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A MODEL TO GENERATE STOCHASTIC NOWCASTS OF RAINFALL

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1. INTRODUCTION

Quantitative nowcasts of rainfall are based on the advection of the observed rain field forwards in time. The accuracy of the nowcasts depends on the rate at which the field is developing in Lagrangian coordinates, the accuracy and resolution of the advection estimates, and the accuracy of the radar reflectivity to rainfall conversion. In any event, the accuracy of the nowcasts rapidly becomes pedestrian indeed and the maximum lead-time is generally less than one hour for quantitative rainfall forecasts. An alternative strategy is to admit that it is not possible to forecast rainfall quantitatively at high space and time resolutions and to attempt to provide probabilistic nowcasts either in the form of a probability distribution that is conditioned on the current rain field, or as an ensemble of nowcasts, each equally likely.

This paper reports on a feasibility study to explore the possibility of generating an ensemble of stochastic nowcasts that are conditioned on the current rain field. The next section provides a truncated account of the method used to generate stochastic nowcasts, and the preliminary results based on a rain event in Melbourne are presented thereafter.

2. METHOD

The method used to generate stochastic nowcasts is based on the S-PROG model (Seed, 2002) and due to space limitations only a very brief account of the method will be presented here. The basic premise of S-PROG is that rain fields are scaling and a rain field consists of a hierarchy of features over a wide range of spatial scales, and the lifetime of a feature is a power law of the scale of the feature.

A field (size *L* x *L* pixels) of decibels radar reflectivity can be decomposed into an additive cascade structure,

$$dBZ_{i,j}(t) = \sum_{k=1}^{n} X_{k,i,j}(t) \qquad (1)$$

$$i = 1,...L,\ j = 1,...L,\ L = 2^n$$

where the k^{th} level in the cascade $X_k(t)$ represents (in the spatial not frequency domain) the variability in the original field with frequencies within the range $1/L2^{-k} < w_k < 1/L2^{-(k+1)}$ pixel^{-1} at time *t*.

The *dBZ* field is transformed into the spectral domain by means of an FFT transformation, thereafter each level $X_k(t)$ of the cascade is calculated by using a band-pass filter based on a Gaussian window that passes the appropriate frequencies and calculating the inverse so as to transform each Fourier component of the field back into the spatial domain. Finally, the cascade levels are normalised for convenience using

$$Z_{k,i,j}(t) = \frac{X_{k,i,j}(t) - m_k(t)}{s_k(t)} \qquad (2)$$

where $m_k(t)$ and $s_k(t)$ are the mean and standard deviation of the k^{th} level respectively, and are assumed to be constant throughout the forecast period. The $Z_k(t)$ fields are referred to as "levels" in the cascade and represent the Fourier components transformed into the spatial domain and then normalised to have zero mean and unit variance. Figure 1 shows a field of radar reflectivity and Figure 2 shows the first three levels in the cascade for this image.

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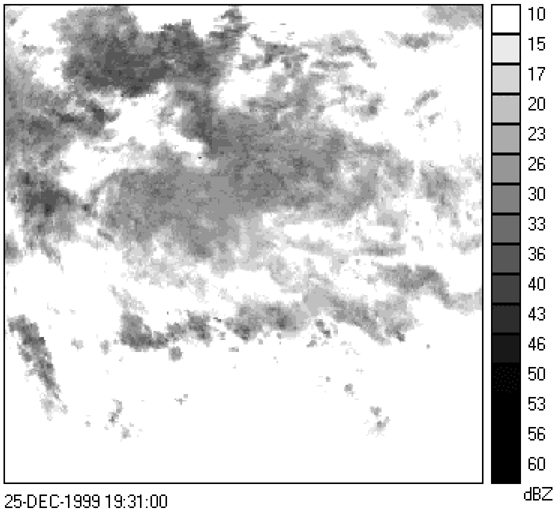

Figure 1. A field of radar reflectivity. The Melbourne (Laverton) radar is at the centre; the image is a 256 km x 256 km field with 1 km resolution data.

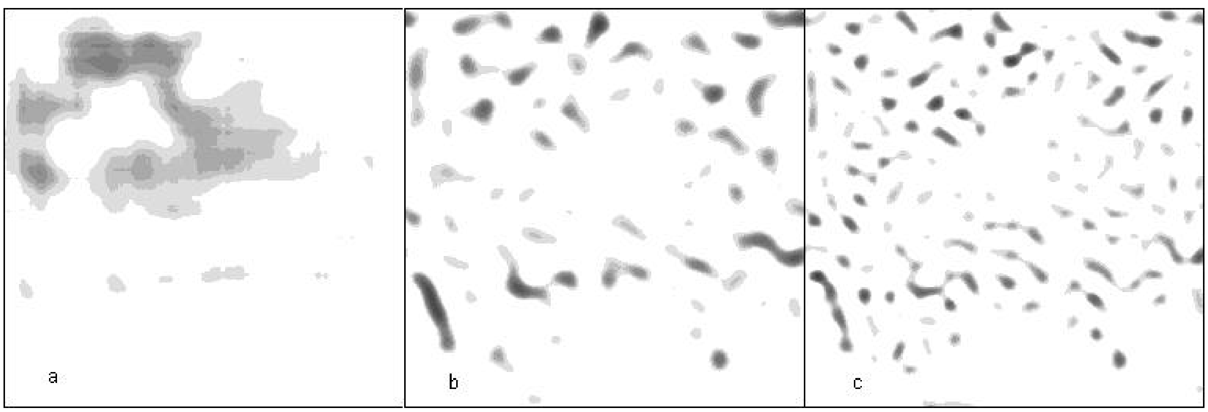

Figure 2. The first three spectral components of the field in Figure 1. a) 128 – 256 km, b) 64-128 km, c) 32 – 64 km wavelengths.

The advection of the field is resolved on an 11 x 11 grid that covers the field, and the advection vector for a particular pixel is calculated using a bi-linear interpolation of the surrounding advection grid points. A hierarchical approach is used to calculate the advection vectors. The average advection over the large scale is calculated by breaking the image into 4 sub-areas. Thereafter, the size of the domain is reduced by a factor of 2 and the advection for this smaller area is calculated if echoes are present in this sub-area, or the advection from the scale above is assigned to that sub-area if there are not enough echoes for tracking. This process is continued until each of the 11 x 11 grid points is at the centre of a sub-area. The 11 x 11 advection vectors are smoothed in time by using a weighted average with the vectors from the previous time step.

The calculation of the advection vectors is based on the optical flow technique, which assumes Taylor's Hypothesis and solves

$$-\frac{\partial dBZ}{\partial t} = \frac{\partial dBZ}{\partial x} v_x + \frac{\partial dBZ}{\partial y} v_y \qquad (3)$$

(Thévenaz, 1990), where *dBZ* is the radar reflectivity field and v_x, v_y are the *x* and *y* velocity components. This is done by using finite differences to estimate the gradients, and then using least squares to estimate v_x, v_y over the sub-area.

The Lagrangian development each field in the cascade is assumed to be able to be modelled by an Autoregressive model whereby the forecast fields for each level in the cascade are built up iteratively using

$$Z_{k,i,j}(t+n+1) = f_{k,1}(t) Z_{k,i,j}(t+n) + f_{k,2}(t) Z_{k,i,j}(t+n-1) + e_{k,i,j}(t+n+1) \qquad (4)$$

The $e_k(t)$ fields are a cascade of stochastic noise and represent the new development that takes place in the rain field during the forecast period. The noise fields have the following properties:

$$Var(e_k(t)) = \frac{1+f_{k,2}(t)}{1-f_{k,2}(t)}\left[\left(1-f_{k,2}(t)\right)^2 - f_{k,1}(t)^2\right]$$

$$E\left[e_k(t)\right] = 0 \qquad (5)$$

(Salas *et al*., 1980).

Finally, the output forecast field at time *t*+*n*+1 is calculated using

$$dBZ_{i,j}(t+n+1) = \sum_{k=1}^{n} m_k(t) + s_k(t) Z_{k,i,j}(t+n+1) \qquad (6)$$

assuming that the m_k (t) and $s_k(t)$ are constant during the forecast period and are estimated at time t, the time that the forecasts are made.

In effect, the forecast is a deterministic forecast based on advection blended with stochastic noise, which progressively dominates the larger scales as the features in the various levels in the cascade perish. The crucial part of the method is to specify an appropriate set of $e_k(t)$ fields at the start of the forecast period and to update these noise fields during the forecast period. At present an *ad hoc* scheme to test the ideas was used to initialise the noise fields. The first level in the noise cascade is generated as a field that has a correlation of 0.7 with the first level in the observed cascade, or 50% of the variance in the noise field can be explained by the observed field. An examination of the observed cascade revealed that there was a weak correlation between pairs of levels in the cascade, so the lower levels in the noise cascade were generated as fields of noise that were correlated with the previous cascade level using the estimated correlation between the observed cascade levels, and smoothed with an appropriate band-pass filter. The noise cascade is updated from one time step to the next in the forecast period using the AR(2) models derived for the observed field (Equation 4), and then advecting the noise using the observed advection field.

3. RESULTS

These ideas were tested using data from a significant widespread rainfall event that was recorded by the Melbourne radar on 22 April 2001. Figures 3 shows an observed field, and Figure 4 shows the 10,20,30,60,90,120 minute forecasts respectively.

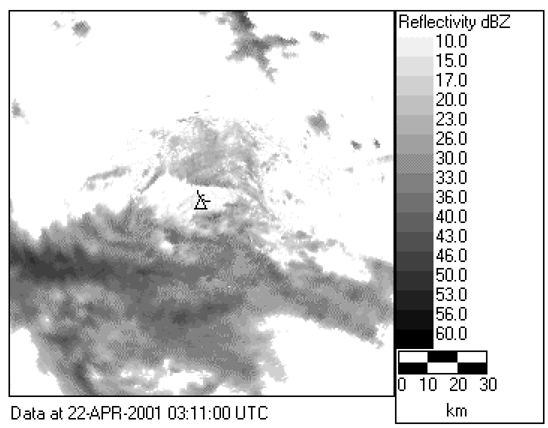

Figure 3. Observed field of radar reflectivity

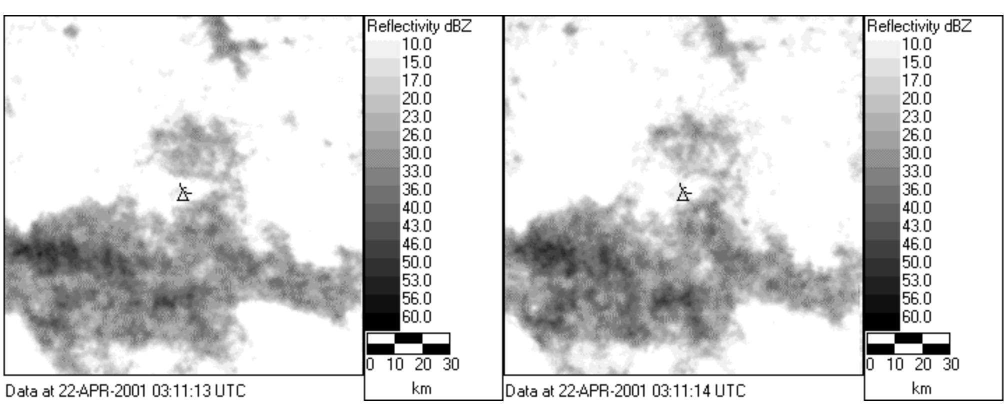

Figure 4a,b 10 minute and 20 minute forecasts

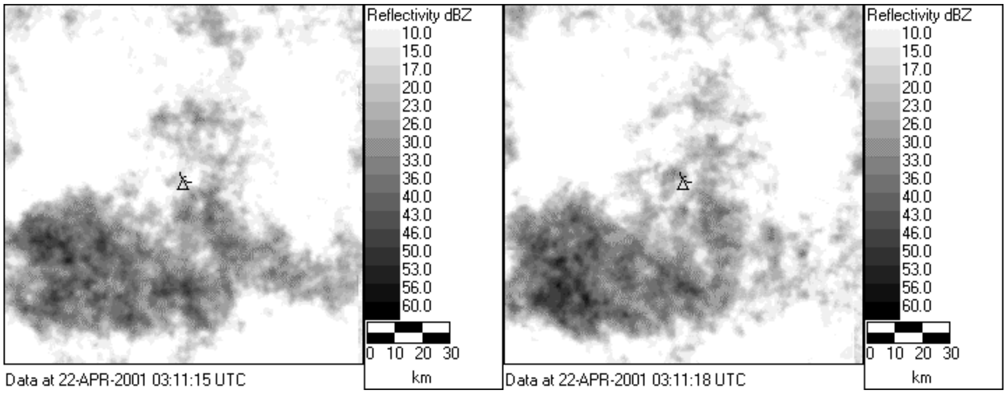

Figure 4c,d 30,60 minute forecasts

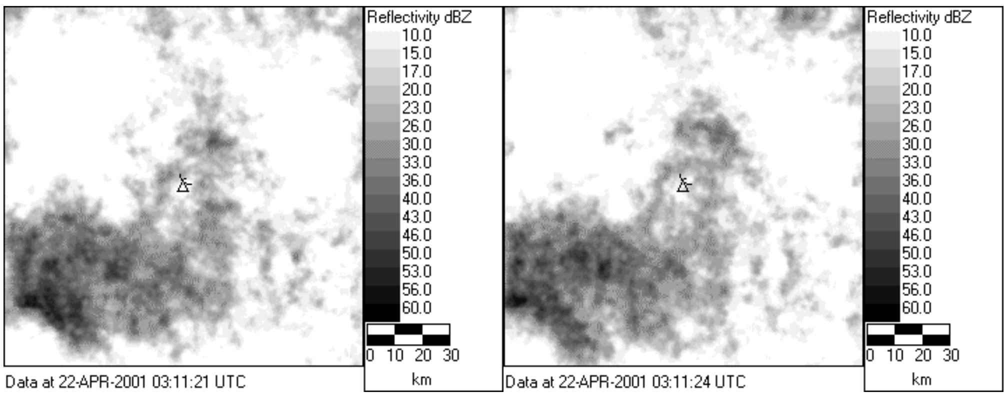

Figure 4e,f 90, 120 minute forecasts

4. REFERENCES

Salas, J.D., J.W. Delleur, V. Yevevich, and W.L. Lane. 1980. *Applied Modelling of Hydrologic Time Series*, Water Resources Publications, 106-117.

Seed, A.W.. 2003. A dynamic and spatial scaling approach to advection forecasting. *J. Appl. Meteor.*, **42**, 381-388.

Thévenaz, P..1990. Motion Analysis. *Pattern recognition and image processing in physics*. Vaughan, R.A.. Ed. Proceedings of the Thirty-Seventh Scottish Universities Summer School in Physics, NATO Advanced Study Institute.