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Understanding the Analysis Error Pattern of a 3DVAR System Using Streamfunction and Velocity Potential

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1. INTRODUCTION

Streamfunction and velocity potential are widely used as background in three-dimensional variational analysis (3DVAR), which, for convenience is referred to as an S-V 3DVAR system. Such a 3DVAR system produces analyses possessing large-scale motions of background field and retrieving small-scale motions from the observations (Xie et al. 2002). This makes an S-V 3DVAR analysis similar to an OI analysis in most cases if similar covariances/correlations are applied to retrieve motions smaller than what observations of velocity can resolve. Global constraints or balances help the 3DVAR to be a better analysis tool than OI. If there are large-scale differences between background and observations, this 3DVAR may miss the opportunity to correct the large-scale motions. However, when accurate background fields of velocity on large scales exist or scales that observations can resolve, an OI can achieve as good an analysis as an S-V 3DVAR without global constraints. In our numerical experiences, an S-V 3DVAR system may add some nonphysical errors to the analyses after examining the error patterns of the increment fields.

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In this presentation, we examine the error patterns in detail and explain the cause of these patterns. In light of a theoretical explanation on these error patterns, we recommend the use of other control variables in designing 3DVAR systems, for example, to use velocity fields or vorticity and divergence in background fields. A 3DVAR using velocity background fields, referred to as a U-V 3DVAR system, with a recursive filter can avoid the nonphysical errors from an S-V 3DVAR for analyzing scales which cannot be resolved by an observation network, a. If there is useful large-scale information in the differences between observations and background fields, a 3DVAR system using vorticity and divergence background fields, referred to as a V-D 3DVAR system, could be a good candidate.

2. THEORY

Consider an ideal case where an observation of U is at the grid point (i, j, k) for simplicity. An S-V 3DVAR using a second order finite difference scheme has a cost function with the following term o^2 as it is a term from the observation part J_o of the cost function,

$$\cdots + \Bigg(\underbrace{-\frac{\psi_{i,j+1,k} - \psi_{i,j-1,k}}{\Delta y} + \frac{\chi_{i+1,j,k} - \chi_{i-1,j,k}}{\Delta x} - U^o_{i,j,k}}_{o}\Bigg)^2 + \cdots$$

The derivatives of this term are

$$\begin{aligned} \frac{\partial o^2}{\partial \psi_{i,j+1,k}} &= -\frac{o}{\Delta y}, \quad \frac{\partial o^2}{\partial \psi_{i,j-1,k}} = \frac{o}{\Delta y} \\ \frac{\partial o^2}{\partial \chi_{i+1,j,k}} &= \frac{o}{\Delta x}, \quad \frac{\partial o^2}{\partial \chi_{i-1,j,k}} = -\frac{o}{\Delta x}. \end{aligned}$$

If o is negative, for instance, a gradient-dependent minimization algorithm will reduce $\psi_{i,j+1,k}$ and increase $\psi_{i,j-1,k}$, a negative gradient direction. If rotational wind dominates the velocity, this results in negative increments on $U_{i,j+2,k}$ and $U_{i,j-2,k}$. Without grid correlations, this can be seen clearly in Fig. 1. Increasing the smoothing effect of a recursive filter causes this wave to become longer, but the negative increments remain and are expanded beyond (i, j, k), (see Fig. 2). If waves are longer than the distance between observations, the negative effect may go away; otherwise these negative increments become nonphysical errors to the increment field. Thus an S-V

3DVAR leaves nonphysical errors if it is used to resolve shorter waves than what observations can resolve.

This problem cannot be blamed on the finite difference scheme used. Since streamfunction and velocity potential are integrals of a velocity field, a background using streamfunction and velocity potential means that we require that the integrals of a velocity field are constants at the same values as the background. Thus, if a U component of the velocity field approaches an observation that is different from the background, it will change the integrals. If there are no other observations in the neighborhood to resolve the wavelengths, the U values in the neighborhood must move the opposite direction so that the integrals remain the same. Figure 3 demonstrates this clearly in one-dimensional space. When the C curve tries to fit an observation, there are always two opposite curvatures associated with it locally.

3. CONCLUSIONS

Based upon the discussion, an S-V 3DVAR may have a problem providing a good analysis on small scales. An S-V 3DVAR not only tends to have higher weights on observed short waves, but also has nonphysical errors in its increments. If the observations contain long wave information, an S-V 3DVAR may discard this information unless a strong recursive filter or correlation is applied; otherwise it could result in more nonphysical errors.

Before a data assimilation is performed, there should be a clear goal regarding an analysis. If larger scale differences exist between background and observations, a V-D 3DVAR would be a good candidate as it provides an analysis with different scales based upon the information provided by an observation network. It provides an analysis with larger weighting functions for observations of long waves, and background for short waves. Figure 4 demonstrates what the analyses of all 3DVAR systems look like when analyzing a single observation, in which a recursive filter with a smoothing parameter of 0.5 is applied to an S-V and V-D 3DVAR.

When a model background can accurately describe long waves, a U-V 3DVAR with a recursive filter or other covariance schemes would be a better choice although in this case, OI also may be a good candidate to use.

Even though other data assimilation techniques are available now, a

3DVAR analysis is simple, direct and efficient. In choosing a proper form of 3DVAR, it could yield an analysis which the other techniques have difficulty outperforming. In the future, we will evaluate how good a 3DVAR analysis is if it is chosen properly.

REFERENCES

Xie, Y. F. , C. G. Lu, and G. Browning, 2002: Impact of formulation of cost function and constraints on three-dimensional variational data assimilation. *Mon. Wea. Rev.*, **130**, 2433–2447.

a) b)

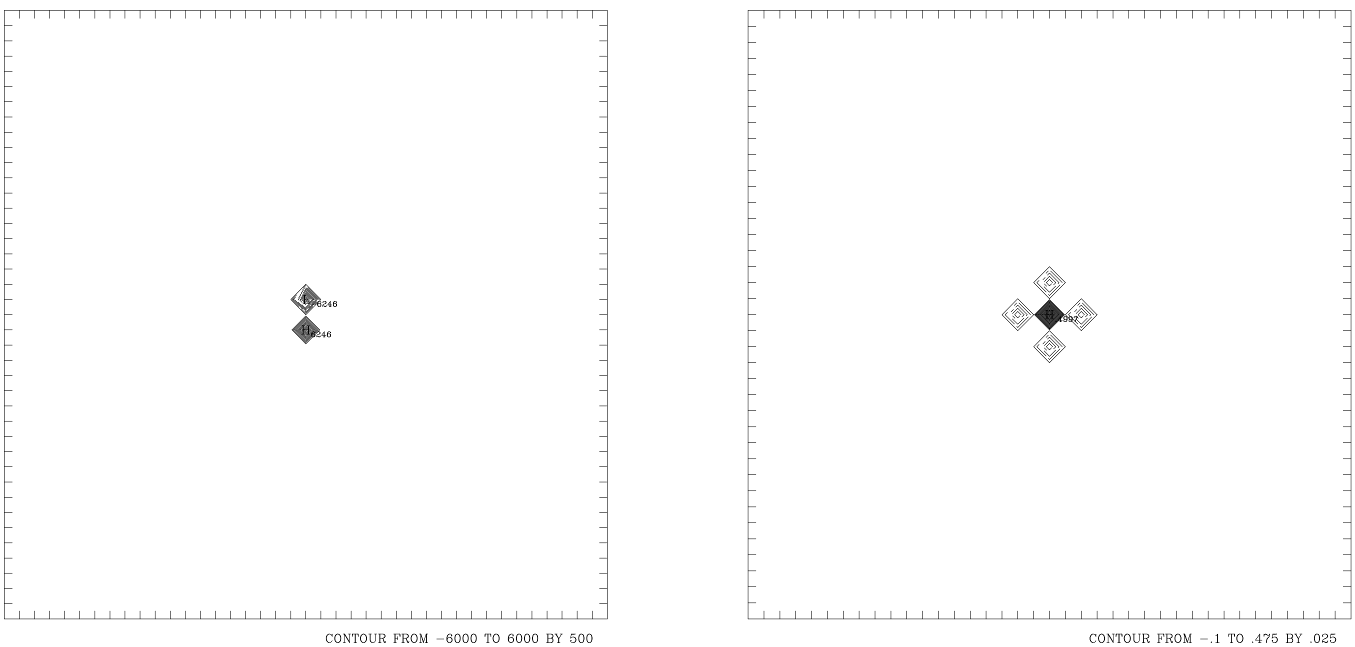

Figure 1: a) Streamfunction and b) U from an S-V 3DVAR for a single point observation without recursive filter, where darker areas are positive and lighter areas are negative.

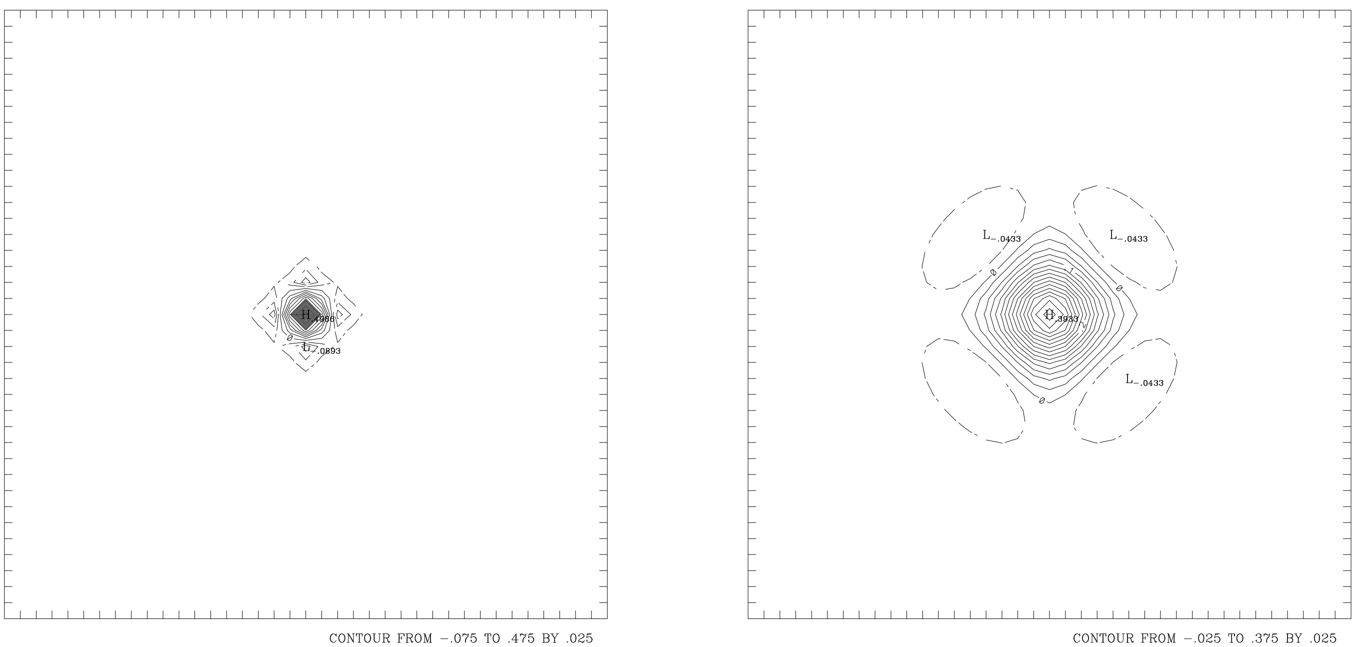

Figure 2: U for a single point observation from an S-V 3DVAR with smooth parameter 0.1 and 0.5.

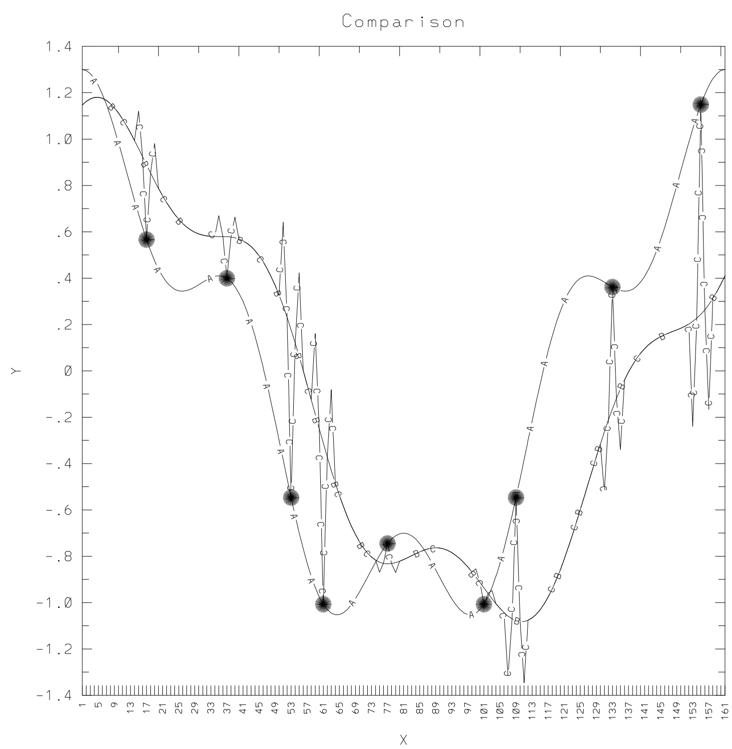

Figure 3: An “S-V” 1DVAR analysis with observations marked with black dots. Curve A is the observation curve; B the background curve and C the 1DVAR analysis.

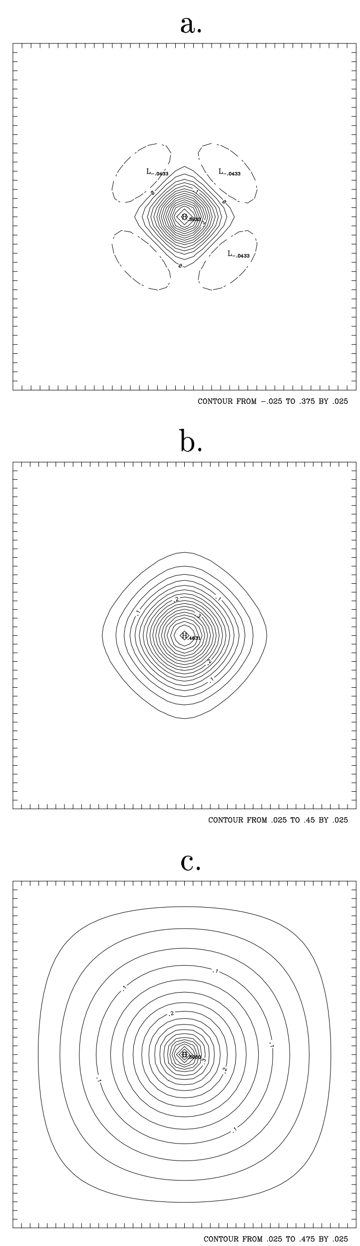

Figure 4: U components for a single point observation from a) S-V, b) U-V, and c) V-D 3DVAR.