

## **“Rabbit Rules” – An Application of Stephen Wolfram’s “New Kind of Science” to Fire Spread Modeling**

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### **1. INTRODUCTION**

Fire spread is the outcome of complex interactions between fire, fuels, terrain, and weather. Most models created to predict fire spread fall into one of two classes, empirical models and physical models. For empirical models (Finney, 1998), fire spread is measured under controlled conditions and a statistical relationship found between fire spread and each variable tested (Rothermal, 1972). The model typically is represented in two dimensions through predetermined geometry, for example, overlapping ellipses (Alexander, 1985). Empirical models have skill at low to moderate winds. However, at higher wind speeds and under conditions that produce erratic fire behavior, statistical/empirical methods are less skillful because the equations were not derived for such conditions.

Physical models describe fire spread as heat transfer between burning and unburned fuel through coupled differential equations (Clark, et al, 1996; Linn, 1997; Linn and Harlow, 1998). Physical models can explain mathematically how combustion processes in heterogeneous fuels under variable atmospheric condition translate to fire behavior and thence to fire spread. In addition, physical models explain nonlinear processes such as complex fire atmosphere feedbacks that can account for extreme fire behavior. Physical models can generate complex patterns of fire spread over complex terrain in heterogeneous fuels.

The one signal disadvantage of physical models is the enormous number of calculations required to model fire spread. These models are computationally intensive, far beyond the capabilities of today’s desktop PCs, which makes their operational use for the foreseeable future problematical.

An alternative is to recast the fire spread problem to take advantage of the simplicity of the empirical models yet capture the complexity delivered by physical models. The “Rabbit” model described below is an application of this modeling philosophy.

Stephen Wolfram (2002) showed that initial value problems described by complex partial differential equations can be recast as a set of simple computer programs and solved recursively. The computer programs can produce complex solutions such as fractal fire fronts, breaks in fire lines, waves

(or bulges), “heads,” and flanking lines. Wolfram noted (p.109.), “Over and over again the single most important principle that I have learned is that the best computer experiments are ones that are as simple and straightforward as possible. And the principle applies both to the structure of actual systems one studies- and to the procedures that one uses for studying them.”

### **2. THE “RABBIT” MODEL**

The Rabbit Model follows Wolfram’s computer programming strategy, but not cellular automata to model fire spread as done previously by Karafyllidis and Thanailakis (1997) and Berjak and Hearne (2002). In the Rabbit Model, each element, a rabbit, is an autonomous agent (Flakes, 2000) not constrained by the definition of the underlying grid (raster) domain. The rabbit lives, moves, and dies as the outcome of a set of simple intuitive rules each stating a physical process. The rules must be foundationally simple or they will not work.

Because the physical/mathematical problem is recast as a set of simple computer programs, it is advisable to change terminology so that the rules will not be confused with the mathematical and physical statements that describe empirical and physical fire spread models. Therefore, the rules are cast in terms of “rabbit behavior.” Rabbit behavior has an analogy to fire behavior. For example, the fundamental physical principles of fire spread can be stated thusly: fire consumes fuel, fire jumps between adjacent fuel elements, and fire spreads. In terms of rabbit behavior; rabbits eat food, rabbits jump, and rabbits reproduce.

The development of the Rabbit Model to date has emphasized environmental rules for rabbit behavior - terrain and weather. That is because, once food (fuel) conditions are set, these are the major determinants of rabbit (fire) behavior. The examples to follow show that the Rabbit Model indeed has power to explain complex behavior often seen in fire spread.

#### **2.a. The Primary Rules for Rabbit Vitality**

The three Primary Rules for Rabbit Vitality are:

V1) Rabbits eat.

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V2) Rabbits jump.

V3) Rabbits reproduce.

Although these rules are foundational to the Rabbit Model, they need some clarification through secondary rules. For example, in Rule V1, rabbit behavior depends on what rabbits eat and how long it takes to eat it. In Rule V2, the ability to jump depends upon food eaten, local terrain and weather. In Rule V3, a minimum number of rabbits need to be reproduced to maintain the integrity of the model solution at each recursive step but not so many rabbits that the computational load becomes excessive.

## 2.b. The Primary Rules for Rabbit Mortality

Tests with Rule V3 showed that reproduction rates of four rabbits per adult per recursive step is required to maintain the integrity of the solution. After fifteen iterations, the Rabbit Model produced enough rabbits to circle the globe 200 times. Therefore, additional rules must be implemented to reduce the number of rabbits.

The Primary Rules for Rabbit Mortality are:

M1) Rabbits are territorial – a rabbit cannot jump onto a space occupied by another rabbit.

M2) Rabbits starve upon jumping onto a space eaten by another rabbit.

M3) Rabbits starve upon failure to jump off a space they have eaten.

As for the Rabbit Vitality Rules, the Rabbit Mortality Rules are flexible and can be further clarified with secondary rules.

## 2.c. The Secondary Rules

To date, the Secondary Rules identified for the Rabbit Model are:

### Rabbit Terrain

T1) Rabbits enjoy jumping up slope; rabbits are afraid to jump down slope.

T2) Rabbits may fall down steep slopes.

### Rabbit Weather

W1) Rabbits are carried by the wind in proportion to how high they jump and how much they weigh.

W2) Rabbits “hunker down” in rain or snow.

W3) Rabbits abhor high humidity.

### Rabbit Hazards

H1) Rabbits drown when jumping into water.

H2) Rabbits become roadkill when jumping onto roads.

### Rabbit Food

F1) Rabbits gain weight in proportion to the water content of their food.

F2) Rabbits jump in proportion to the height of their food.

F3) Rabbits may delay jumping if food is plentiful.

### Rabbit-Atmosphere Feedbacks

A1) Rabbits “work up a sweat” while eating, jumping and reproducing.

Rule A1 is a rabbit-atmosphere feedback rule

designed to match feedback in coupled fire-atmosphere physical models. Each rabbit gives off heat depending on its vigor. The greater the number of rabbits in close proximity, the greater the heating and the greater the impact on the local winds.

## 3. EXAMPLES FROM THE RABBIT MODEL

### 3.a. Example from the primary rules

Figure 1 shows what happens when the Rabbit Model is run with the Primary Rules. Green areas represent food. Black represents areas where food has been eaten. There is no terrain and no wind. Food distribution is homogeneous.

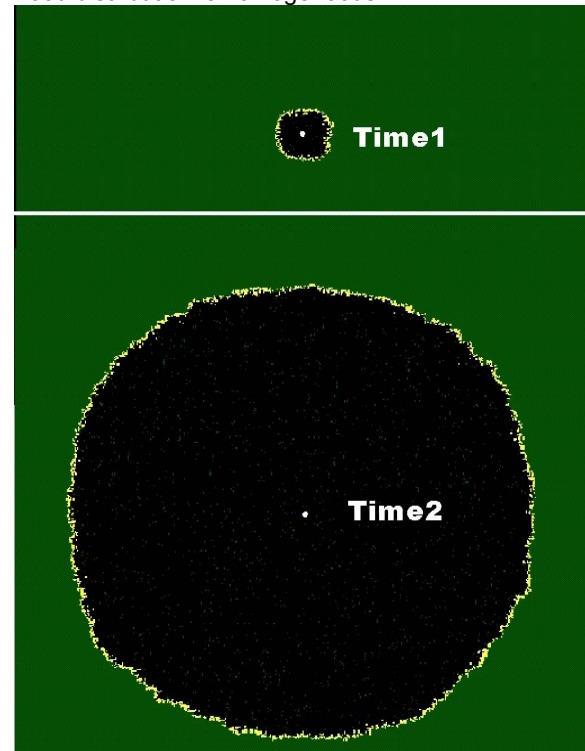


Figure 1. The Rabbit Model for the six primary rules. Time1 is shortly after a single rabbit starts at the white dot. Time2 is later. Time is non-dimensional. Green/black represent uneaten/eaten food. The yellow dots identify rabbits.

The yellow dots represent rabbits that are descendants of a single rabbit initially located at the white dot in Figure 1. Each rabbit can jump in any direction. The height/distance and direction of each jump is determined randomly. The wavy, fractal-like appearance of the “perimeter” of rabbits is an outcome of longer distance jumps and subsequent fill in by offspring.

Though each rabbit is an autonomous agent, it turns out that each rabbit is highly dependent on the location of its neighbors and the history of the spread of rabbits. If a rabbit lands on another rabbit, it is eliminated via Rabbit Mortality Rule M1. If a rabbit lands on a black space, it is removed from the list of

rabbits as a consequence of Rabbit Mortality Rule M2.

The ring of rabbits in Figure 1 is an outgrowth of the recursive application of the Primary Rules. No predetermined geometry, such as an ellipse, or ensemble of ellipses, has been assumed. The spread of rabbits is therefore unconstrained. Therefore the shape of the perimeter of rabbits may become exceedingly complex through the recursive application of the Rabbit Rules.

### 3.b. Rabbit Terrain

Rabbit Rule T1 (rabbits are afraid to jump down slope) can be converted into a simple program that is the product of slope and a coefficient that assigns a "fear factor." If the fear factor is "No Fear", the result is a circular spread pattern as seen in Figure 1. Setting the fear factor to "morbid" yields rabbit behavior shown in Figure 2. There is no wind in these simulations and food distribution is homogeneous.

The Rabbit Model consists of a set of subroutines embedded within the operational smoke prediction model, PB-Piedmont (Achtemeier, 2001). PB-Piedmont uses multiples of the 30-m USGS DEM data for the surface boundary. Figure 2 shows a portion of the elevation map for a mountain range in north Georgia at a 300 m grid interval. Shading interval is 80 m.



Figure 2. Spread of rabbits up a mountain in north Georgia. Elevation difference between mountain and valley is 600 m. Shading interval is 80 m. Fear Factor is "Morbid."

At Fear Factor "Morbid", rabbits will not jump down slope. All that remains is for the rabbits to run up slope as is shown at Time1 and Time2. By Time3, no higher slopes remain. All the rabbits have perished according to the Rabbit Mortality Rules.

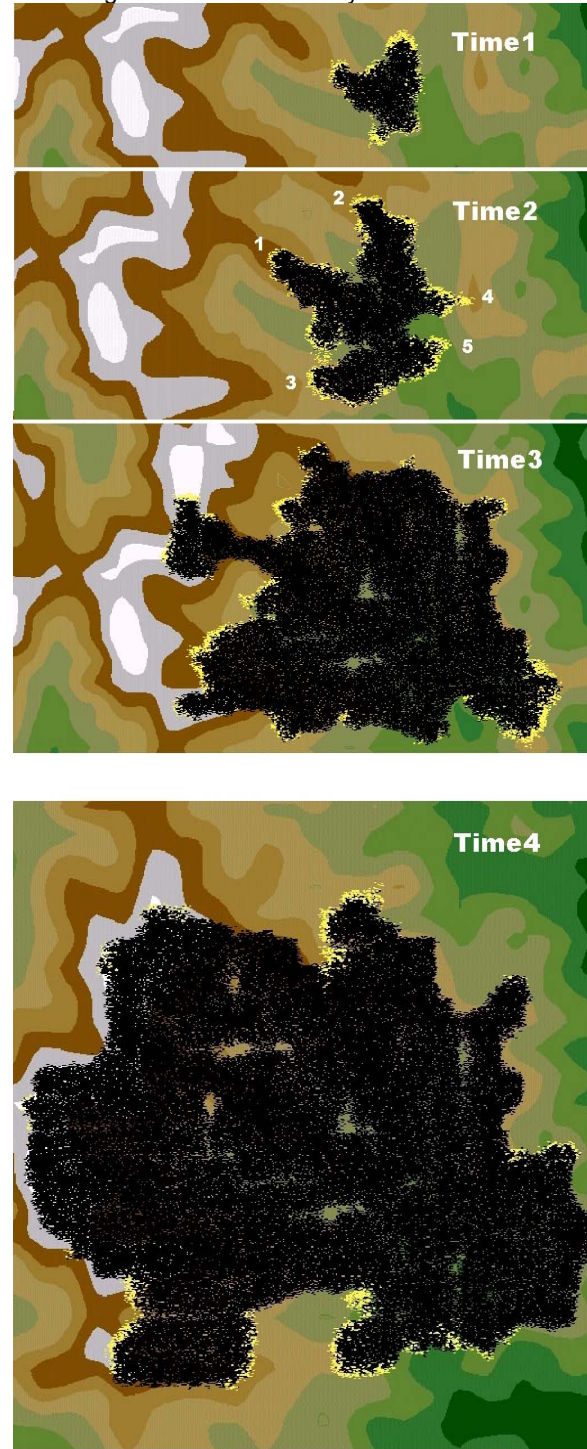


Figure 3. Spread of rabbits in complex terrain in north Georgia. Fear Factor is "Cautious."

Setting the Fear Factor to “Cautious” gives perhaps a more realistic analogy with fire spread in complex terrain. Rabbits will jump down slope if the slope is not too steep. Figure 3 shows the spread of rabbits over the area of the same mountain range as in Figure 2. At Time1, a rabbit run has commenced up the nearby slope as for Figure 2. However, rabbits have also eaten across relatively level ground to start runs toward other up slope terrain. There are five separate runs by Time2, each headed toward high ground. Run 1 has, by Time3, reached the mountain ridge. Other rabbits are approaching the base of the mountain and will commence runs to the ridge. Meanwhile, rabbits have eaten to the ridge on the right hand side of Figure 3. Most have perished as the down slope was too steep to jump.

Figures 2 and 3 show that the Rabbit Model is able to generate rabbit spread of complexity at least equivalent to what can be obtained with parameterized empirical models. The recursive application of simple intuitive rules yields complexity found in physical fire/atmosphere interaction models (see for example, Coen and Clark, 2001).

### 3.c. Rabbit Wind

The potential of the Rabbit model as a rule-driven model of fire behavior/fire spread is demonstrated again by the application of the Rabbit Weather Rule W1 – Rabbits are carried by the wind in proportion to how high they jump and how much they weigh. Figure 4a gives an example of rabbit behavior in strong wind. Shifting wind directions during the period of solution explain the lack of symmetry.

The salient feature of Figure 4a is the ellipse-like shape complete with head and flanking lines. The flanking lines are maintained by rabbits that jump from eaten spaces to adjacent uneaten spaces. The head is maintained both by rabbits jumping to adjacent uneaten spaces and by rabbits that are carried downwind several spaces before landing. Offspring from these rabbits fill out the head to produce the observed thick band of rabbits there.

Results from two other tests using Rabbit Weather Rule W1 and modifications of Primary Rule M2 and M3 are shown in Figure 4. Rule M3 was modified to allow rabbits to remain on a space they have eaten so long as there exists an uneaten adjacent space. The outcomes of this modification are more continuous and thicker flanking lines (Figure 4b).

In addition, Rule M2 was modified to permit a fraction of the rabbits to survive when jumping onto a space eaten by another rabbit. Figure 4c shows the rabbit spread field that results when 15 percent of the rabbits in Figure 4b that jump onto eaten spaces survive.

## 4. Discussion

The potential of the Rabbit Model as a fire behavior/fire spread model lies in its capability to link

the simplicity of empirical fire spread models with the complexity of physical fire spread models based on coupled differential equations. Complex physical processes can be recast as a set of simple intuitive rules that are translated into computer programs to be solved recursively. Application of the Rabbit model to complex terrain and winds show that further development as a fire spread/fire behavior model is warranted. The model shows potential for a fire spread model through its simulation of complex rabbit perimeters. The model shows potential for a fire behavior model through its simulation of the thickness of the rabbit perimeters.

The Rabbit Model can generate features observed in real fire lines without constraints of prespecified geometry. Using simple intuitive rules, the Rabbit Model can generate spread perimeters in complex terrain that are similar to the complexity found in physical fire spread models.

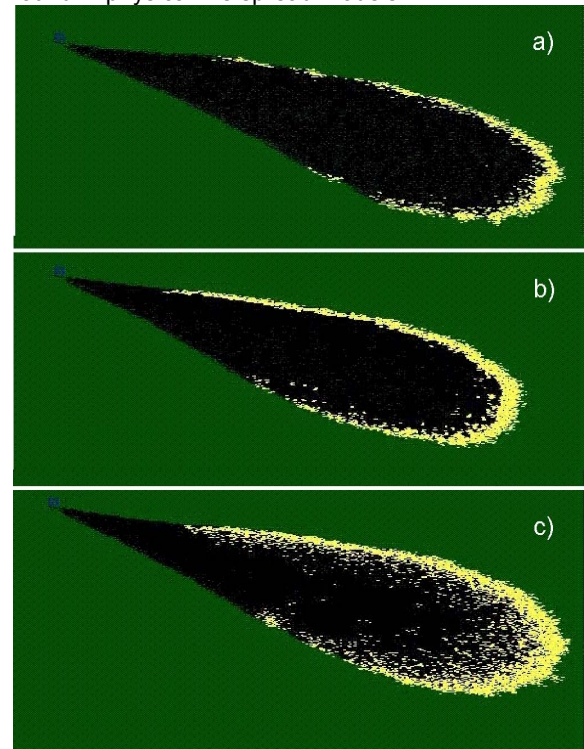


Figure 4. Examples of rabbit distribution in strong winds. a) Primary Mortality Rules, b) modified Rule M3, and c) modified Rule M2 added to modified Rule M3. See text for details.

The Rabbit Model is not constrained to produce active rabbits everywhere along the spread perimeter. This feature is consistent with observed patterns of fire spread.

In conditions of strong winds, the Rabbit Model produces spread structures typified by a head with flanking lines. The head is produced when some rabbits are carried by the wind to locations in front of the perimeter. Offspring fill in the gaps. This is analogous to fire brands carried in strong winds igniting unburned fuels ahead of the fire front.

The additional information regarding the width of the band of active rabbits along the perimeter is critical to programming Rabbit-Atmosphere Feedback Rule A1. Rule A1 is an intuitive statement regarding atmospheric response to continuously changing local heat sources. Implementation of Rule A1 requires knowledge of the number of rabbits per unit area.

Because of its simplicity, the Rabbit Model should be considered as a "screening model." The goal is to make the Rabbit Model a baseline model for fire spread models. In other words, in comparisons with sophisticated fire spread models, the Rabbit Model would become the model to "beat." The Rabbit Model is a modification of the operational smoke model, PB-Piedmont. It uses an existing user interface and data assimilation. Therefore, the Rabbit Model can be brought to operational capability relatively quickly.

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