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1. INTRODUCTION

Resonant damping theory of Schecter, Montgomery, and Reasor (2002) and Schecter and Montgomery (2003) has been applied to the alignment of hurricane like vortices by Reasor and Montgomery (2001) and Reasor, Montgomery, and Grasso (2004). The theory tracks the evolution of discrete vortex-Rossby-waves that describe the tilt of a symmetric-barotropic vortex. When the Rossby waves decay, the vortex becomes vertically aligned. This occurs through resonant damping by interaction of the cyclonically rotating Rossby wave with the swirling motion of the vortex at the critical radius where their angular speeds match (typically at radii between 200 and 800 km). Schecter and Montgomery extended the theory to rapidly rotating vortices. Damping and vortex alignment always occur whenever the symmetric vortex potential vorticity (PV) gradient at the critical radius has the same sign as the vorticity gradient near the inner core of the vortex. Amplification or misalignment occurs when these gradients are opposite. For the Gaussian PV vortex considered by Reasor, Montgomery and Grasso alignment always occurs on the f plane. Real atmospheric vortices may have a PV minimum outside the inner core. S. Jones (1995) considered such a vortex and the vortices considered here also have a PV minimum. This is done here by having the vortex winds decay exponentially beyond the radius of maximum wind.

In this paper we consider isolated initial vortices that are resonantly damped on the f plane by their initial state, but which become unstable on the beta plane, as a result of nonlinear interaction between the symmetric vortex and the beta gyres. In our experiments both the critical radius and the PV minimum migrate inward. The PV minimum moves faster, so that the sign of the PV gradient changes at the critical radius. In order to apply the barotropic theory, the initial model states are made nearly barotropic by having the same wind in each of the two layers, while the mass gyres differ. Experiments not reported here suggest that it is reasonable to do this.

2. MODEL

The model descends from the Vortex Tracking Semispectral (VTSS) model of Willoughby (1994).

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The model has two homogeneous incompressible layers with a free surface patterned after Ooyama (1969), but may be capped with an inert third layer. The flow is partitioned into a fixed environment and the vortex. The time varying vortex is, in tern, a superposition of an axisymmetric vortex in gradient balance and asymmetries that vary sinusoidally in azimuth. Radial variation is by finite differences. Spectral representation in azimuth reduces the nonlinear problem to a series of linear equations, linked by the nonlinear wave-wave interactions and forced by interactions with the environment. The coupling between the layers is through the hydrostatic pressure force that depends upon the depth and density of the layers. Horizontal mixing is present. The coordinates are cylindrical and the origin follows the lower vortex center by the method of alpha-gyre removal. Wavenumber three truncation is currently used on a 4000 km domain with a 4 km mesh interval. The densities are 10 and 9 for the lower and upper layers, respectively. The stability factor is 0.1 for the lower layers and is 1.0 for the free surface. The layer depths are 4 km each, so that the external gravity wave speed is 276 m s^{-1} , while the internal mode is 45 m s^{-1} .

3. RESULTS

The initial vortex has 30 m s^{-1} winds at the maximum wind radius of 40 km in each layer. The wind varies linearly with radius to the maximum and decays exponentially beyond. The decay factor is 400 km. The fluid depth at the center is 3744 m, and 4000 m for the lower and upper layers, respectively. When this vortex is initially aligned on the f plane with no environment current present, nothing happens. When the upper vortex is tilted 14.1 km to the northeast, the vortices rapidly accelerate towards each other while rotating cyclonically and converging toward a point about 4.4 km east and 4.7 km north of the initial position of the lower vortex. The rotation period of the vortices is about 160 h. The critical radius is near 630 km while the PV minimum is near 650 km. Thus the theory predicts resonant damping of the Rossby wave that represents the tilt. By 10 days, the vortices are only 235 m apart. It should be noted that as observed by Reasor and Montgomery (2004) the initial rapid acceleration of the vortices toward each other is mostly due to the symmetrization of the asymmetric PV of the tilt gyres by the radial gradient of the vortex winds outside of the radius of maximum wind (their spiral windup). Resonant damping explains the much slower final approach to alignment.

The tilted experiment may be run on the beta plane, but the spiral windup phase produces significant amplitude inertial oscillations that make

evaluation of the subsequent resonant damping phase more difficult. Thus on the beta plane, the vortices are initially aligned. They are forced apart by the vertical gradient of vortex PV and they begin to co-rotate cyclonically about each other. The vortex pair is also accelerated north-northwest by the beta gyres. Due to the development of the beta gyres, the vortex winds diminish in the region of the initial critical radius. This results in the inward migration of the critical radius and would seem to predict resonant damping. However, the PV minimum also migrates inward and by 72 h, the PV minimum is near 450 km, while the critical radius is near 500 km, so that resonant amplification is predicted. By 3 days, the vortices are 2 km apart. The maximum winds are about 29 m s^{-1} near 50 km radius.

By 5 days, the vortices are 6 km apart and the winds are 28 m s^{-1} at 50 km for the lower layer and a little less at the upper layer at the same radius. By 8 days the vortices are 60 km apart. The maximum winds are 27 m s^{-1} at 65 km for the lower layer and 22 m s^{-1} at 90 km for the upper layers. As the vortex tilts, some of the energy of the mean flow goes into the tilt. Some wind speed loss is also due to the horizontal mixing coefficients which are $2500 \text{ m}^2 \text{ s}^{-1}$ for the asymmetric motion and $10 \text{ m}^2 \text{ s}^{-1}$ for the symmetric winds. The present mixing coefficients are designed for use in shear flow where vortex scale instabilities must be suppressed. The current experiments, however, may be done with an order of magnitude weaker mixing.

Resonant damping theory was applied by finding the positions of the PV minimum and the critical radius at each level each 24 h out to 192 h. Through 48 h damping is predicted. After that time, amplification is predicted except for the upper layer at 192 h. By that time the increasing baroclinicity of the vortex probably negates application of the theory.

Experiments have been run with differing maximum winds and maximum wind radii. In all

cases to date, an initially stable state for the f plane, becomes unstable before 10 days elapsed time on the beta plane. Application of resonant damping theory, however, sometimes gives damping after amplification is observed. This is especially true for stronger vortices and may be the result of increasingly large baroclinicity that is not accounted for by the theory. Nevertheless, it is concluded that many real vortices may misalign, if there is no other mechanism for alignment. It may be that coupling between atmospheric layers by the convectively driven secondary circulation is required to keep tropical storm like vortices vertically aligned.

4. REFERENCES

- Jones S. C., 1995: The evolution of vortices in vertical shear. I: Initially barotropic vortices. *Q. J. R. Meteor. Soc.*, **121**, 821-851.
- Ooyama K., 1969: Numerical simulation of the life cycle of tropical cyclones. *J. Atmos. Sci.*, **26**, 3-40.
- Reasor P. D., and M. T. Montgomery, 2001: Three-dimensional alignment and corotation of weak, TC-like vortices via linear vortex Rossby waves. *J. Atmos. Sci.*, **58**, 2306-2330.
- Reasor P. D., M. T. Montgomery and L. D. Grasso, 2004: A new look at the problem of tropical cyclones in vertical shear flow: Vortex resiliency. (to appear) *J. Atmos. Sci.*
- Schechter D. A., M. T. Montgomery and P. D. Reasor, 2002: A theory for the vertical alignment of a quasigeostrophic vortex. *J. Atmos. Sci.*, **59**, 150-168.
- Schechter D. A. and M. T. Montgomery, 2003: On the symmetrization rate of an intense geophysical vortex. *Dyn. Atmos. and Oceans*, **37**, 55-88.
- Willoughby H. E., 1994: Nonlinear motion of a shallow water barotropic vortex. *J. Atmos. Sci.*, **51**, 3722-3744.