P1.19 THE STEEPNESS LIMIT TO VALIDITY OF APPROXIMATION TO PRESSURE GRADIENT FORCE: ANY SIGNS OF AN IMPACT?

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Abstract

Even though the basic problem of the terrain-following vertical coordinates has been in a step-by-step fashion revealed, for the most part, in a sequence of papers by Rousseau and Pham (1971), Janjic (1977), Mesinger (1982), and Mesinger and Janjic (1985), the problem appears to have received little attention by the NWP community. In numerous ambitious model-development projects the issue has been ignored, in spite of the widespread movement toward higher resolution, which should make the resulting errors more severe. The nature of the problem is summarized, aiming for a statement more general but also more focused than the one of Mesinger and Janjic (1985).

One model that radically circumvents the problem is the Eta model, using quasi-horizontal coordinates. A summary and an update on the performance of the Eta at NCEP is presented, including precipitation scores, rms fits to raobs as a function of time and season, and accuracy in placement of centers of major storms. In a number of aspects the verification results reported, viewed in comparison with those of NCEP's terrain-following coordinate models, are consistent with and suggestive of the possibility of the eta coordinate making a significant contribution to the favorable performance of the Eta. The most recent of these are precipitation scores for the first 12 months of NCEP three-model scores, including the very heavy rainfall over the U.S. high topography western region during the late fall 2002 and winter 2002-2003, with the Eta 12 km model considerably outperforming an 8 km resolution terrain-following coordinate model. There is on the other hand little difference in the scores of the two models in the eastern U.S. region, not having a pronounced topography.

Given that with typical numerical formulations what one is facing with increasing realism of topography and thus increased steepness of coordinate surfaces is loss of information needed to calculate the slopes of pressure surfaces, it is suggested that this issue may well be deserving much more attention that it presently receives.

1. Vertical coordinates and pressure-gradient force

As a consequence of the experiments of Gallus and Klemp (2000), and others, it seems to be generally accepted by the NWP community that the eta coordinate system is "ill suited for high resolution prediction models" (e.g., Schär et al. 2002; Steppeler et al. 2003: Zängl 2003). The only

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evidence on which this belief is based appears to be much better performance of the terrain-following ("sigma") formulations on several cases of downslope windstorms and idealized simulations of flow up and down two-dimensional obstacles. Comprehensive evidence of significantly better results using the eta (e.g., Mesinger and Black 1992; Mesinger et al. 2002b) is brushed aside on account of these being obtained at – for today's standards -- not very high resolutions. It is apparently generally believed that as the resolution of NWP models is being increased the performance of the terrain following coordinates will increasingly improve relative to the eta, the downslope windstorm and idealized two-dimensional cases indicating that this indeed is taking place.

The same belief in increasingly better performance of sigma may be the reason why an alternative rather complex system of "shaved cells" (Adcroft et al. 1999) is not having many followers. Yet another option, of an eta-like system but with partial steps (Tripoli, personal communication) seems to be attracting even less interest. Thus, for example, all three major WRF dynamical core development efforts are based on various versions of the sigma coordinates.

There are numerous both reasons as well as signs suggesting that regarding the vertical coordinate these efforts are misdirected.

One reason suggesting this is the nature of the pressure-gradient force problem of the terrain following coordinates. The problem appears not to be generally understood, or appreciated, or both. This is spite of its understanding being in a step-by-step fashion arrived at, for the most part, in a sequence of papers ending by Mesinger and Janjic (1985). The essence of the issue is that none of the schemes in use or proposed avoid either increasingly using information that is physically inappropriate, or increasingly not using information which is physically appropriate, as the steepness of the coordinate surfaces is increasing. And the steepness of the terrain-following coordinate surfaces will increase as the resolution is increased, assuming that resolution of topography is increased as well -- a customary thing to do and one of the principal reasons for increasing the resolution in the first place.

We shall expand on the situation at some length here, given that published analyses (e.g. Mesinger and Janjic 1985; Lin 1997) do not summarize the situation quite in the light of the points made above. We restrict ourselves to hydrostatic systems, given that there seems to be no justification to expect the nonhydrostatic part to change the situation significantly. At the same time, we also take the governing equations in pressure coordinates as the basic framework in which to view the situation, given that they are the equations various terrain-following systems are derived from. We shall return to this latter point at the end of this section. With these restrictions, perhaps all pressure gradient force/hydrostatic equation schemes can conveniently be classified into three groups as follows.

a. Schemes with the hydrostatic equation analog that relates geopotentials used to calculate the pressure gradient force to temperatures both below and above the considered pressure level. An example of such schemes would be those relating geopotentials to temperatures of vertical columns by fitting of some kind, say a spline fitting;

b. "Level schemes", those with geopotentials used to calculate the pressure gradient force obtained by vertical integration of temperatures from the ground only up to the considered coordinate surface. Examples of such schemes are straightforward Lorenz grid isentropic coordinate schemes, or the Corby et al. scheme, used for the numerical examples of Mesinger (1982, also in Mesinger and Janjic 1985);

c. "Layer schemes", those using layer temperatures to define geopotential increments through layers. With these schemes, it is straightforward to design analogs for the pressure gradient term of the pressure gradient force that are consistent with the analogs of the hydrostatic equation. Examples are the Burridge-Haseler scheme (numerical examples referred to above), schemes used in the Eta, and also the "finite-volume" scheme of Lin (1997). Note however that the "improved contour integration" used for the numerical example of Lin (1998) represents a departure from this group of schemes, since information was used above the finite volume of the velocity considered.

Recall that according to the continuous equations pressure gradient force is equal to the gradient of the wind point's constant pressure surface. On the other hand, vertical integration of the hydrostatic equation from the ground up to a chosen constant pressure surface gives

$$\Phi = \Phi_s - R_d \int_{p_s}^{p} T_v \, d \ln p, \qquad (1)$$

with symbols having their usual meaning. Thus, according to the continuous equations, pressure gradient force should not depend on variables above the considered constant pressure surface.

Looking at various pressure gradient force schemes from the point of view of this simple requirement seems to have failed to receive attention so far. The schemes of the group (a) above clearly violate the requirement even with no slope of the constant pressure surface considered. The schemes of group (b) do not have that degree of a problem but still increasingly depart from the requirement as the slope of the coordinate surface is increasing. Only the schemes (c) have no problem as long as the slope of the coordinate surfaces, s=const, remains within the condition

$$|\partial_x p|_s \, \delta x \le |\partial_s p| \, \delta s, \tag{2}$$

same as the requirement for the hydrostatic consistency.

In Fig. 1 illustration is given of a situation when the slope of the constant s (or, σ) surfaces exceeds the condition (2) by a full layer thickness on each of the two geopotential columns shown. Compared to the information that the continuous equations need for the geopotentials of the constant pressure surface shown, a standard layer scheme will on the right side use two temperatures that the continuous equations do not need. The influence of one, that of the layer considered, will be removed by the hydrostatically consistent calculation of the pressure gradient term of the pressure gradient force, but the influence of the other, of $T_{j+1/2,k+1}$, will not. On the left side, the scheme will fail to take account of the information that the continuous pressure gradient force needs, that of $T_{j-1/2,k-1}$.

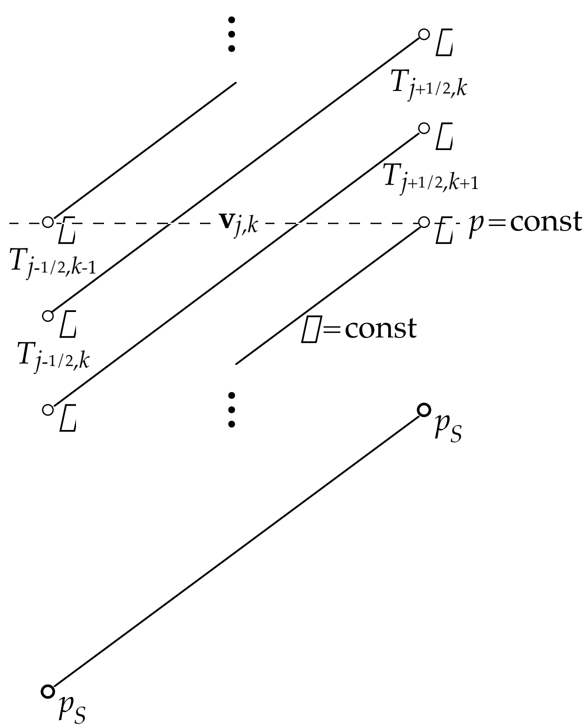

Fig. 1. Schematic illustrating the pressure gradient force problem described in the text.

The resulting error, in principle, can be just about arbitrarily large. Factors conducive to large errors according to the preceding analysis are large temperature variations in the vertical, high vertical resolution, and, as a sine qua non requirement, steepness of coordinate surfaces. The steepness of coordinate surfaces with terrain-following coordinates will tend to increase as the horizontal resolution is increased. While the occurrence of large errors as a result might be rare, it is certain to occur at some places some of the time. These occurrences are likely to happen for situations which are significant, having perhaps somewhat of an outlier character in one way or another. Either way, if one is really intent on improving the skill of weather and climate prediction, a cavalier attitude toward this class of errors is unjustified.

We now return to the issue of our having used the pressure coordinate framework to arrive at (1). Note that one could have concerns in this regard, given that hydrostatic "continuous equations" could also be cast in z coordinate, and the pressure gradient force expressed as proportional to the gradient of pressure with constant z. Vertical integration can then be performed from the top of the atmosphere downward, and it would thus appear that the pressure gradient force should only depend on the atmosphere *above*, and not below the level considered! It is however felt that these concerns should not be looked upon as weakening the arguments made. The terrain-following coordinate systems *are* based on pressure coordinate equations, and geopotentials of whatever coordinate surfaces get subsequently chosen *are* calculated by vertical integrations upward. Thus, considerations as made leading to (1) and to the schematic of Fig. 1 are what results in pressure gradient force errors that actually take place in various atmospheric models.

2. Any signs of the significance of the error?

There is a multitude of signs suggesting that the impact of the resulting errors is overall quite significant already at resolutions used today. A list and discussion of some of these follows.

In two recent conference papers (Mesinger et al. 2002a; Mesinger et al. 2002b) evidence was presented on the operational Eta precipitation scores, and 500-mb height and 250 mb wind rms fits to raobs, as functions of time out to 3.5 days, compared to those of its driver Avn (now GFS) model. The idea is that as the Avn/GFS 6-h old lateral boundary data are advected in and the initial condition flushed out, one would expect the Eta skill to go down in relative terms compared to that of the driver model. The evidence indicates that it does not; in fact, to the contrary, and in particular in winter, it seems to improve with time. An explanation consistent with this evidence is that the Eta contains a design feature or features that increasingly positively impact the result as the forecast time increases, compensating for the increase in the error due to the advection of the lateral boundary data. The eta coordinate is a good candidate for such a feature, given that the Eta resistance to the loss of skill relative to its driver model is greatest in winter, when the prevailing jet stream is at its southernmost latitudes -- those of the contiguous United States (ConUS) where the majority of the raobs are.

The two papers also present information on the accuracy of the Eta vs that of the Avn in placing the centers of major storms, east of the Rockies, and over land, at 2.5 days forecast time. Results have been compiled for the winter 2000-2001. The Eta is shown to have done considerably better. It is generally understood that such lows are placed mostly depending on the position of the jet stream; thus, this is also suggestive of the advantage the Eta may be deriving from its eta coordinate.

Finally, both of the papers also show the result of an experiment in which the Eta was run in its eta and in its sigma mode, everything else being the same. Switching the model to the sigma coordinate, the position error of the center of a major low in the lee of the Rockies at 48-h forecast time increased by 100 km, from 215 to 315 km.

To this body of evidence one should add the emerging statistics of the performance of the operational 12-km Eta compared to that of the NCEP 8-km NMM model. The NMM is operationally run on six domains which include three ConUS domains, "East", "Central" and "West". The domains can be seen at http://wwwt.emc.ncep.noaa.gov/mmb/mmbpll/nestpage/overlays.gif Guided by the results and views pointed out in the beginning of this proposal, the NMM developers have chosen to use the terrain-following coordinate. Precipitation scores on the three ConUS domains are available starting with September 2002, just a bit over a year at the time of this writing. Scores for the first 12 months of the availability of these scores will be shown here, contrasting those for the East, where there is no major topography, against those of the West, where the topography is of course prominent. As can be seen on the gif referred to above, the two domains touch each other at two of their northern corners, so that most of the ConUS is included. The 12-month equitable threat and bias scores for the two nests, East and West, are shown in Figs. 2 and 3, respectively.

Over the "East", with no major topography, Fig. 2, there is very little difference between the equitable threat scores of the operational 12-km Eta and that of the 8-km NMM. Thus, no benefit from higher resolution is visible in the NMM scores. It could well be that at the operational resolutions used today, on the order of 10 km, resolution is not among the leading obstacles to improving the NWP synoptic-scale skill, to the extent that increasing the resolution further is of little or even no help. This is of course meant to refer to the skill in placing systems responsible

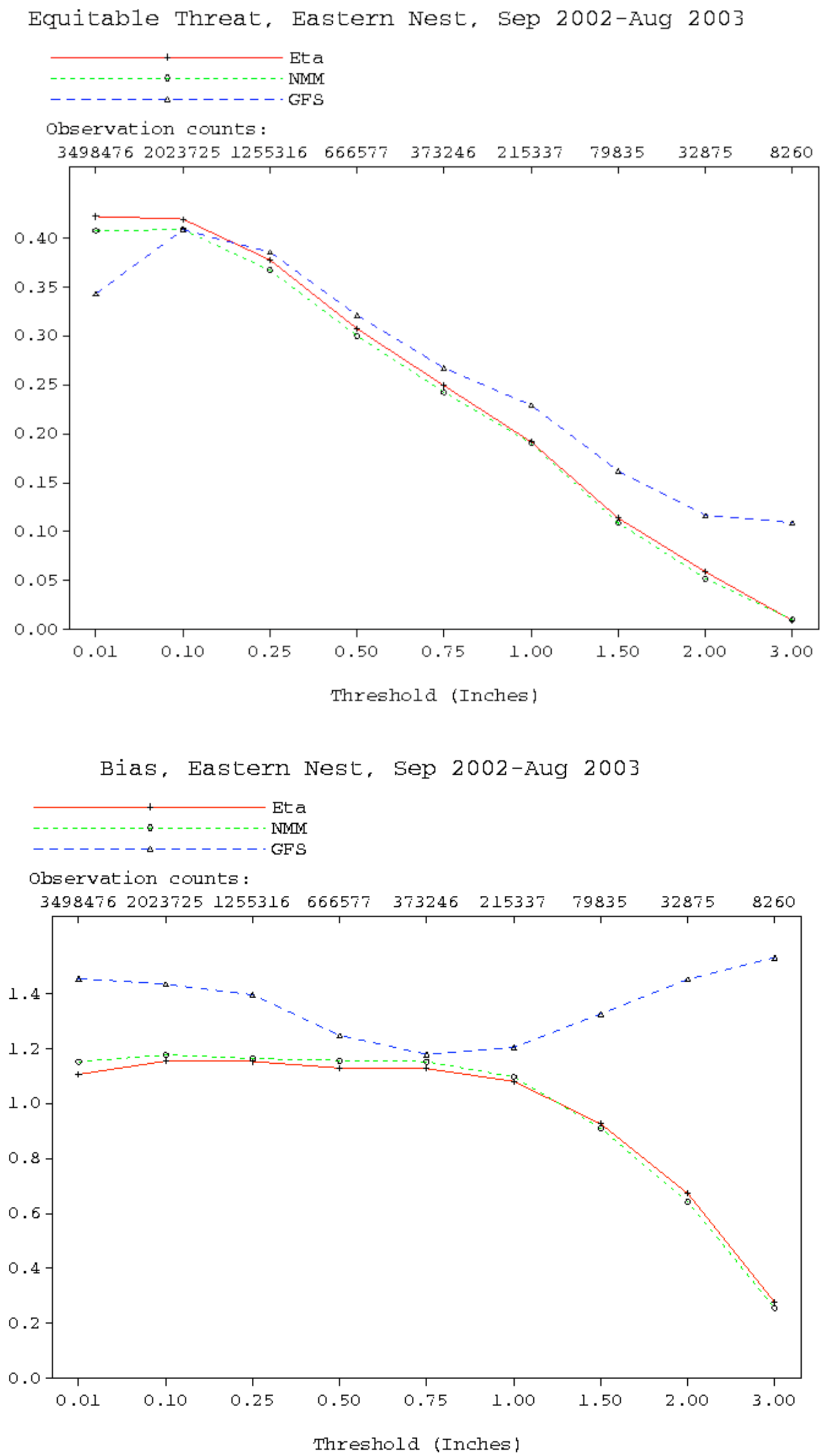

Fig. 2. 12-month precipitation equitable threat and bias scores for three NCEP operational models, "Eastern Nest". See text for further detail.

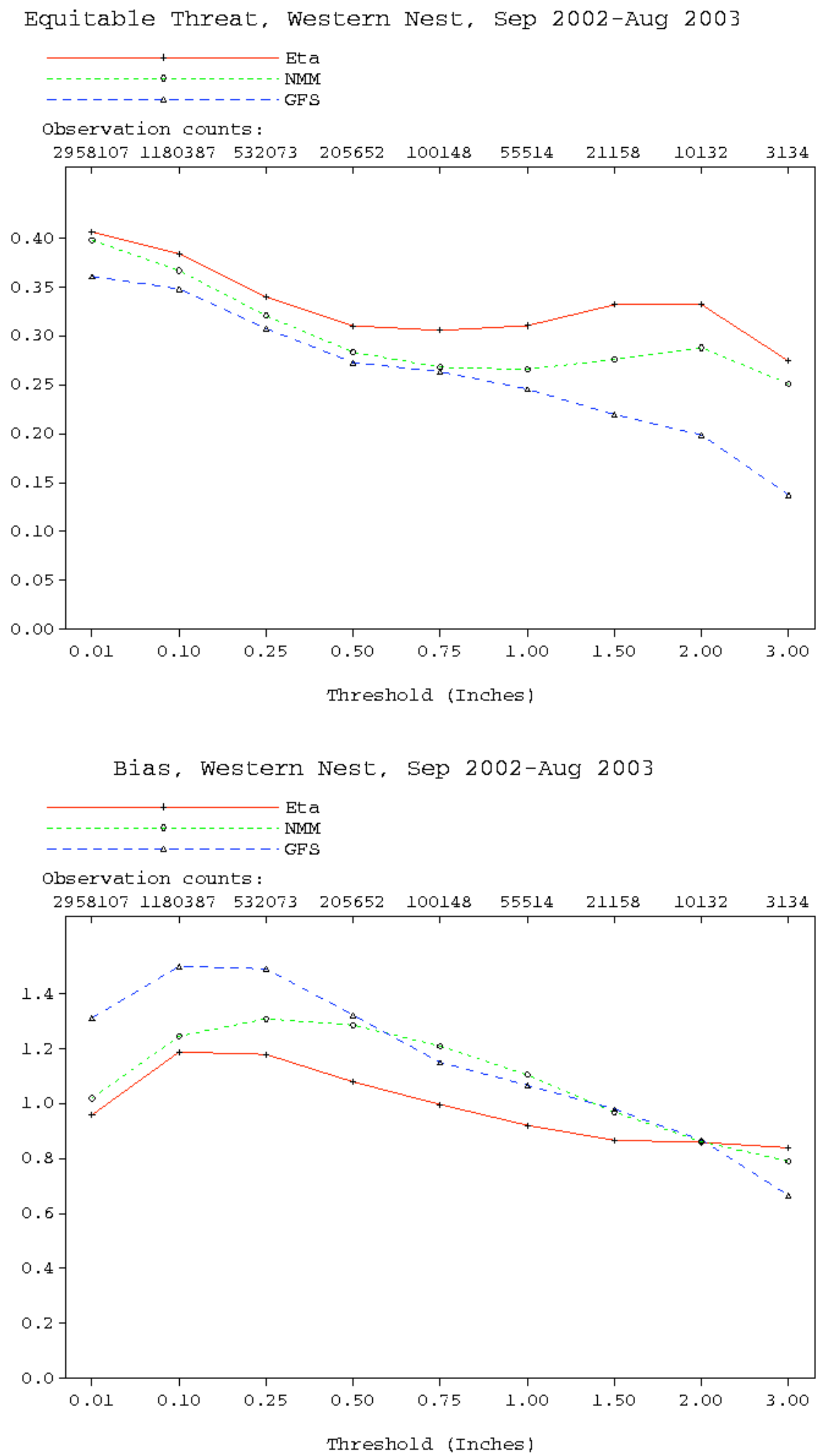

Fig. 3. 12-month precipitation equitable threat and bias scores for three NCEP operational models, "Western Nest". See text for further detail.

for precipitation, such as storms and fronts, in a statistical sense. Certainly, improvements in local features resulting from greater orographic detail, such as in local winds, would occur even if systems are not generally placed more accurately.

If this is so, that increasing the resolution of present state-of-the-art models beyond about 10 km is only of limited help as suggested above, then identifying the leading obstacle or obstacles to increasing the NWP synoptic-scale skill is obviously of primary importance. It has been argued in Mesinger (2002b), and earlier (Mesinger 2001), that reasons exist to believe that the inconsistency in the treatment of dynamics and physics in NWP models is an excellent candidate for such a leading obstacle, referring here to grid point values being treated as valid at points in dynamics, and treated as grid-box averages in physics. Note that a similar opinion, that "improvements in forecasting may have to come from better numerical representations of the structures that are supposedly already resolved", has been expressed earlier by Lorenz (1993).

Over the "West", Fig. 3, where one would expect the higher resolution topography of the 8-km model to be of a particular advantage, the Eta 12-km model is seen to have equitable threats substantially better than the 8-km NMM. The Eta advantage was particularly striking in situations of intense precipitation over the western United States in November and December 2002 (http://wwwt.emc.ncep.noaa.gov/mmb/ylin/pcpverif/scores/), with five events of HPC analyzed precipitation over 4 inches/24 h, and two of these with precipitation over 6 inches/24 h. At these events, the highest precipitation monitored of 3 inches/24 h, would typically be analyzed over individual and separated mountain ranges, Cascades, Coast Ranges, and Sierras, with elongated and clearly topographically defined patterns that are generally considered an extraordinary QPF challenge.

Equitable threat scores are generally intended to show the accuracy of models in placing precipitation. A problem though is that they are affected by bias. Thus, typically they are presented along with bias plots, as done here and at many other places (e.g., Ebert et al. 2003), enabling a subjective assessment of the impact of bias. In our situation, for example, one may wonder if the NMM and the GFS equitable threats, over the West, were adversely affected by increased model biases compared to those of the Eta. To address this problem of the bias dependence of the threat scores, recently two methods have been proposed to normalize the equitable threats to perfect bias (Mesinger and Brill, this CD). The two methods are now included as options in the NCEP Forecast Verification System (FVS). Bias normalized equitable threats corresponding to those of the upper panels of Figs. 2 and 3 were inspected; the two methods gave very similar results. Here in Fig. 4 bias normalized equitable threats obtained using the second of the two methods, that proposed by Keith Brill, are shown. Brill's method is

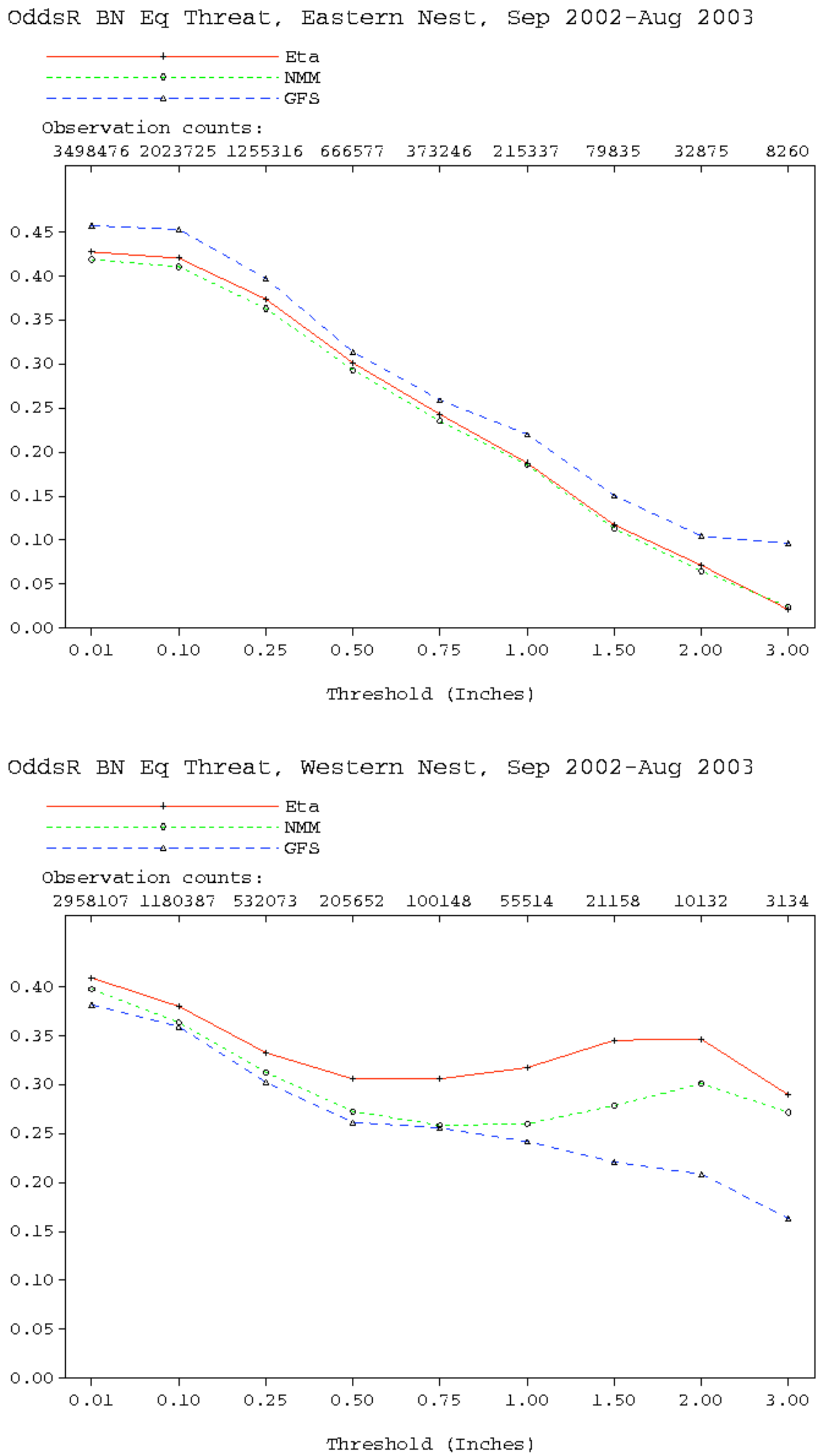

Fig. 4. Equitable threat scores as in the upper panels of Figs. 2 and 3, but normalized to remove the effect of bias using "odds ratio method" for bias normalization. See text for further detail.

based on assuming that the odds ratio (Stephenson 2000) is independent of bias; this allows evaluation of hits that the model, with this assumption, would have had were its bias equal to one.

With the correction for bias, the advantage of the Eta over the NMM in the West is seen to be overall even somewhat greater than without the correction. Comparison of the two models against the GFS is also worth taking note of. In the East, the GFS has clearly done better. The two meso models can take some comfort in the fact that they run first, using "old" lateral boundaries. In the West, the two high resolution models have done much better -- presumably benefiting from their higher resolution -- but the NMM not nearly as much as the Eta.

This is not a result that was expected. Note the statement from the NOAA-wide e-mail of 19 July 2002 announcing the operational implementation of the six nests, referring to the choice of the vertical coordinate: "This choice will avoid the problems encountered at high resolution (10km or finer) with the step-mountain coordinate with strong downslope winds and will improve placement of precipitation in mountainous terrain".

As to the former of these two expectations, there seems little room for doubt that the step-mountain approach does have problems with downslope winds: note also a Santa Ana wind case recorded recently (Janjic et al. 2003) in which the operational Eta had obvious problems getting the strong winds into its lowest layer as observed, while the 8-km NMM did well. However, the right approach, which we find lacking, is to try to understand these problems, and if we can remedy them, which we believe can be done. A method that is expected to remedy these problems, but yet keep the simplicity and other favorable properties of the eta coordinate, is outlined in the following section.

Problems of the terrain-following coordinates, on the other hand, have proven resilient to efforts at being remedied, in spite of more than three decades of trying, and about 6-7 techniques proposed. The recent efforts of Schär et al. (2002) and Zängl (2003) do not really aim to solve the problem, they are only trying to reduce it, by reducing the steepness of coordinate surfaces locally. The latest and perhaps much respected pressure-gradient scheme, the finite-volume scheme of Lin (1997) just as well suffers from the problem of the increased loss of information needed to calculate the slopes of pressure surfaces as the resolution and thus steepness of coordinate surfaces is increased. Thus, given that it is not addressing the pressure-gradient problem referred to, it does not represent an improvement in that sense over typical other hydrostatically consistent schemes.

3. A look ahead: sloping steps?

One unexplored effort that we feel holds considerable promise is a refined discretization of the eta coordinate, so as to use steps that have slopes.

Note that the eta coordinate (Mesinger 1984, also in Mesinger et al. 1988) is defined by an equation, relating eta to pressure and surface pressure. The steps are only the simplest discretization of topography using the coordinate. A considerably more refined discretization of the shaved cells type (Adcroft et al. 1997) has been explored, although not much if at all in an atmospheric context. Two assumptions are made in the step-mountain eta, compared to the shaved cells method. One, the cells are either ground, or air. Two, the tops of the topography cells are horizontal.

The first assumption is removed in the Tripoli's partial steps approach. We have tested using partial steps in the Eta and have decided against it. We have a number of reasons for this decision. The primary among those is that partial steps, and even more general, atmospheric cells of clearly unequal volume inside a horizontal layer of model cells, from an Eta-like model enforcing a variety of Arakawa conservation principles remove the feature of the model being approximately a finite-volume model. This finite-volume feature of the Eta might well have been an important ingredient of the overall a very successful performance of the Eta.

Removal of the second assumption is what we feel is an attractive option. This could be done by keeping the center of each uppermost topography cell at its interface elevation, but allowing it to have slope. Vertical velocity is then needed at the surface pressure grid points. It can be obtained by taking account of the slope and the horizontal velocity obtained by centered averaging of the surrounding horizontal velocities -- some of them zero as a result of being underground. With this system, horizontal winds at the vertical edges common to four neighboring cells that are fully blocked in the current Eta would be partially unblocked, depending on how much of the vertical edge is open.

It is expected that this is a robust approach, that would have an advantage over the shaved cells by avoiding cells of very unequal volume in horizontal layers. It is known that such cells, of a small volume, can be a cause of numerical difficulties (Robert Walko, personal communication). It is also believed that the proposed system would remove the current weakness of the Eta, since it would permit vertical motions next to the sloping model's ground surface. Finally, with the Eta numerics more or less as it is, it would maintain the current approximately finite-volume nature of the Eta.

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