P1.67 Damping and Pumping of a Vortex Rossby Wave in a Monotonic Cyclone: Critical Layer Stirring Versus Inertia-Buoyancy Wave Emission

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1. Introduction

This abstract summarizes a new theory for discrete vortex Rossby (VR) waves in hurricane-like vortices. Such waves include precessing tilts and elliptical deformations. By now, it is well-known that

(i) a VR wave can decay by stirring of potential vorticity (PV) in its critical layer, and

(ii) a VR wave can grow by exciting outward-propagating inertia-buoyancy (IB) waves in the environment.

Past theoretical work considered parameter regimes in which either critical layer stirring or IB wave emission was negligible. Here, we derive a formula for the growth rate of a VR wave that accounts for *both* competing processes.

Our new theory provides a more complete framework for analyzing any aspect of tropical cyclone dynamics that involves discrete VR waves, such as response to environmental shear.

2. Model

For this study, we consider a relatively simple model of a tropical cyclone. The unperturbed cyclone is barotropic and in gradient balance. It is characterized by a radius r_v, a vertical thickness H, and a Rossby number $Ro > 1$. The angular velocity $[\bar{\Omega}(r)]$ and PV $[\bar{q}(r)]$ decrease monotonically with radius r. The atmosphere is dry and stably stratified. The Coriolis parameter f and buoyancy frequency N are both constant. The top and bottom boundaries of the cyclone have uniform potential temperature. The boundary at $r = r_v$ allows for the radiation of energy and angular momentum. Perturbations are governed by 3D linearized primitive equations, which employ the adiabatic, hydrostatic and Boussinesq approximations.

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3. Growth Rate Formula

In Ref. [1], we derive a formula for the growth rate of the amplitude a of a discrete VR wave, as a corollary to conservation of wave activity (angular pseudomomentum). The detailed result is given below:

$$\gamma \;\equiv\; \frac{1}{a}\frac{da}{dt} \;=\; \frac{\epsilon_{rad} - \epsilon_{cl}}{M}, \qquad (1)$$

in which

$$\epsilon_{rad} \;\equiv\; r_v^2\, \Re[UV^*]_{r_v}, \qquad (2)$$

$$\epsilon_{cl} \;\equiv\; -\frac{\pi}{n}\left[\frac{r^2\;|U|^2 d\bar{q}/dr}{|d\bar{\Omega}/dr|}\right]_{r_*}, \qquad (3)$$

and

$$M \equiv -⨍_0^{r_v} dr \left\{ \frac{r^2|Q|^2}{d\bar{q}/dr} \;+\; \frac{2(\pi m r)^2}{(NH)^2}\Re[V\Phi^*]\right\}. \quad (4)$$

Here, $U(r)$, $V(r)$, $\Phi(r)$ and $Q(r)$ are the radial wave functions for radial velocity, azimuthal velocity, geopotential height and PV, respectively. The integers m and n denote the vertical and azimuthal wave-numbers, and $\Re[\ldots]$ is the real part of the quantity in square brackets. The symbol r_* denotes the critical radius, defined by

$$\bar{\Omega}(r_*) \equiv \omega/n, \qquad (5)$$

where ω/n is the angular phase-velocity of the wave. It is assumed that $r_* < r_v$. The radial integral $⨍$ excludes a thin critical layer, centered at r_*. Note that we have used the following definition of PV: $q \equiv (\vec{\zeta} + f\hat{z}) \cdot \nabla\partial_z(\phi/N^2)$, where $\vec{\zeta}$ is the vorticity of the horizontal flow, ϕ is the geopotential height, and ∇ is the 3D gradient operator.

The weight M of a discrete VR wave is generally positive. Accordingly, the sign of $\epsilon_{rad} - \epsilon_{cl}$ determines the sign of the growth rate γ. The radiation term ϵ_{rad} is proportional to the outward flux of relative angular momentum at r_v. This flux is positive, since angular momentum leaves the vortex by a frequency-matched, outward propagating IB wave. The critical layer term ϵ_{cl} is positive, since $d\bar{q}/dr_*$ is negative. Moreover, it increases with the magnitude of $d\bar{q}/dr_*$. As a result, *there is a critical*

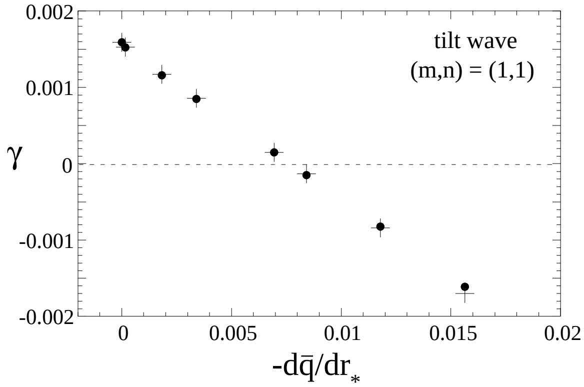

Figure 1: Growth rate of a discrete VR wave in a monotonic cyclone, with variable PV gradient at r_*. The Rossby number (Ro) and Froude number (Fr) of the cyclone are both 10, assuming the definitions $Ro \equiv 2\bar{\Omega}(0)/f$ and $Fr \equiv 2\pi r_o \bar{\Omega}(0)/NH$, where r_o is the radius of the vortex core. In the plot, the growth rate γ is normalized to $2\bar{\Omega}(0)$, and the PV gradient $d\bar{q}/dr_*$ is normalized to $2\bar{\Omega}(0)/r_o$. The text explains the difference between circles and cross-hairs.

magnitude of $d\bar{q}/dr_*$, below which the wave will grow ($\gamma > 0$), and above which it will decay ($\gamma < 0$).

4. Verification

Figure 1 shows the growth rate of a VR wave in a monotonic cyclone, with variable PV gradient at r_*. This particular wave represents a precessing tilt. The plot shows two solution sets for the growth rate. The circles were obtained from a numerical solution to the vortex eigenmode problem.† The cross-hairs were obtained from the analytical growth rate formula [Eq. (1)] of the previous section. Apparently, this formula is correct.

Moreover, this plot nicely illustrates the competition between critical layer stirring, and IB wave emission. If there is zero PV gradient at r_*, the VR wave slowly destabilizes by emitting IB waves into the environment. As the PV gradient at r_* increases, γ decreases. Once the magnitude of $d\bar{q}/dr_*$ exceeds a small threshold, the VR wave decays.

5. Balanced Dynamics

Despite their neglect of IB waves, balance models seem to adequately describe the dynamics of VR waves in some hurricane-like vortices.[2,3] This result is a bit surprising, because VR waves in a hurricane can (in principle) resonantly excite IB waves in the environment. Equation (1) provides a possible explanation. We posit that, for the vortices examined, $d\bar{q}/dr_*$ is sufficiently large that critical layer damping dominates the positive feedback of IB wave emission; i.e., $\epsilon_{cl}/\epsilon_{rad} >> 1$.

†For cases in which $\gamma < 0$, the wave is actually a quasi-mode. The complex frequency of a quasi-mode is found by the solution to a generalized eigenmode problem, described in Ref. [1].

6. Current Research

Although linear theory provides useful insight, there are various nonlinear processes that merit future investigation. For example, nonlinear stirring in the critical layer decreases the magnitude of the radial PV gradient at r_*. If the initial wave amplitude is sufficiently large, this gradient might eventually drop below the stability threshold. We are currently developing numerical simulations, with nested grids, to accurately and efficiently examine the nonlinear damping and pumping of VR waves. Preliminary results should be available by the time of this conference.

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