13.5 THE IMPORTANCE OF THE PRECIPITATION MASS SINK IN TROPICAL CYCLONES

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1. INTRODUCTION AND BACKGROUND

Traditionally, the concept of mass conservation is used in conceptual and NWP models of the atmosphere. Mathematically, this concept takes the following form of the mass continuity equation:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0. \qquad (1)$$

Applying eq. (1) to a control volume, the mass inflow (outflow) per unit volume must equal the rate of mass increase (decrease) per unit volume (e.g. Holton 1992, section 2.5.1). Atmospheric mass is not conserved, however, in a moist atmosphere in which phase changes of water and/or precipitation are occurring. Therefore, eq. (1) should be rewritten to include these effects, as shown by Dutton (1986, section 8.1) and by Trenberth (1991) in the following generic form:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = \mathrm{Evap} - \mathrm{Precip}. \qquad (2)$$

Assuming a hydrostatic atmosphere, the pressure at a given height is proportional to the total mass of the overlying air. Therefore, water mass removal via downward precipitation flux will lower the pressure. Note that evaporation from the surface will add mass and thereby increase the pressure, but evaporation tends to occur over a broader area than precipitation.

2. APPLICATION TO TROPICAL CYCLONES

Although the authors acknowledge that the dynamics of tropical cyclogenesis are dominated by air-sea interaction and latent heat release (e.g. Emanuel 1986), the possibility that precipitation mass removal may also provide a non-negligible contribution to these dynamics is explored here. A simple schematic of the precipitation mass sink effects on a tropical cyclone is shown in Fig. 1. On average, the most concentrated area of heavy precipitation occurs near the eyewall, with less heavy precipitation out towards the fringes of the storm (except perhaps in the spiral bands). Also, evaporation from the ocean is restricted in areas where the lower atmosphere is nearly saturated. This horizontal precipitation gradient leads to an unbalanced horizontal pressure gradient force (HPGF) with a localized pressure fall maximum near the eyewall. Two possible feedback mechanisms then emerge in response to the unbalanced HPGF: 1) air and moisture convergence towards the eyewall and 2) vorticity generation and a tangential wind speed increase. Note that the high inertial stability and small Rossby radius of deformation in tropical cyclones favors feedback 2 over feedback 1 (Ooyama 1982). Therefore, the hypothesis advanced here is that the heavy precipitation in tropical cyclones removes a non-negligible amount of atmospheric mass, leading to dynamical feedbacks including but not limited to a hydrostatic surface pressure decrease and a tangential surface wind speed increase.

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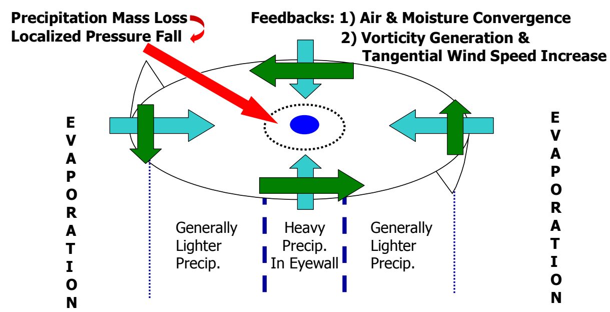


Figure 1. Precipitation mass sink in tropical cyclones.

3. TROPICAL CYCLONE MASS BUDGET: HURRICANE LILI (2002) MM5 SIMULATION

A mass budget was performed using a PSU/NCAR MM5 simulation of Hurricane Lili (2002) during part of the actual hurricane's intensification stage (00 UTC 01 to 12 UTC 02 October 2002). Model simulation parameters include 36-km/12-km one way nested grid spacing (Fig. 2), 38 vertical levels, GFS 95-km initialization and boundary conditions updated every 6 hours, Betts-Miller cumulus parameterization, Goddard cloud microphysics, the MRF PBL scheme, and the cloud-radiation scheme. The purpose of this simulation was to create a physically realistic precipitation dataset, not necessarily to simulate the actual hurricane with great accuracy, but Figure 3 is provided to show that the model simulated hurricane was reasonably consistent with the actual hurricane.

The MM5 model does not explicitly account for the precipitation mass sink effect on the pressure tendency (Dudhia 2002, personal communication). Therefore, if a cylinder is drawn around the model storm center that has a given radius and extends from the surface to the top of the model atmosphere, the change in average sea-level pressure over time within that cylinder should be approximately equal to the net lateral mass flux into and out of that cylinder, assuming that no atmospheric mass can enter or leave the cylinder vertically, as shown in eq. (3):

$$\left(\frac{d\,\overline{p}_{sfc}}{d\,t}\right)_{\text{model\_storm}} = \int_0^{H_{top}}\oint g\rho u_r \; ds\; dz, \qquad (3)$$

where $\overline{p}_{sfc}$ is the average sea-level pressure within the cylinder and u_r is the radial wind component. A complete mass budget for the storm, however, should include the effect of precipitation on hydrostatic pressure reduction within the cylinder in addition to the net lateral mass flux, as shown in eq. (4):

$$\left(\frac{d\,\overline{p}_{sfc}}{d\,t}\right)_{\text{complete}} = \left(\frac{d\,\overline{p}_{sfc}}{d\,t}\right)_{\text{model\_storm}} - g\,\rho_\ell\,\overline{R}, \qquad (4)$$

where ρ_l is the density of liquid water and $\overline{R}$ is the average rain rate within the cylinder. Note that evaporation from the sea surface within the cylinder can be neglected relative to precipitation (see Palmén and Riehl 1957, section 6).

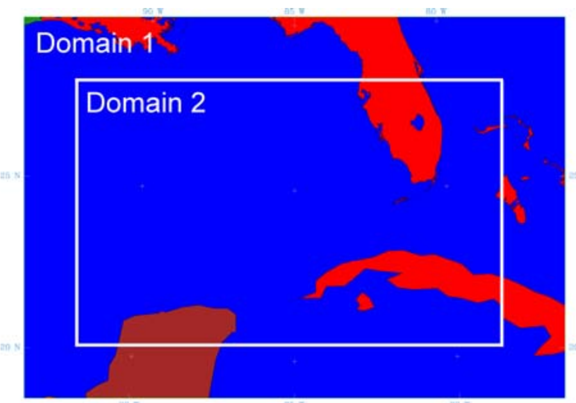


Figure 2. Nested domain configuration for MM5 simulation; domain 1 has 36-km grid spacing and domain 2 has 12-km grid spacing.

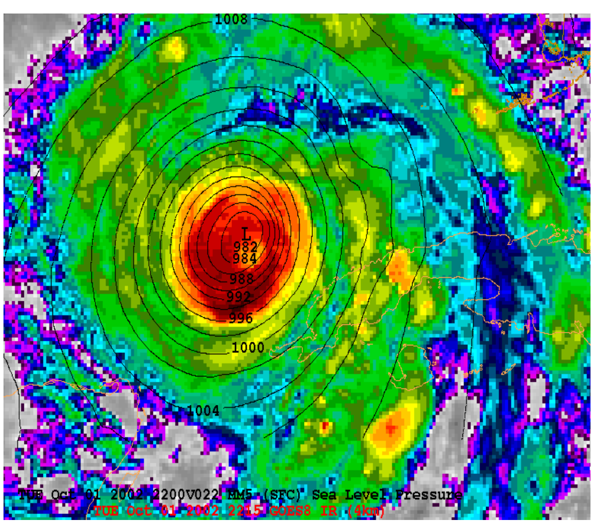


Figure 3. MM5 sea-level pressure field (hPa) at 22 UTC and GOES-8 IR image at 2215 UTC 01 October 2002.

During hours 30 through 35 of the MM5 simulation, the average sea-level pressure within a cylinder of 100-km radius around the storm decreased by 2.29 hPa. During that same time period, the hydrostatic pressure decrease needed to explain the mass loss due to precipitation was 7.25 hPa. Although the pressure decrease due to precipitation would not be fully realized because of compensating mass convergence, the fact that the amount of atmospheric mass removed via precipitation exceeded that needed to explain the model sea level pressure decrease means that the precipitation mass sink should not be neglected for a developing hurricane, and the physical effects of the precipitation mass sink are worthy of further investigation. To quantify the actual effects of the precipitation mass sink, it is necessary to perform model sensitivity experiments with and without the precipitation mass sink term(s), a task better suited for models other than MM5 (Dudhia 2002, personal communication).

4. MODEL SENSITIVITY EXPERIMENTS

The neglect of moisture sources and sinks in NWP models has been questioned by Trenberth (1991), Gu and Qian (1991), Qiu et al. (1991), Qiu et al. (1993), Van den Dool and Saha (1993), Savijärvi (1995), and more recently by Davies et al. (2002) and Lackmann and Yablonsky (2004). Dr. Kerry Emanuel has generously provided us with the use of the idealized nonhydrostatic axisymmetric numerical model of Rotunno and Emanuel (1987), which we have since modified to include the precipitation mass sink. Also, Dr. Fedor Mesinger has recently created a modified workstation version of the NCEP Eta model (e.g. Mesinger et al. 1988) that includes water vapor and hydrometeor sources and sinks, and he has generously provided us with its use.

4.1 *Nonhydrostatic Axisymmetric Numerical Model*

The nonhydrostatic axisymmetric numerical model of Rotunno and Emanuel (1987) is based upon the original model developed by Klemp and Wilhelmson (1978) and modified by Willoughby et al. (1984). In its present form, the model uses the governing equations for compressible, axisymmetric flow on an f-plane in cylindrical coordinates. The conservation of mass equation, which in essence is a nondimensionalized pressure tendency equation, is as follows:

$$\frac{\partial \pi}{\partial t} + \frac{\overline{c}^2}{c_p \overline{\rho}\overline{\theta}_v^2}\left\{\frac{1}{r}\frac{\partial\left(ru\overline{\rho}\overline{\theta}_v\right)}{\partial r} + \frac{\partial\left(w\overline{\rho}\overline{\theta}_v\right)}{\partial z}\right\} = 0. \qquad (5)$$

Eq. (5) can be modified to include the precipitation mass sink effect as follows (neglecting evaporational effects):

$$\frac{\partial \pi}{\partial t} + \frac{\overline{c}^2}{c_p \overline{\rho}\overline{\theta}_v^2}\left\{\frac{1}{r}\frac{\partial\left(ru\overline{\rho}\overline{\theta}_v\right)}{\partial r} + \frac{\partial\left(w\overline{\rho}\overline{\theta}_v\right)}{\partial z}\right\} = \left.\frac{\partial \pi}{\partial t}\right|_{\text{mass\_sink}}, \qquad (6)$$

where $$\left.\frac{\partial \pi}{\partial t}\right|_{\text{mass\_sink}} \approx \frac{R}{c_p P_0^{R/c_p}} P^{R/c_p - 1} \left.\frac{\partial p}{\partial t}\right|_{\text{mass\_sink}} \qquad (7)$$

and $$\left.\frac{\partial p}{\partial t}\right|_{mass\_\sin k} \approx -g\int_z^{z_T} \overline{\rho u}\frac{\partial q_l}{\partial r}dz + g\overline{\rho}_z\left(w - V_T\right)_z q_{lz}. \qquad (8)$$

In this way, the pressure is now modified at each model level to account for both the vertically integrated lateral advection of hydrometeors in the overlying column and the hydrometeor flux through the bottom (i.e. the current model level) of the column.

Sensitivity experiments were performed using the default input parameters by running the model without code modifications (CTRL) and with code modifications to account for the precipitation mass sink in the model pressure tendency equation (MSNK). The difference in central pressure between the runs is shown in Fig. 4.

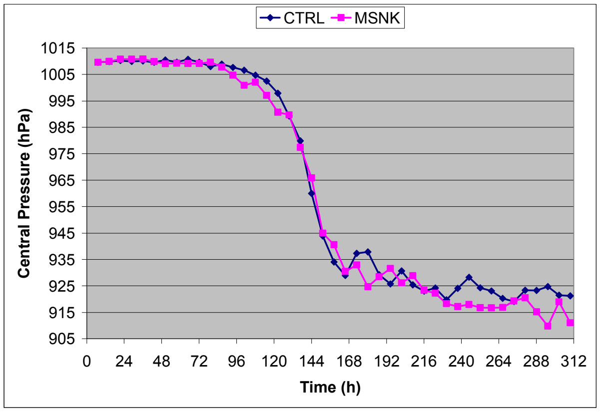


Figure 4. Nonhydrostatic axisymmetric numerical model results of central pressure vs. time for CTRL and MSNK.

MSNK begins to deepen earlier than CTRL (~F096), but during the most rapid intensification period, neither MSNK nor CTRL is systematically deeper than the other. Between F168 and F216, the central pressures in both runs oscillate between relatively higher and lower values, but after F216, MSNK remains deeper than CTRL. Also, MSNK continues to oscillate at the end of the model run, suggesting possible future deepening, whereas CTRL approaches a steady state. Note that at times, the difference in central pressure between the two runs is ~15 hPa.

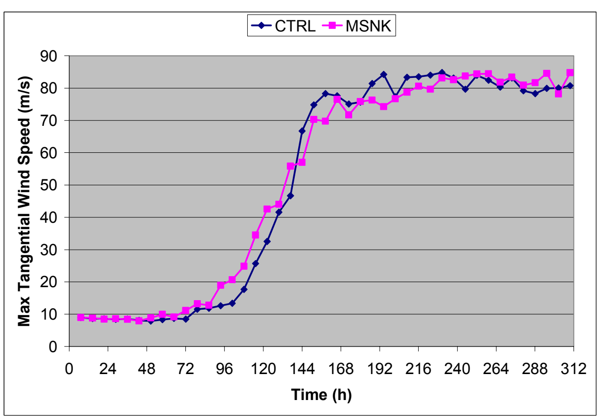


Figure 5. Nonhydrostatic axisymmetric numerical model results of maximum tangential wind speed vs. time for CTRL and MSNK.

The difference in maximum tangential wind speed between the runs is shown in Fig. 5. Note that the radius of maximum winds also varies between the runs (not shown). Analogous to the central pressure differences, the maximum winds in MSNK increase earlier than CTRL (~F096), but during the rest of the run, neither MSNK nor CTRL has systematically stronger maximum winds than the other. Again, CTRL approaches more of a steady state than MSNK by the end of the run. Note that at times, the difference in maximum tangential wind speed between the two runs is ~10 m s^{-1}.

4.2 *Workstation Version of the NCEP Eta Model: Hurricane Isabel (2003) Simulations*

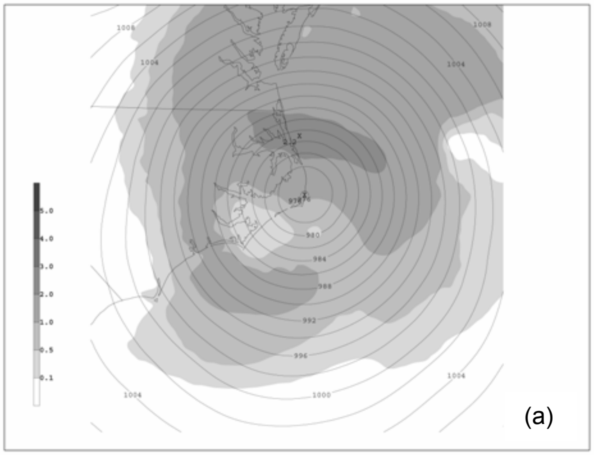
The workstation version of the NCEP Eta model is chosen because it has a hydrostatic option, which simplifies the inclusion of the precipitation mass sink effect. Assuming hydrostatic balance, eq. (1) can be expressed in terms of the vertical coordinate η (e.g. Kasahara 1974, Mesinger et al. 1988, Davies et al. 2002) as follows:

$$\frac{\partial}{\partial \eta}\left(\frac{\partial p}{\partial t}\right) + \nabla \cdot \left(\mathbf{v}\frac{\partial p}{\partial \eta}\right) + \frac{\partial}{\partial \eta}\left(\dot{\eta}\frac{\partial p}{\partial \eta}\right) = 0, \qquad (9)$$

where
$$\eta = \frac{p - p_T}{p_s - p_T}\eta_s \qquad (10)$$

and
$$\eta_s = \frac{p_{rf}(z_s) - p_T}{p_{rf}(0) - p_T}. \qquad (11)$$

The inclusion of water vapor and hydrometeor sources and sinks in the Eta model involves modification of not only eq. (9), which is eq. (2.5) of Mesinger et al. (1988), but also of eq. (2.7-2.9) of Mesinger et al. (1998). The details of these model equation modifications will not be shown here, but they are given in an unpublished document generously provided by Dr. Mesinger (2003).
Sensitivity experiments were performed with (MSNK) and without (CTRL) the precipitation mass sink modifications for a simulation of Hurricane Isabel (2003) during the hours up to and including landfall (12 UTC 17 to 00 UTC 19 September 2003). Model simulation parameters include 15-km grid spacing without nesting, 60 vertical levels, GFS 95-km initialization and boundary conditions updated every 6 hours, Betts-Miller-Janjić cumulus parameterization, and the hydrostatic option. The differences in the sea-level pressure fields and the 3-hourly rainfall rates between the two model runs at forecast hour 33 (21 UTC 18) is shown in Fig. 6. Figure 6(b) reveals that the greatest difference in sea-level pressure between the runs (~2 hPa lower in MSNK than CTRL) is near the center of the model hurricane, which is consistent with most of the other hours near the end of the model forecast cycle (not shown). Also, asymmetries in the model pressure field difference tend to coincide with the regions of greatest model QPF disparity between the runs (Fig. 6(b)), suggesting the varying degree of interaction between rain rate and pressure tendency in MSNK and CTRL.



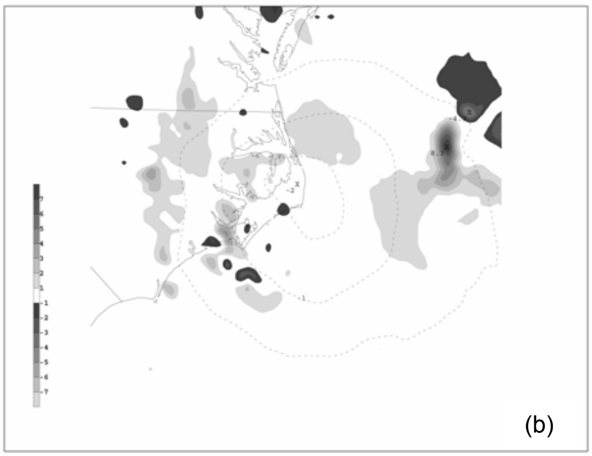


Figure 6. Workstation version of the NCEP Eta model sea-level pressure (hPa) and 3-hourly rainfall (shaded as in the color bar) for a) CTRL, rainfall in cm, and b) MSNK-CTRL, rainfall in mm.

5. CONCLUSIONS AND FUTURE WORK

The theory and initial MM5 mass budget results are discussed in greater detail in Lackmann and Yablonsky (2004). All of the model sensitivity experiments using the idealized nonhydrostatic axisymmetric numerical model and the workstation version of the NCEP Eta model should be considered preliminary as of the writing of this preprint, but the results thus far reveal that although the precipitation mass sink is not a dominant effect, it is non-negligible. Also, the feedbacks may be significant, including locally enhanced precipitation. Continuing research involves adding evaporational effects on the pressure tendency to the idealized model, as well as performing additional simulations with the Eta model, the details of which still need to be scrutinized for possible omissions and/or inconsistencies such as the precipitation mass sink effects in the cumulus parameterization scheme, for example. Also, certain feedbacks may become more important in the early stages of tropical cyclogenesis or in extratropical heavy precipitation events where the lack of inertial stability would allow for more moisture convergence. Future results and those presented here will be included in Richard Yablonsky's master's thesis and perhaps future publications.

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