6.7 THE CALCULATION OF THE STATISTICAL ERROR OF MEASURED TURBULENT FLUXES - CONSIDERATIONS AND IMPROVEMENT

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1. INTRODUCTION

The eddy correlation method is the usual procedure to calculate turbulent fluxes from airborne and many ground-based in-situ measurements. For the calculation of the statistical errors of these fluxes a method established that is based on the determination of the integral scale (Lumley and Panofsky 1964; Lenschow and Stankov 1986; Lenschow *et al.* 1994). Two important assumptions are usually made:

1. The statistics of the measured time series follow Gaussian distributions.
2. The integral length scale of the flux $\lambda_{w\theta}$ (in this example for the sensible heat flux) can be estimated by the length scales of the single turbulent quantities e.g., the vertical wind and the temperature (λ_w and λ_θ).

For the field-experiment strategy the results of the error calculations have two consequences:

1. The relative statistical errors of turbulent fluxes measured in the convective boundary layer (CBL) are often around 100 %, when measured with averaging lengths L of only some 10 km.
2. To decrease these errors a large distance (airborne systems) or fetch (ground stations like towers etc.) has to be recorded. Normally this consumes too much time to disregard the instationarity of the CBL.

During the field experiment LITFASS 1998 (Beyrich *et al.* 2002) an arrangement of several ground-based, remote sensing, and airborne measurement systems was used to determine turbulent fluxes with different methods. Above the heterogeneous site 60 km southeast of Berlin the airborne measurement system Helipod and the research airplane Do 128 (both Technical University of Braunschweig) flew simultaneous missions in the CBL (Bange *et al.* 2002a). The Helipod is an autonomous meteorological probe attached to a 15 m rope under a helicopter and operates at 40 ms^{-1} airspeed (see e.g., Bange and Roth 1999). The Do 128 is a twin-propellered research aircraft that travels at 60 ms^{-1} (Hankers 1989; Corsmeier *et al.* 2001). To meet the ratio of airspeeds the two systems flew two different square-shaped flight patterns - one with 10 km legs, the other with 15 km legs - simultaneously around a common central area.

Within this central area, several ground stations, a 99 m meteorological tower, a scintillometer, and a wind profiler with RASS (Engelbart and Bange 2002) completed the experimental arrangement. As a result the area-averaged turbulent fluxes of heat (Fig. 1), humidity, and momentum measured by the Do 128 and the Helipod were nearly identi-

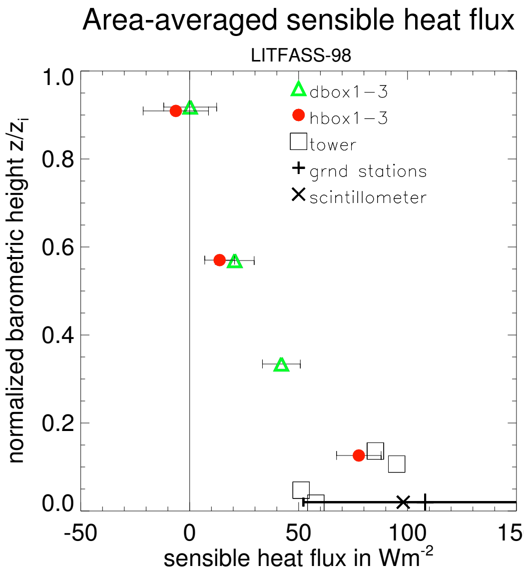

Fig. 1: *Sensible heat fluxes and their statistical errors were calculated from airborne measurements (Helipod: triangles, Do 128: filled circles) by the usual method. Ground based measurements: The cross (x) without error bar depicts the averaged scintillometer measurements. The plus sign and squares in the figure represent data from micro-meteorological ground stations and from tower measurements, respectively.*

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cal (Bange *et al.* 2002a; Bange *et al.* 2002b). The discrepancy was in the order of a few Wm^{-2} and 0.01 Nm^{-2}, respectively, while the statistical errors as calculated with the usual method were clearly larger (see error bars in Fig. 1) Since this was systematic and in order to increase the significance of the flux measurements, a closer look at the error calculation seemed to be appropriate.

2. CONSIDERATIONS AND IMPROVEMENT

2.1 Direct calculation of the statistical errors of turbulent fluxes

The turbulent flux F is defined by (Lumley and Panofsky 1964)

$$\begin{aligned} F &= \mathrm{Cov}_f \\ &= \int_{-\infty}^{\infty} \Big(\big(w - E(w)\big) \cdot \big(s - E(s)\big) \Big) \cdot \phi_f \, df \quad . \end{aligned} \tag{1}$$

In this equation w is the measured time series of the vertical wind (after subtraction of its mean value and linear trend, see e.g., Chatfield 1982), and s is the time series of a scalar quantity transported by the vertical wind (e.g., the air temperature or humidity). The product of w with s is here defined as $f = w \cdot s$. The frequency distribution of f is described by the distribution-density function ϕ_f.

The variance

$$\sigma_F^2 = \int_{-\infty}^{\infty} \Big(F - \langle F \rangle \Big)^2 \phi_F \, dF \tag{2}$$

defines the statistical error of the turbulent flux. Here, the flux F was calculated using the actually measured time series of s and w (a data selection sampled during a measurement period L), while $\langle F \rangle$ represents the ensemble average of all possible selections. The frequency distribution of F is described by the distribution-density function ϕ_F.

The integral time scale λ_{ws} is defined as

$$\lambda_{ws} = \int_{0}^{\infty} \frac{\gamma_{f'}(\tau)}{\gamma_{f'}(0)} \, d\tau \quad , \tag{3}$$

with the auto-covariance function $\gamma_{f'}(\tau)$ and time lag τ (Lumley and Panofsky 1964) . The prime in f' denotes that the mean value and linear trend were removed from f.

Assuming that the integral scale exists, and that λ_{ws} is much smaller than the averaging duration L used to calculate the flux (mostly the period of the measurement), then the statistical error is given by (Lenschow and Stankov 1986).

$$\begin{aligned} \sigma_F^2 &= 2 \cdot \langle f'^{\,2} \rangle \cdot \frac{\lambda_{ws}}{L} \\ &= 2 \cdot \big(\langle w^2 s^2 \rangle - \langle ws \rangle^2 \big) \cdot \frac{\lambda_{ws}}{L} \quad . \end{aligned} \tag{4}$$

For the field experiment LITFASS 1998, these assumptions were fulfilled. The statistical error of the measured flux was directly calculated using (4), without the use of any estimations (Fig. 4).

2.2 Usual calculation of the statistical flux errors

The integral-scale approximation

Since the auto-covariance function $\gamma_{f'}(\tau)$ sometimes behaves 'wild' (Mann and Lenschow 1994), the integral time scale λ_{ws} (3) is not defined in those cases. A usual method (Lenschow *et al.* 1994) to deal with this problem is to substitute the integral time scale λ_{ws} by the integral time scales of w and s (λ_w and λ_s), whose auto-covariance functions normally behave reasonably. For the systematic statistical error (see Lenschow *et al.* 1994; Bange *et al.* 2002a) this is

$$\lambda_{ws} \leq \frac{\sqrt{\lambda_w \, \lambda_s}}{r_{ws}} \quad , \tag{5}$$

with the correlation coefficient

$$r_{ws} = \frac{\langle ws \rangle}{\sqrt{\langle w^2 \rangle \cdot \langle s^2 \rangle}} \quad . \tag{6}$$

The approximation for the random statistical error (Lenschow *et al.* 1994) is significantly different:

$$\lambda_{ws} \leq \min(\lambda_w, \lambda_s) \quad . \tag{7}$$

The Gaussian-distribution assumption

Usually the time series of w, s, and f are assumed to be Gaussian distributed (e.g., Lenschow and Stankov 1986; Engelbart and Bange 2002). Following Lenschow *et al.* (1994), the upper limit of the standard deviation of the measurement flux is

$$\sigma_F \leq \frac{2}{r_{ws}} \cdot \sqrt{\frac{\min(\lambda_w, \lambda_s)}{L}} \cdot |F| \quad . \tag{8}$$

In the following it will be demonstrated that the distributions of turbulent time series are far away from being Gaussian and that the use of (8) increases the statistical error of the fluxes unnecessarily.

As a first step, the statistical distribution of wind components, humidity, and temperature were calculated. The corresponding time series were measured during several flight experiments in stable and unstable thermal stratification. A Chi-Square test usually showed that the assumption of Gaussian distributions was not appropriate. The histograms of the turbulent properties (named the real distribution) usually displayed a distribution with finite kurtosis and skewness (e.g. Fig. 2).

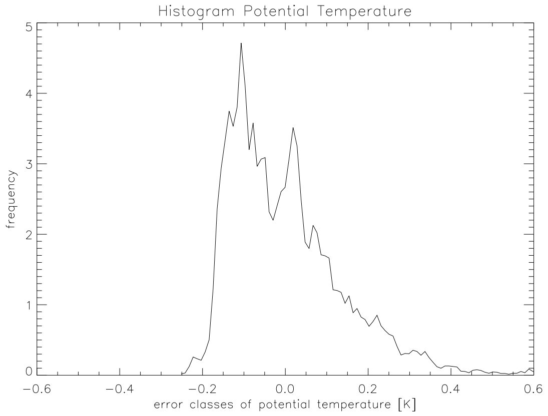

Fig. 2: *The histogram of the potential temperature measured on a 10 km flight leg shows a non-Gaussian distribution.*

Then the seek for a better description than the Gaussian distribution began. The three-parameter IDB(α, β, γ) distributions (Hjorth distributions)

$$f(t) = \frac{\alpha\, t\, (1 + \beta\, t) + \gamma}{(1 + \beta\, t)^{\gamma/\beta + 1}} e^{-\alpha\, t^2/2} \qquad (9)$$

yield promising results. Special cases of this distribution type are the Weibull, Rayleigh, and the exponential distribution. The parameters α, β, and γ were calculated by solving a non-linear system of equations. An example for the approximation of a real distribution with the IDB distribution is displayed in Fig. 3. The discrepancy is still large. To date no satisfying solution was found, mainly due to the difficulties connected with the solving of the system of non-linear equations. Besides, the turbulent fluxes calculated using the real, the IDB, and the Gaussian distribution in Eq. (2) were about the same. The choice of the distributions effects mainly the statistical error of the flux.

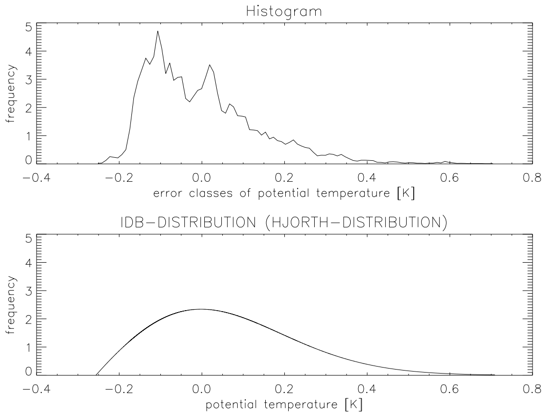

Fig. 3: *The distribution density of potential temperature on a 10 km flight leg (upper diagram) was approximated by an IDB($\alpha = 15$, $\beta = 16$, $\gamma = 0$) distribution (lower diagram).*

3. RESULTS AND CONCLUSIONS

The data gained during the LITFASS 1998 flights were used to demonstrate the difference in the flux errors calculated using the 'usual' (8) and the 'direct' (4) method. The auto-covariance function $\gamma_{f'}(\tau)$ behaved reasonable so that the integral (3) existed. The Fig. 1 shows the results using the

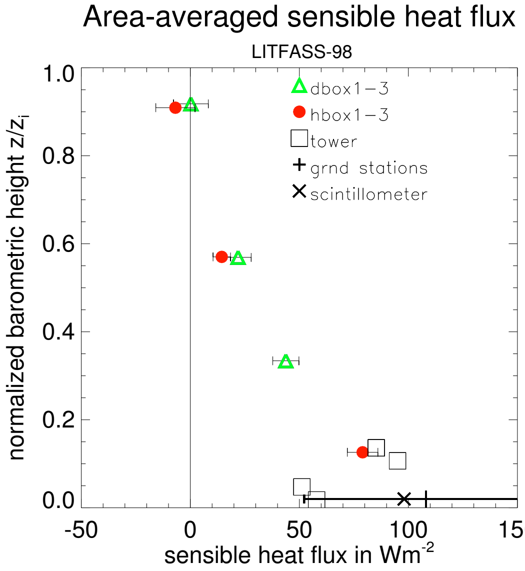

Fig. 4: *These are the same measurements as introduced in Fig. 1. But this time the statistical error was determined with the direct method.*

'usual' method and the Fig. 4 depicts the results of the 'direct' calculation.

The mean average of the sensible heat fluxes remained unchanged, but the statistical errors were considerably reduced with the direct method (4) by about 35 %. This is also demonstrated by Fig. 5, which displays the direct comparison of the flux errors calculated from all LITFASS 1998 flight legs. Thus it is proved to be advantageous to determine the statistical error without estimations or the assumption of Gaussian distributed time series.

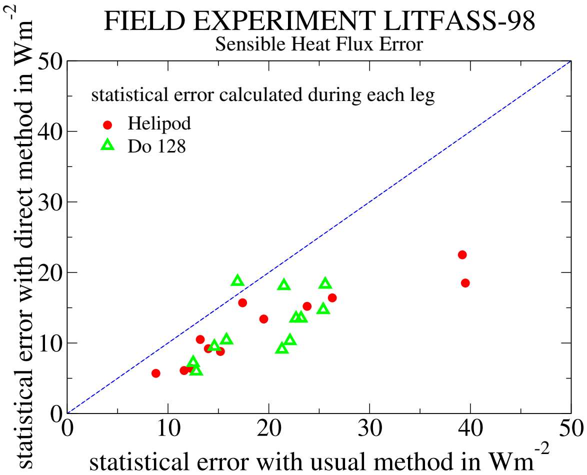

Fig. 5: *Direct comparison of the calculated statistical errors using the LITFASS 1998 flight measurements. A linear interpolation of the data gives a line (not drawn) with about 0.7 slope*

4. OUTLOOK

The next step will be to determine the individual effects of the approximation (8) and the assumption of Gaussian distributed time series on the flux error calculation. The statistical error of the measured latent heat flux were also reduced using the direct method (not shown here). Additionally, the method will be adapted to the momentum fluxes. Further analysis will follow in the future. When it comes true, that the Gaussian assumption increases the flux errors significantly, the finding of an adequate distribution function for turbulent time series in various thermal stratifications would be helpful. Then an effective method is needed to solve the non-linear system of equations that delivers the parameters of the IDB(α, β, γ) distribution in an acceptable processing time.

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