ON THE CHANCES OF BEING STRUCK BY CLOUD-TO-GROUND LIGHTNING

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1. INTRODUCTION

Gated, wideband lightning sensors (Krider 1976) similar to those used in the U.S. National Lightning Detection Network (NLDN) (Cummins 1998a, Cummins 1998b) have been used in many countries for many years, and the resulting data provide accurate measurements of the area-density of cloud-to-ground (CG) lightning flashes under individual storms and over larger regions on monthly, seasonal, and annual time scales (see, for example, Orville 2001 and the references therein). Here, we will show how knowledge of the average area density of strikes, N_g, in a region can be used to estimate the chances that the nearest strike will occur within a specified distance of any origin in that region. We will also show how the "nearest-neighbor" can be generalized to higher orders.

2. NEAREST-STRIKE TO A POINT

We begin by supposing that the average area-density of strikes is N_g, and we want to know the chances that any strike has occurred within a distance, R, of any origin (chosen at random) in that region. We assume that each strike is a random event and that the spatial pattern of the strike points has a homogeneous Poisson distribution, i.e., N_g has complete spatial randomness. With this assumption, we can use the method outlined in Krider (1988,2003) to derive the probability density, $w(r)$, for the *nearest* strike being within a distance r and $r + dr$ of the origin,

$$w(r)dr = \begin{Bmatrix} \text{Probability that NO} \\ \text{strike is within r} \end{Bmatrix} \begin{Bmatrix} \text{Probability there IS a strike} \\ \text{between r and r + dr} \end{Bmatrix}$$

or

$$w(r)dr = \left(1 - \int_0^r w(r')dr'\right)\left(2prdr\ N_g\right) \text{ when} \int_0^\infty w(r')dr' = 1$$

Solving this, we obtain the well-known *nearest-neighbor distribution*,(NND)

$$w(r)dr = 2prdr\ N_g \exp\left(-N_g p r^2\right) \quad (1)$$

Using (1), it is straightforward to show that the most probable nearest-neighbor distance is $r_{MP} = \frac{1}{\sqrt{2pN_g}}$, the mean is $\bar{r} = \frac{1}{2\sqrt{N_g}}$, and the variance is

$$Var(r) = \frac{(4-p)}{4pN_g} = \frac{0.0683}{N_g}.$$

The integral of (1) describes the probability that the *closest* strike is within a distance R,

$$P(\le R) = \int_0^R w(r')dr' ,$$

or

$$P(\le R) = 1 - \exp(-N_g pR^2)$$

(2)

Now, if P is specified, (2) can be solved for R,

$$R = \left[-\ln(1-P)/pN_g\right]^{1/2} \quad (3)$$

It should be noted that in cases where $N_g pR^2 << 1$, equation (2) reduces to

$$P(\le R) \approx N_g pR^2 .$$

It is straightforward to show that the 2nd, 3rd,… nth nearest-neighbor distances distance is given by

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$$w_n(r) = \frac{2(pN_g)^2}{(n-1)!} \exp\left(-N_g r^2\right) r^{2n-1} dr \qquad (4)$$

where n=1,2,3,… (Thompson 1956) Now, with (4), the most probable distance are,

$$r_{MP,n} = \sqrt{\frac{2n-1}{2pN_g}}\text{ , the mean, } \overline{r}_n = \frac{1}{\sqrt{N_g}} \frac{(2n)!\,n}{\left(2^n n!\right)^2},$$

and the variance,

$$Var_n(r) = \frac{n}{pN_g} - \left(\frac{1}{\sqrt{N_g}} \frac{(2n)!\,n}{\left(2^n n!\right)^2} \right)^2$$

and the cumulative distributions can be evaluated numerically. We will now see how well these equations describe some lightning measurements.

3. COMPARISON WITH LIGHTNING DATA

Figure 1 shows the spatial pattern of 7202 CG lightning flashes that were recorded by the NLDN near Denver, Colorado, over a 5-year period. Here, each dot shows the most probable location of the first return stroke in each flash (see the Appendix in Cummins 1998b), and in the following we will refer to these points as *events*. (Note: we have made no corrections for an imperfect NLDN detection efficiency or for the multiple attachment points that commonly occur in CG flashes (Valine and Krider 2002)

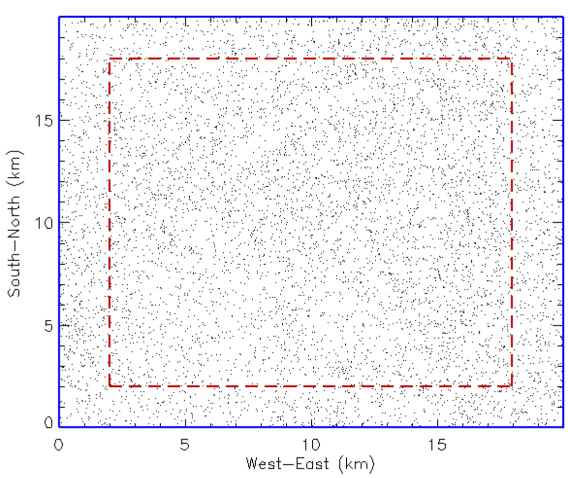

Figure 1 Plot of CG lightning locations over a 5-year period near Denver, CO (January, 1995, through December, 1999). There are a total of 4786 strikes in the analysis sub-region shown by the dashed red line.

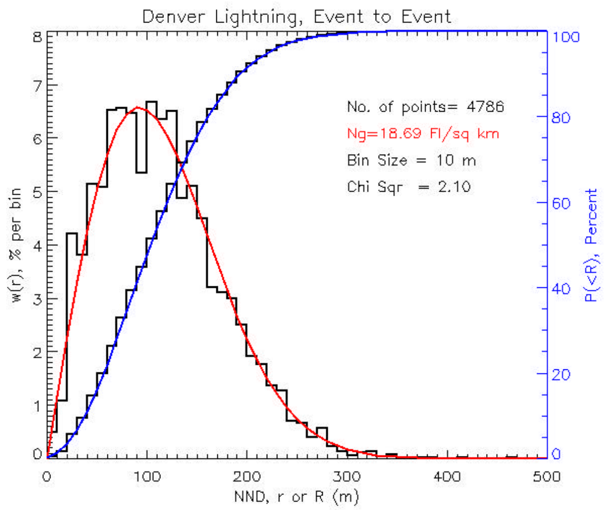

Figure 2 The event-to-nearest-event distribution for the spatial pattern that is within the dashed analysis area of Figure 1. The mean and variance of the *r* data are 115 m and 3720 m^2, respectively.

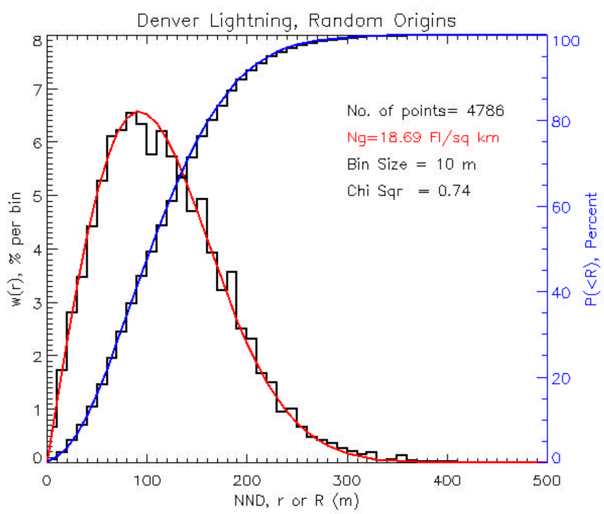

Figure 3 The random-origin-to-nearest-event distribution for the same pattern of events as in Fig. 2. The mean and variance of the *r* data are 115 m and 3910 m^2, respectively.

The data in Figure 1 have a numerical precision of four decimal digits in latitude and longitude, which translates to a spatial resolution of about 10 m, but random and systematic errors in the NLDN typically produce location errors that are of the order 0.5 to 1 km (Cummins 1998a, Cummins 1998b). There were 4786 events within the 16x16 km^2 analysis sub-region (to avoid edge effects) that is shown by the dashed red line in Figure 1. The average value of N_g over the sub-region is 18.7 flashes per km^2.

Figure 2 shows the measured distribution of event-to-nearest-event distances in the analysis sub-region of Figure 1 together with the measured cumulative distribution, and the red and blue curves show plots of equations (1) and (2) with N_g = 18.7 flashes per km^2, respectively. The mean and variance of the experimental data are 115 m and 3720 m^2, respectively, and the corresponding values predicted by equation (1) are 116 m and 3650 m^2.

Another way to characterize a spatial pattern of events is to place a series of random origins within the analysis area, and then to compute the distribution of the origin-to-nearest-event distances for a large number of origins. (Note: if the spatial distribution of events is truly uniform and random, this distribution should be the same as the event-to-nearest-event distribution that is shown in Figure 2.) Figure 3 shows a random-origin-to-nearest-event distribution that was computed for the same pattern of events as Figure 2, using the same number of random origins as there are events in Figure 2. The mean and variance of the measured NND's are 115 m and 3910 m^2, respectively, and the corresponding values predicted by equation (1) are 116 m and 3650 m^2. The mean and variance obtained for the random origins in Figure 3 are very close to the event-to-nearest-event distances in Figure 2. (mean=115 m, variance=3720 m^2)

Figure 4 shows the higher order neighbor distances that were also computed for the region outlined in Figure 1. Both the event-to-event and random-origin-to-event methods produced very good agreement, with the prediction of equation (4).

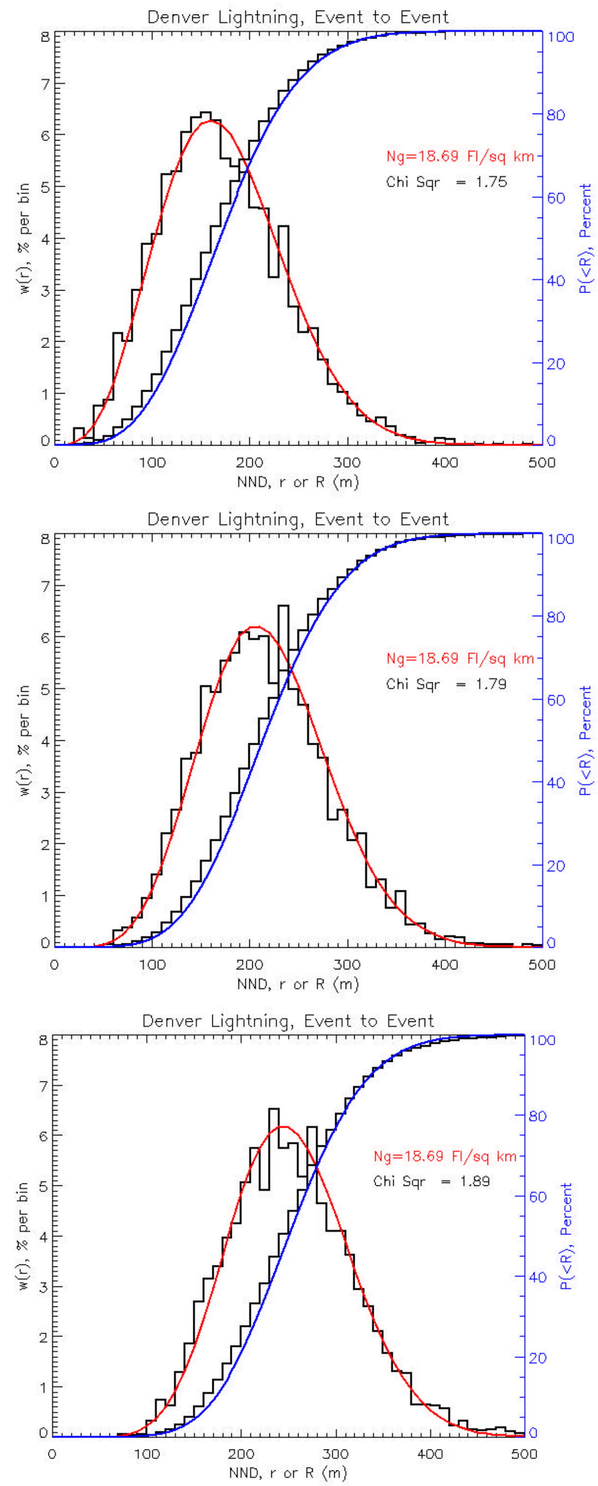

Figure 4 The Event-to-event distributions for 2nd (top), 3rd (middle), and 4th (bottom) nearest neighbor.

4. DISCUSSION

Figures 2, 3, and 4 show that equations (1-4) do describe the measured, long-term patterns of the nearest-strike distances rather well, but of course, such tests are limited by the accuracy of the NLDN measurements on small spatial scales. The most probable distances are in good agreement with equation (1) , and the values of the reduced chi-square, a "goodness of fit" parameter, are excellent. The sample means and variances are also in good agreement with model predictions; in fact, they are well within the 0.5 to 1.0 km location accuracy of the NLDN.

As examples of possible applications of the above, let us consider a region that has an average area density of 6.0 CG strikes per km^2, a representative value for the *annual* area density over much of the U.S. From equations (1) and (3), we can say that, in such a region, the most probable nearest-strike distance from any point (or person) will be about 160 m, there is a 50-50 chance of a strike within 190 m, and there is a 10% chance of a strike within 75 m.

In practice, the above estimates will really only be valid over spatial scales that range from a few tens of meters on the low end to tens of kilometers on the upper end. At smaller distances, the primary factors controlling the lightning strike probability (in addition to the presence of a lightning leader) are the number and lengths of the upward connecting leaders, and these will depend on the size and geometry of the strike object, the presence and size of any other objects in the local vicinity of the strike point, and the strength of the electric field under the downward-propagating leader (Bazelyan and Raizer, 2000; Petrov 2002; Rakov and Uman 2003). At larger distances, N_g may not be spatially uniform (Finke 1999).

If the measurements of N_g show clusters of strike points, such as might occur if there has been an unusually active storm in the region, then the event-to-nearest-event distributions will contain more events at short distances than our model predicts, and if N_g contains holes or regions of reduced area density, then the nearest-neighbor distributions will contain more large distances. In any case, even if N_g is not completely uniform, the assumption of complete spatial randomness can still be used as the null hypothesis when applying various statistical tests to identify and quantify the underlying spatial pattern and to find the optimum value of N_g (see, for example, Diggle 1981, Ripley 1981, Ripley 1988, Cressie 1993). The second, third, and higher order distributions also show good agreement with the measurements, and the higher order equations also provide a way of testing for spatial randomness, etc.

5. ACKNOWLEDGEMNTS

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