b_3 are the Fourier series coefficients described in Harasti and List (2001). Harasti (2002) showed results of an application of (10) to Hurricane Bret that looked very promising. However, based on shared results between HEVAD and HVVP, and the ground truth provided by Dodge et al (2002), the author is postulating that (10) may only be appropriate in those situations when V_r is weak and/or positive in sign (e.g., in regions of the hurricane where outflow is present). Such was the case encountered by the KBRO radar. The KCRP radar on the other hand was located in a region of strong convection and significant inflow ($V_r < 0$). Although the HEVAD method attempts to minimize the contamination of (10) owing to the vertical motions, it is possible that the combination of strong vertical motions and asymmetries in a region of strong inflow bias (10) beyond that allowed for in the formulation of the HEVAD method. This conjecture should be confirmed with more case studies. In the meantime, an alternative to (10) was derived from a simplification to the axisymmetric tangential momentum equation shown in Willoughby (1995, equation 2.11) by applying the analytic model (2) and comparing the powers of ζ on both sides of the equation for the two cases considered by Willoughby. The result is the two expressions that relate X_t to X_r shown in (11), which are strictly only valid in those situations in which $V_r < 0$.

The proposed procedure to estimate X_t is therefore based on the diagnoses of V_r: if (6) indicates that V_r is weak (say, < 1 ms^{-1} in absolute magnitude) and/or positive in sign, then (10) is used, otherwise, (7) is used to evaluate (11). The only problems foreseen with this approach are that the simplifying assumptions that led to (11) may not always be valid, and a significant downwind shear in V_t may contaminate (6) and (7), and therefore bias the result for (11) as well. However, it may be possible to eliminate this problem by correcting the biases to (6) and (7) from GBVTD estimates of δ_n and σ_n using bias correction expressions (not shown).

The HVVP method was applied to the volume scan data of KCRP and KBRO near 23:43 UTC August 22 1999 using the first three elevation tilts at 0.5°, 1.5° and 2.5°. This volume of data provided HVVP solutions for the 500 m layers between 0.5 and 2 km altitude, which will be shown at the conference. The main conclusion is that HVVP retrieves the vertical profiles of (5)-(11) with greater accuracy than the application of HEVAD to the same case study. The values and trends found within the profiles compare well with extrapolated results from Dodge et al. (2002). The anti-cyclonic shear in $\mathbf{V_m}$ found to exist by HEVAD was also confirmed by HVVP but to a much smaller extent, and with a value closer to what would be expected in the environment. For example, the results for $\mathbf{V_m} = (u_m, v_m)$ derived from KCRP and KBRO at 1.5 km altitude are (–7.56, –0.76) ms^{-1} and (–9.06, –3.02) ms^{-1}, respectively. The differences between these results and Dodge et al. (2002) are, respectively, (–0.46, 1.84) ms^{-1} and (–1.96, –0.42) ms^{-1}. These results from Hurricane Bret are very encouraging and suggest that HVVP may provide sufficiently accurate vertical profiles of the wind field components in the lower troposphere not only for use with GBVTD but also for input into numerical weather prediction models. Future work will include the evaluation of HVVP in different situations, both using a Doppler velocity model for (2) and in a wide range of real hurricane data sets.

5. ACKNOWLEDGEMENTS

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