10.4 EXTENSION OF THE MIXED-LAYER CONCEPT TO STEEP TOPOGRAPHY

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1. INTRODUCTION

Over flat terrain, convective boundary-layer (CBL) growth is well described by mixed-layer models. Within the bulk of the CBL strong thermally-driven turbulent mixing keeps the vertical gradient of potential temperature small. Thus the CBL can be treated as a single layer, characterized by a mean potential temperature and specific humidity. The main advantage of the approach is that it reduces the dimensionality of the problem, thereby simplifying the governing equations. The mixed-layer concept has also been applied to complex terrain, such as northeast Colorado (Wilczak and Glendening 1988; Dempsey and Rotunno 1988), or the Los Angeles Basin (Glendening 1990). In both cases a smoothed, envelope-type model topography was used, with maximum terrain inclinations on the order of 10% and elevation differences of up to 2500 m. However, these studies focused on the mesoscale wind field within the foreland/basin rather than the mixed-layer growth characteristics over the ridge and mountain peak areas.

Here we investigate the fundamental interaction between thermally driven upslope flows and mixed-layer growth under conditions of negligible ambient flow. It is shown how ambient stability and topographic characteristics lead to distinct mesoscale patterns of mixed-layer height.

2. MIXED-LAYER MODEL

In a two-dimensional framework a simplified version of the mixed-layer equations can be written

$$\frac{\partial \theta}{\partial t} = -u\frac{\partial \theta}{\partial x} + \frac{(1+a)F}{h}, \qquad (1)$$

$$\frac{\partial u}{\partial t} = -u\frac{\partial u}{\partial x} + \frac{g}{\theta_0}\left(\theta' \frac{\partial (z_s+h)}{\partial x} + \frac{h}{2}\frac{\partial \theta}{\partial x}\right) - \frac{C_D}{h}|u|u, \qquad (2)$$

where F is the surface sensible heat flux divided by specific heat capacity and density, $a\approx 0.2$ is a non-dimensional coefficient accounting for entrainment warming, h is the mixed-layer height, z_s is the height of the topography, and C_D is a nondimensional drag coefficient. The bulk variables θ and u are mean values of potential temperature and flow speed within the mixed-layer. The terms in the heat budget equation (1) represent along-slope advection, and diabatic heating by the surface heat flux. The inversion capping the CBL is not explicitly modelled. Likewise, h is not calculated from a prognostic equation but determined diagnostically from θ via

$$\theta = \theta_0(z_s + h), \qquad (3)$$

where $\theta_0(z)$ is the ambient potential temperature profile.

Note that in this latter respect the present model fundamentally differs from previous mixed-layer models. Using (3) in the presence of horizontal mass-flux divergence within the CBL means that the *mixed-layer top is not treated as a material surface* with regard to that divergence. Justification for this approach is based on observational evidence, and physical considerations. Let us assume 3 m s^{-1} and 300 m, respectively, as the typical speed and depth of the upslope flow on a moderately steep mountain. If the slope flow air converges at ridgetop to rise in an adiabatic core of, say, 3 km total width (Braham and Draginis 1960), it would form an updraft on the order of 0.5 m s^{-1}. A similar value for the maximum updraft speed was derived from observations by Raymond and Wilkening (1980). If the mixed-layer top were indeed a material surface it would rise for several hours by more than 2 km per hour due to this mass-flux convergence, which is clearly unrealistic.

Moreover, unless the ambient stratification is close to dry-adiabatic, surface heating of the CBL would not be sufficient to keep up with such rapid convergence-driven growth, and a very strong capping inversion would develop. This in turn would induce a strong adverse pressure gradient within the CBL, reducing the strength of the upslope flows. (In the real atmosphere we observe some enhanced overshooting of thermals in the adiabatic core but the capping inversion strength rarely exceeds 2-3 K.) It follows that the depth *h* of the mixed-layer is closely tied to, and constrained by, its temperature θ. It also follows that a significant amount of air must be leaving the CBL near ridgetop (venting). Note also that if it is assumed that the air leaves the CBL close to its level of non-buoyancy, the effect on the environmental stratification is minimized. However, neglecting the effect of venting on the ambient atmosphere is approximately valid only for rather isolated topography. In narrow valleys the effect of slope-flow induced subsidence must be considered.

The terms in the momentum equation (2) are the along-slope advection of momentum, the layer-averaged pressure-gradient force, and surface drag (Wilczak and Glendening 1988). For the simulations with a realistic ambient atmosphere, a CBL humidity budget equation is solved analogous to (1)-(2) taking

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into account advection, surface evapotranspiration, and entrainment.

3. SIMULATION SETUP

The mixed-layer equations (1)-(3) are numerically integrated for both idealized and realistic topographic cross-sections and ambient temperature profiles. A sinusoidal surface forcing

$$F(t) = \frac{H_m}{c_p \rho} \sin(\omega t) \quad (4)$$

is prescribed, with a half-period of 12 hours, and a maximum sensible heat flux of $H_m = 500$ W m^{-2}. This corresponds to strong daytime heating typically found in the semi-arid regions of the southwestern U.S. The idealized topographic profile is of the form

$$z_s(x) = z_0 + \frac{z_m}{1+(x/a)^2} \quad (5)$$

with varying mountain heights and half-widths. In the idealized simulations the ambient stratification corresponds to that of the standard atmosphere, i.e. θ increases with height at a rate of 3.5 K km^{-1}.

Realistic topographic cross-sections are taken from a 1 km digital elevation dataset. Two 100 km west-east sections in south-central Colorado are studied, one through the Sangre de Cristo mountains, and one at the same longitude, about 60 km to the north, through the Wet Mountains. Ambient potential temperature and humidity profiles are based on NOAA/FSL Mesoscale Analysis and Prediction System (MAPS) analyses from 13 Aug 1999. On that day, highly selective mountain cumulus initiation was observed in the area. Around 10 MST, cumulus clouds started to form over the Sangre de Cristo range, whereas none were observed over the Wet Mountains which rise to roughly the same elevation.

The model is run at 1 km horizontal resolution. In addition to the forcing terms shown in (1)-(2), some horizontal diffusion is applied to the mixed-layer variables.

4. THE STEEP SLOPE 'BOTTLENECK'

The most fundamental effect of upslope flows on mixed-layer evolution is a retardation, or even complete suppression, of the growth driven by surface heating. This is because in a stable atmosphere any upward motion causes advection of potentially colder air, giving a negative contribution to the heat budget. Figure 1 shows that due to this mechanism the mixed-layer depth at the upper mountain slopes (maximum inclination 15%) has reached less than 100 m after 2 hours of heating, whereas it approaches 1 km over the foreland. Over the upper slopes, the mixed-layer has essentially become a Prandtl-type slope-flow layer, with advection balancing surface heating. The mixed layer top over the foreland rises little as the mountain is approached. To a first approximation it intersects the topography quasi-horizontally.

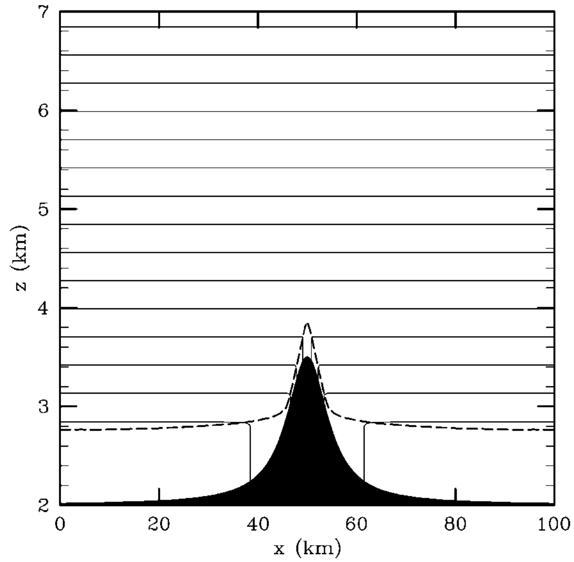

Figure 1: Mixed-layer growth after 2 hours of simulation in a standard atmosphere, over a mountain with 1500 m maximum relative height, and 5 km half-width. Thin lines indicate potential temperature, the dashed line shows the mixed-layer top.

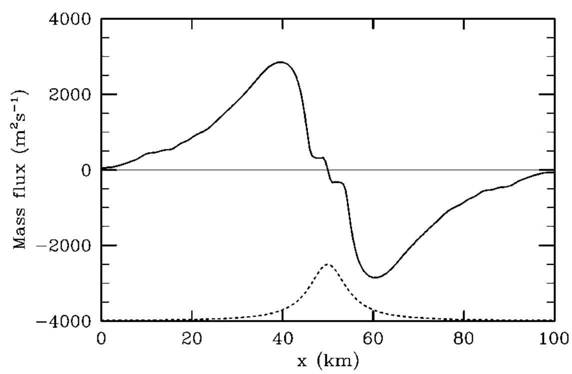

Figure 2: Along-slope mass-flux within the mixed-layer shown in Figure 1. Note the near constant values over the upper parts of the slopes where the mixed-layer is thin, indicating quasi-equilibrium upslope flow. Mountain profile inserted for reference.

Over the foreland, the along-slope mass-flux (Figure 2) increases towards the mountain. As long as the terrain inclination is small (on the order of a few percent or less) a linear regime is present in which slope flow strength increases with increasing slope. The mass-flux reaches a maximum near the foot of the mountain and then decreases sharply towards the steepest slopes. Here, the two terms on the r.h.s. of (1) cancel each other almost completely, resulting in an equilibrium mass-flux given by

$$(hu)_{eq} = \frac{F}{\sin\alpha \, d\theta_0 / dz}, \quad (6)$$

which decreases with increasing slope inclination. In a stably stratified atmosphere, a steep slope acts like a 'bottleneck' on along-slope mass-flux (Vergeiner, 1982). For comparison Figure 3 shows the same simulation for a mountain with halfwidth 15 km, i.e. only a third as steep as the one in the reference run. The reduced CBL growth and associated bottleneck effect is present but less pronounced. There is also a more gradual rise of the mixed-layer top towards the mountain.

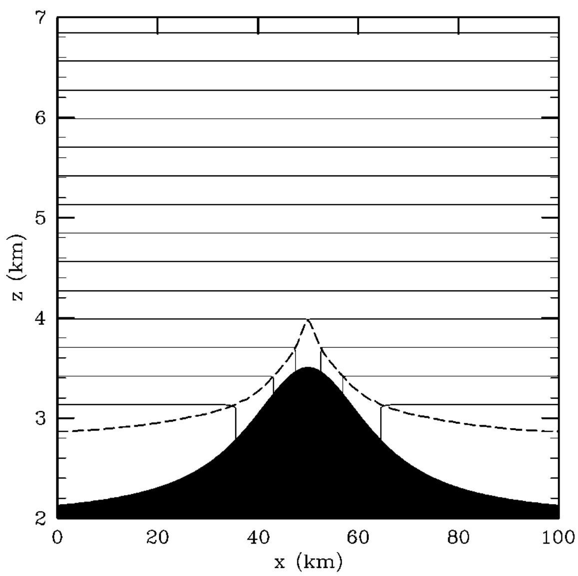

Figure 3: As in Figure 1 but for a mountain half-width of 15 km.

5. STEEPENING OF THE MIXED-LAYER TOP

Simulations were also carried out for realistic topographic cross-sections through the Colorado Rockies, and realistic potential temperature profiles. Typically, the stratification in the morning is more stable than the standard atmosphere at low levels, and only marginally stable above, as a result of vertical mixing on the previous day. Figure 4 shows how this affects CBL growth. As in the idealized case, the mixed-layer is most suppressed over the steepest slopes. However, this applies to the outer slopes only, not those within the interior valleys. Here we see the formation of a broad and rather uniform warm core region with a high mixed-layer top. Enhanced by the two-dimensional geometry of the simulation (no flow perpendicular to the cross-section), the mountain range acts like a plateau. At the easten boundary of the warm core the mixed-layer top drops sharply to the lower values found over the foreland. In the presence of a weak synoptic flow such a steepening of the CBL top could result in the formation of a discontinuity and the subsequent development of an elevated dry-adiabatic layer.

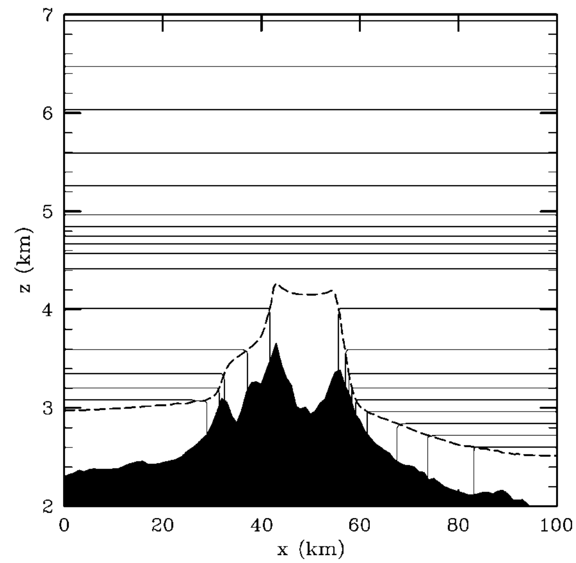

Figure 4: As in Figure 1 but for realistic topography (a west-east cross-section through the southern Colorado Rockies) and stratification (based on the MAPS analysis from 13 Aug 1999, 05 MST), after 3 hours of simulation. Note the sharp drop of the mixed-layer top at the eastern boundary of the warm core.

6. MOUNTAIN CUMULUS INITIATION

Mesoscale variations of the mixed-layer top, if considered together with mesoscale patterns of the lifted condensation level (LCL), can aid in understanding spatial patterns of mountain cumulus initiation. The day simulated here was characterized by cloudless skies at sunrise, and subsequent development of mountain cumulus strongly tied to individual topographic features. At 10 MST (about 5 hours after sunrise), cumulus clouds began to form over the Sangre de Cristo mountain range but not over the Wet Mountains, 60 km to the north. Figure 5 shows the simulated mixed-layer top and LCL for both areas, after 5 hours of simulation. A Bowen ratio of 5, typical of semi-arid conditions was assumed. As expected, the LCL generally rises from the foreland towards the mountains. Note that the steep slope bottleneck has broadened compared to +3 hours (Fig. 4) due to the continuous heating. In the case of the Sangre de Cristo mountains the advective supply of moisture is apparently strong enough to keep the LCL from rising too high, and some of the air in the warm core can rise past it. In the Wet Mountains cross-section the mixed-layer is dryer, and the LCL is just marginally reached close to the peaks. Again, the effect of terrain shape is probably exaggerated due to the two-dimensional nature of the simulations. Nevertheless, they illustrate the basic interplay between CBL growth and associated drying, and advective cooling and humidity supply by slope flows.

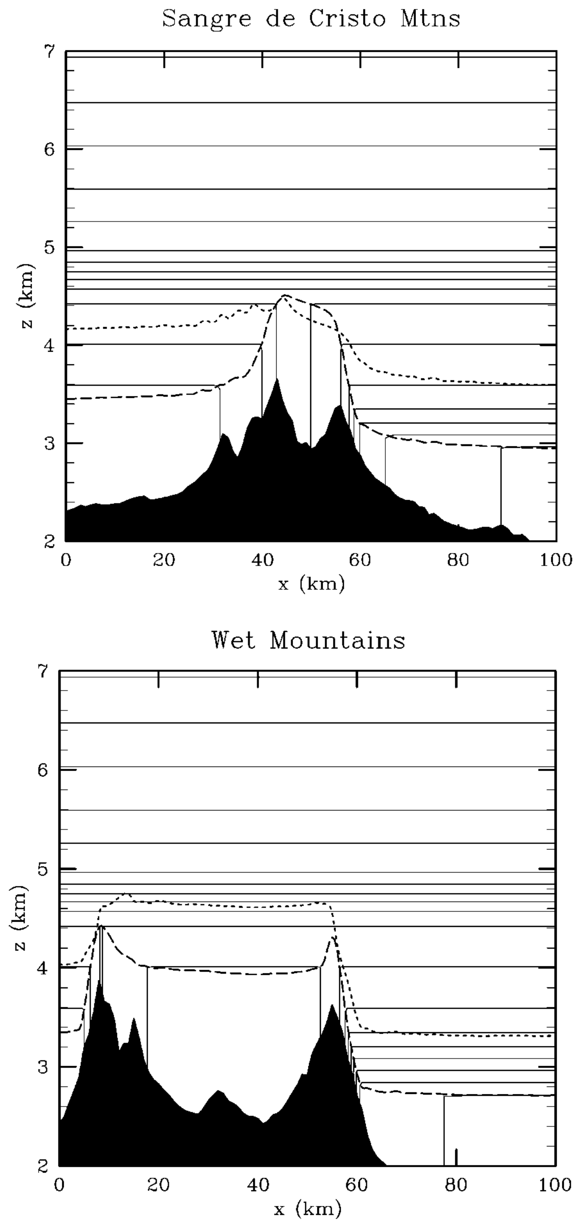

Figure 5: As in Figure 4, but after 5 hours of simulation. Upper panel shows Sangre de Cristo cross-section, lower panel Wet Mountains cross-section. Dotted line indicates height of the LCL.

7. CONCLUSIONS

A simplified mixed-layer model is applied to idealized and realistic topographic cross-sections that include steep mountain slopes. In contrast to previous models of this kind the mixed-layer top is not assumed to represent a material surface with regard to mass-flux divergence within the mixed-layer. The results show strongly reduced CBL growth and Prandtl-type slope flow formation on steep slopes. If the mountain range is sufficiently wide or consists of several peaks, there is a tendency for the development of a warm core above the central parts of the mountain. Formation of the warm core is favoured by the weak stratification typically found in residual layers. Under such conditions the mixed-layer top, rather than rising gradually, exhibits little rise towards the foothills, and then a sharp rise close to the warm core. Simulation of the LCL based on the CBL humidity budget illustrates how mountain shapes may affect cumulus initiation.

The mixed-layer model is highly simplified and may give unrealistic results within narrow valleys, where subsidence sinking can significantly alter the ambient stratification. Further work will focus on three-dimensional simulations, and on the inclusion of the effect of compensating subsidence on the ambient atmosphere.

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