

# MATHEMATICAL MODELING OF LARGE FOREST FIRE INITIATION

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## 1. INTRODUCTION

A large technogeneous or space catastrophe, as a rule, is known to accompany by the initiation of mass forest fires (Glastone (1962) and Goldin (1992)). In connection with the estimate of ecological and climatic impacts of severe fires, the prediction of the process influence on forest phytosenoses and ground layer state of the atmosphere is of interest. Considering that, natural investigations of these problems are merely impossible, methods of mathematical modeling are urgent. The most complete discussion of the problem of forest fire modeling is provided by a group of co-workers at Tomsk State University. A fairly complete bibliography of these works is given by Grishin (1997). In particular, general mathematical model of forest fire based on an analysis of known experimental data and using concept and methods from reactive media mechanics. Within the framework of this model, the forest and combustion products during a fire represent a non – deformable porous – dispersed medium. Based on this model of forest fires the problems of forest fire initiation and spread are studied with due consideration for the effect of a turbulent atmosphere and the actual structure of the forest biogeocenosis. In papers of Grishin (1997), Grishin and Perminov (1993) and Perminov (1995) attention is given to questions of description of the initial stage in the development of a massive forest fires initiated by heat radiation (for example, from a Tunguska celestial body fall).

## 2. THEORETICAL ANALYSIS AND PHYSICAL MATHEMATICAL MODEL

It is known that in the case of entering of body in atmosphere with supersonic the powerful ballistic shock wave is arose at the around stagnation point and the gas temperature has high value (Goldin (1992)). As a result of this the sublimation of celestial body matter is took place and temperature tension is arose. Therefore the celestial body is destroyed in Earth atmosphere or its remains are falling with formation of crater. During the celestial body flying a fraction of its kinetic energy transformed into radiation and the heating of Earth surface and forest phytocenoses are took place. As a rule, the sizes of celestial bodies are small as compare with radius of Earth and thickness of overterrestrial layer, it may be considered to be a point source of radiation (Goldin (1992)). It is supposed, that the celestial body is destroyed as a result of explosion in Earth atmosphere.

Let the radiant energy source be at a height of  $H$  from the Earth surface at the initial moment (Fig.1).

Where  $R_0$  - the distance from source of radiation to vegetation cover,  $h$  - the height of forest massif,  $O$  - epicenter of explosion - is the center of decart coordinate system. An upper boundary  $z=h$  of the forest massif is acted upon by an intensive radiant flux  $q_R(r,t)$ , which defined at the flight stage from (Goldin(1992), Grishin and Perminov(1993)).

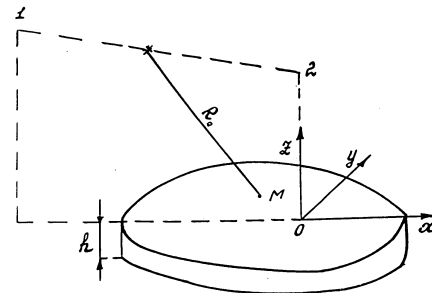


Figure 1.

$$q_R(r,t) = \frac{t_p I \sin L}{4\pi R_0^2}, \quad I = 0.5 C_H \rho V^2 S_m,$$

where  $I$ ,  $V$ ,  $S_m$  – the brightness, velocity and midship section square of Tunguska fireball,  $C_H$  - the fraction of kinetic energy transformed into radiation;  $L$  - angle between radiative heat flux and vegetation cover,  $\rho$  - density of atmosphere at a height  $H$ .

After explosion of celestial body (at moment  $t=t_1$ ) the light flux is defined according to the data Glastone (1962)

$$q_R(r,t) = \frac{t_p P_m \sin L}{4\pi R_0^2} \begin{cases} (t-t_1)/t_m, & t < t_m \\ \exp(-k_0((t-t_1)/t_m - 1)), & t \geq t_m \end{cases}$$

$$t_m = t_1 + 0.032 W_0^{0.5}, \quad P_m = 1.33 W_0^{0.5}.$$

$R_0$  - the distance from the source of radiation to forest;  $t_p$  - atmospheric transmissivity coefficient;  $kT/sec$ ,  $P_m$  - maximum value of heat radiative impulse at moment  $t=t_m$ ;  $W_0$  - weapon yield,  $kT/sec$ ;  $k_0$  - empirical coefficient.

When the radiant energy reaches vegetation cover, it causes heating forest fuels, evaporation of moisture and subsequent thermal decomposition of solid material, with evaporating pyrolysis products liberation. The last material is burning in the atmosphere and interacting with the oxygen of air. Since the intensities of radiant and convective fluxes to the forest canopy in  $x$  and  $y$  directions are low as compared to the  $z$  direction, it enabled the problem to be treated in the quasy- one- dimensional setting up. The forest canopy is considered as a homogeneous, two-temperatures, reacting, non-deformed medium. Temperatures of condensed (solid)  $T_s$  and gaseous  $T$

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phases are separated out. The first includes a dry organic substance, moisture, condensed pyrolysis products and mineral part of forest fuels. In the gaseous phase we separate out only the components  $C_\alpha$  necessary to describe reactions of combustion ( $\alpha=1$  - oxygen, 2 - pyrolysis combustion products of forest fuels (CO and etc.), 3 - the rest of components). The solid phase constituting forest fuels has no intrinsic velocity, and its volumetric fraction, as compared to the gaseous phase, can be neglected in appropriate equations. Radiation is the governing mechanism of the energy transfer in this case. The solid phase mainly absorbs, reflects and reradiates. Diffusion approximation is used to describe the transfer in this specific continuous medium.

The system of equations for the celestial body is

$$\frac{dV}{dH} = \frac{C_x \rho V S_T}{2m \sin \alpha_0} - \frac{g}{V},$$

$$\frac{dm}{dH} = 6 \frac{C_x \rho V^2 S_T}{\sin \alpha_0},$$

$$\frac{d\alpha}{dH} = \frac{C_y \rho S_T}{2m \sin \alpha_0} + \left( \frac{1}{R_z} - \frac{g}{V^2} \right) \text{ctg} \alpha_0, \quad (1)$$

$$\frac{dt}{dH} = -\frac{1}{V \sin \alpha_0}, \quad \frac{d\ell}{dH} = -\frac{1}{\sin \alpha_0},$$

$$S_m = \pi R_T^2, R_T = \left( \frac{3m}{4\pi \rho_T} \right)^{\frac{1}{3}}, \sigma_0 = \frac{2Q}{C_x},$$

where  $m$ ,  $R_T$ ,  $\rho_T$  - mass, radius and density of celestial body,  $C_x$ ,  $C_y$  - coefficients of drag and lifting,  $t$  - time,  $\alpha$  - angle of trajectory inclination,  $\ell$  - the distance along of trajectory,  $g$  - constant acceleration,  $R_{\text{azz}}$  - Earth radius,  $\sigma_0$  - ablation coefficient,  $\Lambda$  - heat transfer coefficient,  $Q$  - celestial body specific energy of ablation. Setting the initial point of considering trajectory  $H=60$  km, the height of explosion - 6.5 km,  $\alpha_0=40^\circ$ . The initial values of velocity and mass of celestial body are set up according to the data Goldin (1992).

To describe convective transfer controlled by the wind and gravity in forest canopy, we use Reynolds equations for the description of turbulent flow taking into account diffusion equations for chemical components and equations of energy conservation for gaseous and condensed phases. For the objective of the present studies, wind (velocity) speed was considered to be relatively not high, and the energy was considered mainly to be transferred due to radiation. Since the intensities of radiant and convective fluxes to the forest canopy in horizontal directions ( $x$  and  $y$ ) are low as compared to the  $z$  direction, it enabled the problem to be treated in the quasi- one- dimensional setting up and we supposed all parameters are depended on  $t$  and  $z$  - vertical coordinate

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial z}(\rho w) = \dot{m}; \quad (2)$$

$$\frac{\partial}{\partial t}(\rho w) + \frac{\partial}{\partial z}(\rho w^2) = -\frac{\partial p}{\partial z} + \frac{\partial}{\partial z}(-\rho \overline{w'^2}) - \rho s c_d w^2 - \rho g; \quad (3)$$

$$\begin{aligned} & \frac{\partial}{\partial t}(\rho c_p T) + \frac{\partial}{\partial z}(\rho w c_p T) = \\ & = \frac{\partial}{\partial z}(-\rho c_p \overline{w'T'}) + k(cU_R - 4\sigma T^4) + \\ & \quad + q_5 R_5 + \alpha_v (T_s - T); \end{aligned} \quad (4)$$

$$\begin{aligned} & \frac{\partial}{\partial t}(\rho c_\alpha) + \frac{\partial}{\partial z}(\rho w c_\alpha) = \\ & = \frac{\partial}{\partial z}(-\rho \overline{w'c'_\alpha}) - R_{5\alpha}, \quad \alpha = 1, 2; \end{aligned} \quad (5)$$

$$\frac{\partial}{\partial z} \left( \frac{c}{3k} \frac{\partial U_R}{\partial z} \right) - k(cU_R - 4\sigma T_s^4) = 0; \quad (6)$$

$$\begin{aligned} & \sum_{i=1}^4 \rho_i c_{pi} \varphi_i \frac{\partial T_s}{\partial t} = q_3 R_3 - q_2 R_2 + \\ & \quad + k(cU_R - 4\sigma T_s^4) + \alpha_v (T - T_s); \end{aligned} \quad (7)$$

$$\rho_1 \frac{\partial \varphi_1}{\partial t} = -R_1, \quad \rho_2 \frac{\partial \varphi_2}{\partial t} = -R_2, \quad (8)$$

$$\rho_3 \frac{\partial \varphi_3}{\partial t} = \alpha_c R_1 - \frac{M_c}{M_1} R_3, \quad \rho_4 \frac{\partial \varphi_4}{\partial t} = 0;$$

$$\sum_{\alpha=1}^3 c_\alpha = 1, \quad p_e = \rho RT \sum_{\alpha=1}^3 \frac{c_\alpha}{M_\alpha},$$

$$R_1 = k_1 \rho_1 \varphi_1 \exp\left(-\frac{E_1}{RT_s}\right),$$

$$R_2 = k_2 \rho_2 \varphi_2 T_s^{-0.5} \exp\left(-\frac{E_2}{RT_s}\right),$$

$$R_3 = k_3 \rho \varphi_3 s_\sigma c_1 \exp\left(-\frac{E_3}{RT_s}\right),$$

$$R_5 = M_2 k_5 \left( \frac{c_1 M}{M_1} \right)^{0.25} \left( \frac{c_2 M}{M_2} \right) T^{-2.25} \exp\left(-\frac{E_5}{RT}\right).$$

The system of equations (1)–(9) must be solved taking into account the following initial and boundary conditions:

$$t = 0: w = 0, \quad T = T_e, \quad c_\alpha = c_{\alpha e}, \quad T_s = T_e, \quad (9)$$

$$\varphi_i = \varphi_{ie};$$

$$z = z_0: \frac{\partial w}{\partial z} = 0, \quad \frac{\partial T}{\partial z} = 0, \quad \frac{\partial c_\alpha}{\partial z} = 0, \quad (10)$$

$$-\frac{c}{3k} \frac{\partial U_R}{\partial z} = \frac{\varepsilon}{2(2-\varepsilon)} (4\sigma T_s^4 - cU_R);$$

$$z = h : \frac{\partial w}{\partial z} = 0, \frac{\partial T}{\partial z} = 0, \frac{\partial c_\alpha}{\partial z} = 0, \quad (11)$$

$$\frac{c}{3k} \frac{\partial U_R}{\partial z} + \frac{c}{2} U_R = 2q_R(r, z).$$

Here and above  $z$  read from the ground cover,  $w$  are the velocity components;  $t$  is time;  $U_R$  - density of radiation energy,  $k$  - coefficient of forest fuel adsorption,  $p$  - pressure;  $c_p$  - constant pressure specific heat of the gas phase,  $c_{pi}$ ,  $\rho_i$ ,  $\varphi_i$  - specific heat, density and volume of fraction of condensed phase (1 - dry organic substance, 2 - moisture, 3 - condensed pyrolysis products, 4 - mineral part of forest fuel),  $\alpha_V$  - coefficient of heat exchange,  $q_i$  - thermal effects of chemical reactions. To define source terms, which characterize inflow (outflow of mass) in a volume unit of the gas-dispersed phase, the following formulae were used for the rate of formulation of the gas-dispersed mixture  $\dot{m}$ , outflow of oxygen  $R_{S1}$ , changing carbon monoxide  $R_{S2}$ ,  $R_i$  - the rates of chemical reactions.

Thus, the solution of the system of equations (1)-(8) with initial and boundary conditions (9)-(11) may result in defining the of velocity, temperature, component concentrations and radiation density. The last conditions in (11) were obtained in  $P_1$ -approximation of the spherical harmonics method. It should be noted that this system of equations describes processes of transfer within the entire region of the forest massif, which includes the space between the underlying surface and the base of the forest canopy (for this region the coefficients  $\dot{m} = \alpha_V = 0$ ), the forest canopy  $h_1 < z < h_2$ , for which  $\dot{m} \neq 0$ ,  $\alpha_V \neq 0$ , and the space above it, for which  $\dot{m} = 0$ ,  $\alpha_V = 0$ .

The thermodynamic, thermophysical and structural characteristics correspond to forest fuels in the canopy of a pine forest, and are given numerically (following Grishin(1997)) during the solution of the first problem.

The system of equations (2) - (8) contains terms associated with turbulent diffusion, thermal conduction, and convection, and needs to be closed. The component of the tensor of turbulent stresses

$\rho \overline{w'^2}$ , as well as the turbulent fluxes of heat and mass  $\overline{w'T'}$ ,  $\overline{w'c'_\alpha}$  are written in terms of the gradients of the average flow properties.

### 3. NUMERICAL METHOD AND VERIFICATION OF THE MODEL

The boundary-value problem (1)-(11) was solved numerically using the method of splitting according to physical processes. In the first stage, the hydrodynamic pattern of flow and distribution of scalar functions was calculated. The system of ordinary differential equations of chemical kinetics obtained as a result of splitting was then integrated. A discrete analog was obtained by means of the control volume method using the SIMPLE algorithm (Patankar (1984)).

The accuracy of the program was checked by the method of inserted analytical solutions. Analytical expressions for the unknown functions were substituted in (2)-(8) and the closure of the equations was calculated. This was then treated as the source in each equation. Next, with the aid of the algorithm described above, the values of the functions used were inferred with an accuracy of not less than 1%. The effect of the dimensions of the control volumes on the solution was studied by diminishing them. The time interval was selected automatically.

### 4. RESULTS AND DISCUSSION

The distribution of temperature of gas and condensed phases, velocity, component mass fractions, and volume fractions of phases were obtained numerically at different distances from the source of radiation to forest and different instants of time. The full energy of celestial body ( $E$ ) is  $10^{16}$  J which consists of kinetic energy ( $K_0$ ) and energy of explosion -  $E_0$ . A fraction of the celestial body energy transformed into radiation equals to 0.1.

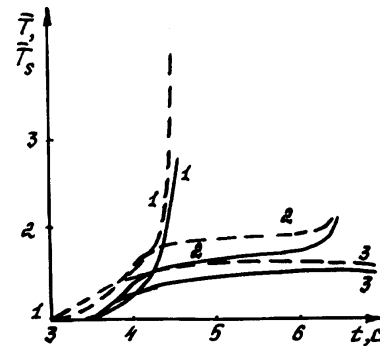


Figure 2.

1 -  $x, y = 0$ ; 2 -  $x = -10$  km,  $y = 0$ ; 3 -  $x = -15$  km,  $y = 0$   
;  $-\bar{T} = T/T_e$ ;  $---\bar{T}_s = T_s/T_e$ ,  $T_e = 300K$ .

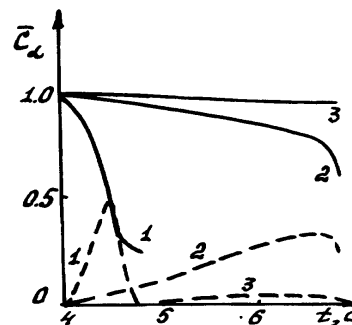


Figure 3.

1 -  $x, y = 0$ ; 2 -  $x = -10$  km,  $y = 0$ ; 3 -  $x = -15$  km,  $y = 0$   
;  $\bar{c}_\alpha = c_\alpha / c_{1e}, c_{1e} = 0.23$ .

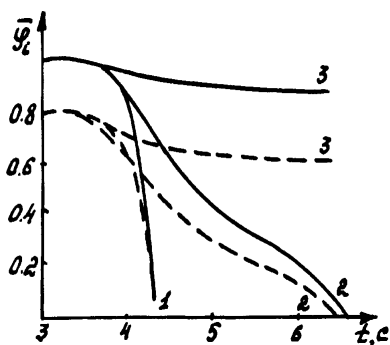


Figure 4.

1-x, y = 0; 2-x = -10 km, y=0; 3-x = -15km, y = 0.

$$- \bar{\varphi}_1 = \varphi_1 / \varphi_{1c}, \quad - - - \bar{\varphi}_2 = \rho_2 \varphi_2 / \rho_c.$$

Figures 2 – 4 illustrate the time dependence of dimensionless temperatures of gas and condensed phases, concentrations of components and relative volume fractions of solid phases at upper boundary  $z=h$  of the forest for various distances from the epicenter (solid curves — temperature of gas phase; dash curves — temperature of solid phase). Fig. 3 (solid curves — concentration of oxygen; broken curves — concentration of combustible products of pyrolysis (CO)) illustrates the distribution of concentrations of components of the gas phase. At the moment of ignition the CO burns away, and the concentration of oxygen is rapidly reduced. The temperatures of both phases reach a maximum value at the point of ignition.

The ignition processes is of a gas - phase nature, i.e. initially heating of solid and gaseous phases occurs, moisture is evaporated. Then decomposition process into condensed and volatile pyrolysis products starts, the later being ignited at the upper boundary of the forest canopy. At the ignition zone boundary gaseous fuel products are also generated, but they are not ignited because of not high enough radiant flux power.

On the basis of data calculated for this problem as the ignition condition, the condition was

$$\left. \frac{\partial^2 T}{\partial t^2} \right|_{t=t_K, x=x_*, y=y_*, z=h} = 0,$$

where  $h$  - is an upper boundary height of the forest canopy, and  $t = t_K$  is the ignition time, which corresponds to the value of time, at which there is a second bending point in the temperature curve  $T|_{x=x_*, y=y_*, z=h} = T_0(t)$ .

From the calculation results of forest canopy ignitions (Fig.2 - 4), it is seen that three conditions are realized: the first is factual combustion, the second is so - called normal state of ignition, and third is non - ignition (non - flammability). Within the framework of the problem mentioned above the sizes of the ignition zones were defined. Contours derived for collision catastrophes look like a circle arc in the neighborhood of epicenter of the explosion and take the form of the

ellipse extended in the flight trajectory projection direction of Tunguska celestial body (Figures 5 – 7). As distinct from collision catastrophes, ignition contours take the form of a circumference as illustrated as the result of numerical experiments for the ignition of a homogeneous vegetation layer by radiation from the air nuclear explosion. Figures 5 – 7 present the dynamic of the development of forest fire contours for different types of forest (pine, larch and birch).

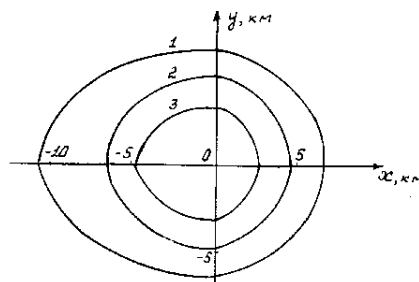


Figure 5.

1 - t=7.0 sec, 2 - t=5 sec, 3 t=4.3 sec

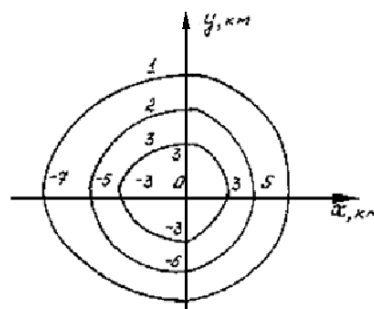


Figure 6.

1 - t=7.0 sec, 2 - t=5 sec, 3 t=4.3 sec.

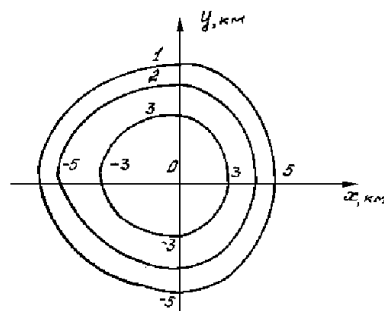


Figure 7.

1 - t=7.0 sec, 2 - t=5 sec, 3 t=4.3 sec.

## 5. CONCLUSION

A mathematical model has been developed for the simulation of the problem on the vegetation ignition as meteorites fall down in the Earth's atmosphere. The results obtained agree with the laws of physics and experimental data of Goldin (1992). Thus, the model can be potentially utilized for the modeling of forest ignition by radiant energy and for the prediction of forest fire contours.

## 6. REFERENCES

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