Nonlinear Retrieval of Single Doppler Radar Wind Field Using Wavelet

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Abstract

In this paper, we develop a novel approach to nonlinearly retrieving wind field from radial wind data, which are obtained by Single Doppler radar. In general, it is impossible to uniquely recover the 2-D (or 3-D) wind field from single Doppler radar, since the problem is underdetermined. In order to overcome the difficulty, according to the characteristic of the wind field distribution, we add the piecewise stationary assumption of the wind field function for the wind retrieval. Furthermore, when the wavelets are applied to approximate the wind field, we found the additional assumption yields the sparse coefficients. This connects the introduction of the entropy function, because their minimization results sparse solution. The corresponding nonlinear optimization can be solved by a modified affine scaling methodology, which is at a lower computational cost due to the behavior of the affine scaling technique. The real experiments demonstrate that the algorithms performs well.

1 INTRODUCTION

Modern Doppler radars have the ability to scan large volumes of atmosphere at high spatial and temporal resolutions, but the direct measurement is limited to the velocity component in the direction of the radar beam. However, in general, the real wind fields are necessary for any wind products. So, there exists a lot of interest for retrieving wind field from radar observation. From the view point of mathematics, the retrieving from radial observation data is an underdetermined problem, infinitely many solutions exist, and additional criteria must be used to select a unique solution. In a study by Peace et al. (1969), the main hypothesis is that the echo region moves across the radar field of view with little modification of the wind field structure, to such an extent that two radar explorations separated in time can be viewed as a simultaneous exploration by two distant radars. Lhermitte and Atlas (1961) suggested the VAD (Velocity Azimuth Display) method and showed how the mean horizontal wind magnitude and direction can be retrieved from radial velocity data around horizontal circles centered along the vertical axis of the radar site. This and other similar techniques, such as volume velocity processing (VVP) (Waldteufel and Corbin,1979), extended VAD (Srivastava et al., 1986; Matejka and Srivastava, 1991), and double VAD (Scialom and Testud, 1986) assume linearity in the observed wind fields and are best suited for the estimation of large-scale wind fields in a stratiform cloud system. The velocity azimuth processing method (VAP)is provided to retrieve the horizontal wind field (Tao, 1992), assuming that the wind vector is equal at a neighboring azimuth. Another single-Doppler analysis technique has been proposed utilizing the temporal correlation of reflectivity and velocity signatures to indicate horizontal wind (Smythe and Zrnic 1983; Tuttle and Foote 1990). Recently, single-Doppler analysis techniques with integrated dynamic models have been developed for the retrieval of thermodynamic parameters in addition to the full wind, such as retrieval by the adjoint technique with a full numerical model (Sun et al., 1991; Liou et al., 1991), retrieval by the simple adjoint method (Qiu and Xu; 1992; Xu et al., 1994, 1995) etc, and others. These methods contribute to the advancement of single-Doppler wind retrieval and show promise for increasing the amount of data recoverable from single-Doppler dada, but they bring extra assumptions associated with the numerical models, which may degrade the effective of the retrieved wind.

Of the above methods, the VAD and VVP methods are used widely. But theirs resolution is affected by non-linearity, such as wind tangent, tornado in the wind field.

In order to provide a more suitable wind retrieval method, we present a nonlinear retrieval idea, based on wavelet analysis and non-smooth optimization. Nonlinear retrieval is an application of nonlinear approximation. In fact, instead of just representing signals with traditional linear approximation, of particular interest to us is the approach of using an overcomplete dictionary to represent a signal, named nonlinear approximation. The motivation for such an approach is that a minimal spanning set of basis vectors is usually only adequate to efficiently represent a small class of signals while forming an overcomplete dictionary using a carefully chosen set of redundant basis vectors that can represent a larger class of signals compactly. Now, let us consider retrieving wind field, as we mentioned above, additional assumption is needed for the uniquely recovery wind field. However, what kind of hypothesis to add is nature? It is well-known that, in general the wind field is randomly stationary in the most of space, even in the extent of time, although there exist some abruptly wind variance at some domains. So, we can assume that the really wind field is piecewise stationary, which is relatively weak constraint and reflect the characteristic of wind field. On the other hand, we found that [18], when applying the wavelet to the approximation problem, the corresponding coefficients of the piecewise stationary signal are sparse, i.e., most of coefficients are zero. It is evident to attain the piecewise stationary through putting the sparse constraints on the wavelet coefficients. Hence, in the present paper, we will first use wavelet to expand the wind field, then form a non-smooth optimization problem of the corresponding coefficients via entropy measure, which minimization subject to the linear constraint results in sparse solutions.

2 FORMULA

Wavelets have emerged in the last twenty years as a synthesis of ideas from fields as different as electrical engineering, statistics and computer science. Wavelet transforms have beautiful and deep mathematical properties, making them a well-adapted tool for a wide range of functional spaces, or equivalently, for very different types of data. Let us first consider the case of a wavelet basis. $L^2[0,1]$ is approximated by a multi-resolution analysis, i.e., a ladder of closed subspaces

$$V_{j_0} \subset V_{j_0+1} \subset V_{j_0+2} \subset \cdots \to L^2[0,1], \tag{1}$$

such that

$$\bigcap_{m\in Z} V_m = \{0\}, \overline{\bigcup_{m\in Z} V_m} = L^2(R) \tag{2}$$

and

$$f \in V_m \Leftrightarrow f(2\cdot) \in V_{m-1} \tag{3}$$

with $j_0 \geq 0$, where V_j is generated by 2^j orthonormal scaling functions $\phi_{j,k}, k = 0,\ldots,2^j-1$, such that $\mathrm{supp}\phi_{j,k} \subset [2^{-j}(k-c), 2^{-j}(k+c)]$($c$ does not depend on j). At each level, the orthonormal complement W_j between V_j and V_{j+1} is generated by 2^j orthonormal wavelets $\psi_{j,k}, k = 0,\ldots,2^j-1$, such that $\mathrm{supp}(\psi_{j,k}) \subset [2^{-j}(k-c), 2^{-j}(k+c)]$. As a consequence, the family

$$\bigcup_{j\geq j_0} \{\psi_{j,k}\}_{k=0,\ldots,2^j-1}$$

completed by $\{\phi_{j_0,k}\}_{k=0,\ldots,2^{j_0}-1}$, constitutes an orthonormal basis of $L^2[0,1]$. Now, let us extend the multi-resolution analysis to two dimensions. Assume that we dispose of one-dimensional multi-resolution analysis, i.e., we have at hand a ladder of spaces V_m, and functions ϕ, ψ satisfying (1)-(3), where ϕ_{0n} and the ψ_{0n} are assumed to be the orthonormal. Define

$$\mathbf{V}_m = V_m \bigoplus V_m.$$

Clearly, the $\mathbf{V}_m$ define a ladder of subspaces of $L^2(R^2)$, satisfying (1) and the equivalent, for R^2, of (2). Then we define an orthonormal basis for $\mathbf{V}_m$ as

$$\Phi(x_1, x_2) = \phi(x_1)\phi(x_2)$$

That is

$$\mathbf{V}_m = \overline{\text{linear span } \Phi_{mn}; n \in \mathbf{Z}^2}$$

where

$$\Phi_{mn} = \phi_{mn_1}(x_1)\phi_{mn_2}(x_2).$$

The orthogonal complement $\mathbf{W}_m$ of $\mathbf{V}_m$ in $\mathbf{V}_{m-1}$ is given by the functions $\Psi^l_{mn}, l = 1, 2, 3$ with

$$\Psi^1(x_1, x_2) = \phi(x_1)\psi(x_2)$$
$$\Psi^2(x_1, x_2) = \psi(x_1)\phi(x_2)$$
$$\Psi^3(x_1, x_2) = \psi(x_1)\psi(x_2).$$

Now, let $v_x(x, y), v_y(x, y)$ denote the x and y axis component of 2-D wind field $v(x, y)$ respectively, and $\theta(x, y)$ represents the radial direction of the location (x, y). Since the observation is along the radial, we consider the radial projection $v_r(x, y)$ of the real wind field $v(x, y)$, that is

$$v_r(x, y) = v_x * cos(\theta(x, y)) + v_y * sin(\theta(x, y)).$$

Denote the observation data by $v_r(i, j)$, then the retrieving wind field should be coincided with the observation data. In simplicity, assume the observation is without error, so we have

$$v_x * cos(\theta_{i,j}) + v_y * sin(\theta_{i,j}) = v_r(i, j) \tag{4}$$

where $\theta_{i,j}$ is the wind direction.

We expand the x, y axis components of the wind field using discrete wavelet function

$$v_x(x, y) = \sum_n c_{x,n}\Phi_{Kn}(x, y) + \sum_{l=1}^{3}\sum_n\sum_{m=1}^{K} d^l_{x,mn}\Psi^l_{mn}(x, y) \tag{5}$$

and

$$v_y(x, y) = \sum_n c_{y,n}\Phi_{Kn}(x, y) + \sum_{l=1}^{3}\sum_n\sum_{m=1}^{K} d^l_{y,mn}\Psi^l_{mn}(x, y) \tag{6}$$

where $c_{x,n}, c_{y,n}, d^l_{x,mn}$ and $d^l_{y,mn}$ represent the corresponding wavelet decomposition coefficients, K is the level of wavelet composition.

For brevity of expression, we can rewrite the equations (5),(6) in the matrix form:

$$V_x = A\xi \tag{7}$$

$$V_y = A\eta \tag{8}$$

where A is an orthonomal matrix formed from the Φ_{mn} and $\Psi^l_{mn}, l = 1, 2, 3$, V_x and V_y are corresponding discrete of $v_x(x, y)$ and $v_y(x, y)$, ξ denotes the vectors of the wavelet decomposition coefficients $c_{x,n}, d^l_{x,mn}$, η is for $c_{x,n}, d^l_{x,mn}$. Consequently, (4) is rephrased as

$$\Lambda_1 A\xi + \Lambda_2 A\eta = v_o \tag{9}$$

here Λ_1, Λ_2 are diagonal matrices, in which the diagonal elements are formed from $\theta_{i,j}$, and v_o is the vector form of the observation $v_r(i,j)$.

Furthermore, notice the piecewise stationary assumption on the wind field, so we introduce the minimization of the entropy

$$\sum_{i=1}^{N} |\xi(i)| + \sum_{j=1}^{N} |\eta(i)| = min \tag{10}$$

All in all, the retrieving problem of the wind field can be formulated a non-smooth optimization (10) subject to the constraint (9). The corresponding problem can be solved by means of FOCUSS [20] algorithm.

3 INITIAL RESULTS

In this section, we will use the method presented in above section to retrieve the RHI wind field. A case of intense storm observation was obtained with the Kun Ming city weather radar, Yun Nan Province, China, at 5.4-cm (C band) wavelength at 2029 LST on 13 August 2001. The well-developed storm about 100 km northwest of Kun Ming, moved to southwest and occurred heavy rainfall and hailstone. Figure 1 shows the RHI scan of Doppler wind field at 283o azimuth at 2102:05 LST. At this time the storm was in the most intensive stage, the clouds extended to above 11 km and consisted of distinct generating updraft cell in the 14 km upper regions at range 65 km from radar center. There were two intensive reflectivity regions near the range of 65 km and 78 km, the maximum reflectivity were respectively 49 dbz and 39 dbz. In the Doppler wind field, two main radial velocity convergence regions were found, which coincided with the intensive reflectivity regions. Figure 2 illustrates two dimension wind field retrieving for RHI using wavelets method. The main convergence updraft flow region coincides with the most intensive reflectivity region, and the weak convergence region is formed near the slightly intensive reflectivity region at 78 km. The retrieving maximum vertical updraft speed is 37m/s. It find that the low-level air flow exists convergence and upward motion below 5 km level, and divergence occurred above 5 km.

4 CONCLUSIONS

In this paper, the nonlinear retrieval of single Doppler radar wind field using wavelet is proposed and applied to retrieve the two dimensional wind field from single Doppler radar observations. The result shows that the present method is an effective method for wind retrieving. In the next step we will focus on the use of dual-Doppler analysis methods to verify the availability of wavelet retrieval. In the meantime, from the idea of present methodology, we foresee that it can be used to recovery 3-D wind field.

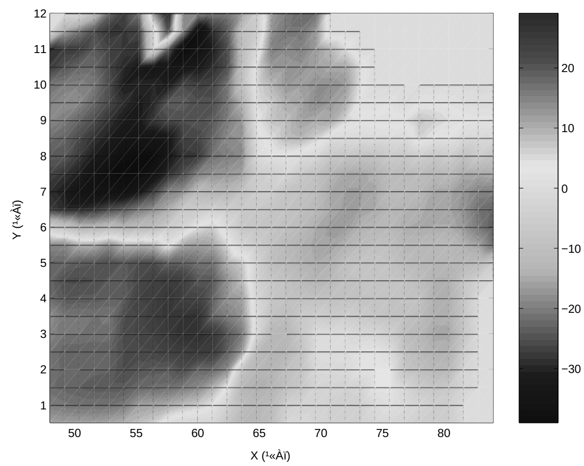

Figure 1: Doppler velocity

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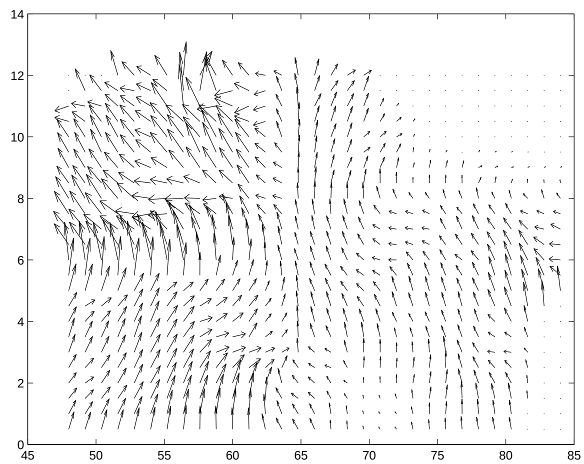

Figure 2: Two dimension wind field retrieving for RHI

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