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ON REYNOLDS AVERAGING AND THE ZERO INTEGRAL SCALE CONSTRAINT

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1. INTRODUCTION

Several investigators (Comte-Bellot and Corrsin 1971; Kaimal and Finnigan 1994, p. 276; Sreenivasan et al. 1978; de Waele et al. 2002) have reported the property of stationary turbulence time series that the integral scale (defined in the classical way) is always zero. This property is strange because it is known that the true autocovariance (defined via ensemble averaging) does not always have zero integral scale. In other words, the autocovariance determined through Reynolds averaging of stationary turbulence time series and the autocovariance determined through ensemble averaging of stationary turbulence time series do not have equipollent properties. Lumley and Panofsky (1964, p.14) state that the "major simplification that stationarity permits is the introduction of time (or space) averages which are meaningful in the sense that their values represent properties of the process and not just of the averaging time." Specifically, after they have been averaged, turbulence data must satisfy other criteria.

Given that the integral scale is a measure of the 'randomness' in turbulence, small integral scale implying a highly random turbulence and large integral scale implying a not-so-random turbulence, questions thus arise as to why the zero integral scale feature perpetually manifests in Reynolds analysis of stationary turbulence time series and what are its dynamical implications. A perpetual zero integral scale suggests that the underlying turbulence is extremely random all the time, a condition not in harmony with characteristics of the boundary layer. Another question is how to reconcile the differences between dynamical implications of the autocovariance determined by Reynolds averaging and the autocovariance determined by ensemble averaging. Lumley and Panofsky (1964, p. 37) state "in order to make sensible interpretations of experimental measurements of a process, we must assume that integral scales exist." The conflict between the Reynolds averaging algorithm and its practical implementation via block averaging is evident.

Before proceeding further, though, it's important to make a distinction between the actual physical phenomenon (i.e., the turbulence), our measurements of that phenomenon (the recorded time series), and a Reynolds average analysis of that time series. The turbulence *should* have an integral scale (cf. Lumley and Panofsky 1964, p. 37). If our measurements are valid, the raw time series will sustain this property. And for our analysis algorithm to have merit, it must display, and not annihilate, this feature. The issue we address here is neither the turbulence nor the recorded time series. It's the analysis algorithm (block averaging) and its integral scale incompatibility with turbulence. Keep in mind, though, that the validity of any analysis algorithm is limited by how faithfully the recorded time series captures the time-dependent properties of the turbulence. Here we assume that all measurements are valid to some specification of numerical accuracy.

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2. REYNOLDS AVERAGING

The answer to the first of the above questions is straightforward—the perpetual zero integral scale feature manifests because Reynolds averaging requires it. Consider an arbitrary time series, say U(t) defined over the domain [0,T] where T is on the order of several hours. In Reynolds averaging, we find the turbulent fluctuations over a succession of adjacent time blocks, each labeled generically as i and each of length ΔT, by averaging U(t) according to

$$\overline{U}_i(\Delta T) = \frac{1}{\Delta T}\int_0^{\Delta T} U_i(t)dt, \quad i = 1,2,3,...,N, \qquad (1)$$

where $\Delta T<T$ and $U_i(t)$ is the portion of the total time series in the i-th block. Turbulent fluctuations for the i-th block are then just

$$u_i(t,\Delta T) = U(t) - \overline{U}_i(\Delta T). \qquad (2)$$

From (1) and (2) it follows that $\overline{u_i(t,\Delta T)} = 0$ and that

$$\left(\frac{1}{\Delta T}\int_0^{\Delta T} u_i(\zeta,\Delta T)d\zeta\right)^2 = \left(\frac{1}{\Delta T}\right)^2 \int_0^{\Delta T}\int_0^{\Delta T} u_i(\zeta_1,\Delta T)u_i(\zeta_2,\Delta T)d\zeta_1 d\zeta_2 = 0 . \qquad (3)$$

Changing variables according to $\zeta=(\zeta_1+\zeta_2)/2$ and $\tau=\zeta_2-\zeta_1$ allows the double integral in (3) to be expressed as (cf. Papoulis 1965, p. 325; Panofsky and Dutton 1984, p. 62f.)

$$\frac{1}{\Delta T}\int_{-\Delta T}^{\Delta T}\left(\frac{1}{\Delta T}\int_{|\tau|/2}^{\Delta T-|\tau|/2} u_i(\zeta-\frac{\tau}{2},\Delta T)u_i(\zeta+\frac{\tau}{2},\Delta T)d\zeta\right)d\tau . \quad (4)$$

The integral within ( ) in (4) is the autocovariance for the i-th block and is denoted as $C_i(\tau,\Delta T)$. Invoking the definition of the integral scale reduces (4) to

$$\frac{1}{\Delta T}\int_{-\Delta T}^{\Delta T} C_i(\tau,\Delta T)d\tau = \frac{2\sigma_i^2(\Delta T)\Lambda_i(\Delta T)}{\Delta T} = 0. \qquad (5)$$

Here $\sigma_i^2(\Delta T)$ is the variance for the i-th block, sometimes called the 'local variance', and $\Lambda_i(\Delta T)$ is the integral scale for the same block, correspondingly called the *local integral scale*. In these, we retain ΔT to remind us of the averaging interval. Since ΔT is finite and the variance is always positive, (5) requires that Λ_i always be zero. In effect, (5) renders questionable the related τ-functional form of $C_i(\tau,\Delta T)$.

This same result is also true for cross-covariances. That is, if we correlate $u_i(t,\Delta T)$ with, say, $w_i(t,\Delta T)$, where $w_i(t,\Delta T)$ is the random part of the vertical velocity and is defined by an equation similar to (2), then, since $\overline{w_i(t,\Delta T)} = 0$, we can likewise formulate equations for cross-covariances similar to (3)-(5), with the result that the net area under the cross-covariances curve is also zero. The same holds true for cross-covariances between a random velocity and a random pressure $p_i(t)$. In fact, any time we correlate two stationary time series, one of which whose time average is zero, we get correlations with zero net area under the curve. Sreenivasan et al. (1978) emphasize this point for autocovariances, viz. "the integral scale…must strictly be zero for a random variable with zero mean."

Suppose now that the turbulence is nonstationary over some block, say, the j-th block. Nonstationarity is often characterized by periods of rapid boundary layer growth or decay or the passage of clouds or fronts. During these periods, a block average only formally obeys traditional Reynolds averaging rules since it is 'bogus' in the sense that, unlike the stationary case, it does not provide any information about the ensemble mean. For example, suppose that in the j-th time block there is a small trend defined by εt. That is, $U_j(t) = u_j(t) + \varepsilon t$, where ε is a 'small' but non-negligible constant. The time average of this $U_j(t)$ is $0.5\varepsilon\Delta T$ where, for convenience, we've assumed that the time average of $u_j(t)$ is zero.

Suppose next that the trend is instead $\varepsilon(\Delta T\text{-}t)$, which is the original trend reflected in the horizontal line $0.5\varepsilon\Delta T$. The time average of this $U_j(t)$ is also $0.5\varepsilon\Delta T$. The ensemble mean of each, however, is different from the other. In the stationary case the ensemble mean of $\overline{U}_i(\Delta T)$ produces a usable estimate of the true mean of $U_i(t)$. In the nonstationary case it doesn't.

For this nonstationary case, let μ_j be the average of $U_j(t)$. If we again use (2) to define the random component in this j-th block, that random component is also bogus because the *same* μ_j is being subtracted from the raw data at each instant within the block whereas the true random part is defined by subtracting the time-dependent mean from the raw data. Finnigan et al. (2003) report the bogus nature of such an average, stating that nonstationarities in the form of "deterministic trends or low frequency components in the [original] record are contained in" $u_j(t)$.

And while the time average of this bogus random part is zero, its expected value is not. That is, even though $\overline{U_j(t) - \mu_j} = 0$ for the j-th block, it doesn't follow that $< U_j(t) - \mu_j > = < U_j(t) > - \mu_j = 0$, where $< >$ denotes ensemble mean, the reason being that the ensemble mean of $U_j(t)$ is not a constant. This non-equality between time averaging and ensemble averaging in the nonstationary case is what marks the departure from the stationary case. It also makes it impossible to assign meaningful physics to any related result. In particular, the integral scale for the j-th block is not necessarily zero; and, as in the stationary case, the resulting τ-functional form of the autocovariance is questionable. Specifically, in the nonstationary case, subtracting a Reynolds average of raw data from the raw data does not produce the exclusively zero ensemble mean random part of the original data. For time averaging to be meaningful, it must produce results equivalent to ensemble averaging. In short, we cannot extract 'meaningful' physics by invoking incorrect mathematics.

3. DYNAMICAL IMPLICATIONS

The immediate implication is that estimates of autocovariances using standard block averaging are suspect. Specifically, for stationary turbulence, the perpetual zero integral scale feature suggests that the τ-functional form of $< u_i(t-\tau/2,\Delta T)u_i(t+\tau/2,\Delta T) >$ does not equal the τ-functional form of $\overline{u_i(t-\tau/2,\Delta T)u_i(t+\tau/2,\Delta T)}$. The same is true for cross covariances such as $\overline{u_i(t-\tau/2,\Delta T)w_i(t+\tau/2,\Delta T)}$, $\overline{u_i(t-\tau/2,\Delta T)u_i^2(t+\tau/2,\Delta T)}$, and $\overline{u_i(t-\tau/2,\Delta T)p_i(t+\tau/2,\Delta T)}$. That is, because the net area under the curve of these types of cross-covariances is perpetually zero, it doesn't follow that the τ-functional form $< u_i(t-\tau/2,\Delta T)w_i(t+\tau/2,\Delta T) >$ equals the τ-functional form $\overline{u_i(t-\tau/2,\Delta T)w_i(t+\tau/2,\Delta T)}$, etc., thus making their respective τ-functional forms equally suspect. Note that these covariances are neither *even* nor *odd* functions of the lag. Therefore, the zero net area under the curve for these cases has a somewhat different physical meaning. In the autocovariance case, block averaging leads to an autocovariance that appears as the blue line in Figure 1 when, in fact, the true autocovariance may appear as the red line in Figure 1. The differences between the two are clear.

These differences combine to produce spectral information whose frequency bandwidth, content, distribution and domain differ. Thus, in spite of stationarity, an FFT analysis of $u_i(t,\Delta T)$ will not produce the requisite spectral characteristics of the turbulence in the i-th block. Moreover, the spectrum of the blue-line autocovariance in Figure 1 is zero at frequency zero while the spectrum of the red line autocovariance in Figure 1 is not. Taylor (1921) says that the type of curve depicted by the blue line "might be due to some sort of regularity in the eddies of which the turbulent motion consists." It appears that the "regularity" is a consequence of the tacit assumption that mean and variance scale in the same way. The two curves can be made to coincide only at lag zero by normalizing each with respect to its own variance. For nonstationary turbulence, block averaging over user-defined windows is inapplicable altogether.

The zero integral scale result is 'generic' in the sense that it has nothing to do with turbulence per se. Thus, to further investigate the turbulence implications of a zero integral scale we need to examine the dynamical evolution of single-point, two-time correlations consistent with the Navier-Stokes equation. Accordingly, for the vertical velocity W we write

$$\frac{\partial W}{\partial t}+\frac{\partial W^2}{\partial z}+\frac{1}{\rho}\frac{\partial P}{\partial z}=0. \qquad (6)$$

The variables P (pressure) and W are taken to be measured at a fixed point in the boundary layer defined by the coordinates X (longitudinal), Y (transverse), and Z (vertical), and the total flow is taken to be homogeneous in x and y but inhomogeneous in z. We also assume high Reynolds number and neutral stratification for demonstration purposes and, therefore, ignore the viscous and gravitational terms normally present in (6).

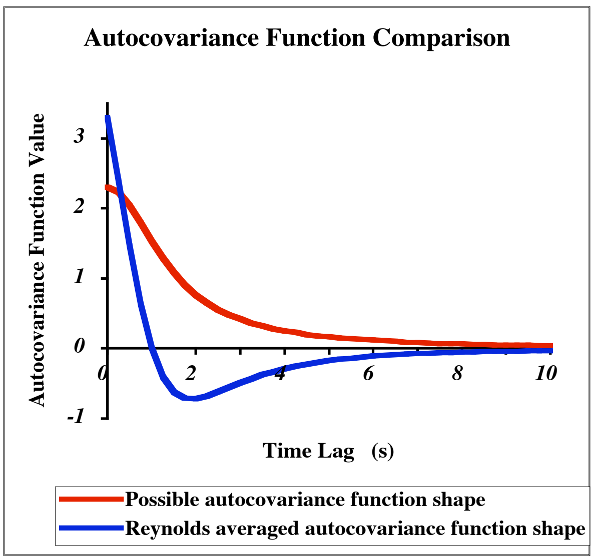

Figure 1. Autocovariances for stationary turbulence. The red curve is a possible autocovariance as would be determined by ensemble averaging. The net area under this curve is not zero. The blue curve is the autocovariance as determined by Reynolds averaging. The net area under this curve is zero. The blue-line autocovariance "might be due to some sort of regularity in the eddies of which the turbulent motion consists" (cf. Taylor 1921).

Letting $W=<W>+w\approx w$ and $P=<P>+p$ in (6) produces

$$\frac{\partial w}{\partial t}+\frac{\partial w^2}{\partial z}+\frac{1}{\rho}\frac{\partial <P>}{\partial z}+\frac{1}{\rho}\frac{\partial p}{\partial z}=0. \qquad (7)$$

Taking the ensemble average of (7) and subtracting this average from the original leaves

$$\frac{\partial w}{\partial t}+\frac{\partial w^2}{\partial z}+\frac{1}{\rho}\frac{\partial p}{\partial z}=<\frac{\partial w^2}{\partial z}>=\frac{\partial \sigma^2}{\partial z}, \qquad (8)$$

where $\sigma^2=<w^2>$. Note that the ensemble average of (8) is zero, as it should be. Evaluating (8) at time t_1 and multiplying by $w(t_2)$ results in

$$w(t_2)\frac{\partial w(t_1)}{\partial t_1}+w(t_2)\frac{\partial[w^2(t_1)-\sigma^2(t_1)]}{\partial z}+\frac{w(t_2)}{\rho}\frac{\partial p(t_1)}{\partial z}=0. \qquad (9)$$

Changing variables according to $t=(t_1+t_2)/2$ and $\tau=t_2-t_1$ converts (9) into

$$w(t+\frac{\tau}{2})\left(\frac{1}{2}\frac{\partial}{\partial t}-\frac{\partial}{\partial \tau}\right)w(t-\frac{\tau}{2})+w(t+\frac{\tau}{2})\frac{\partial[w^2(t-\frac{\tau}{2})-\sigma^2(t-\frac{\tau}{2})]}{\partial z}+\frac{w(t+\frac{\tau}{2})}{\rho}\frac{\partial p(t-\frac{\tau}{2})}{\partial z}=0. \qquad (10)$$

Similarly, evaluating (8) at t_2 and multiplying by $w(t_1)$ results in

$$w(t-\frac{\tau}{2})\left(\frac{1}{2}\frac{\partial}{\partial t}+\frac{\partial}{\partial \tau}\right)w(t+\frac{\tau}{2})+w(t-\frac{\tau}{2})\frac{\partial[w^2(t+\frac{\tau}{2})-\sigma^2(t+\frac{\tau}{2})]}{\partial z}+\frac{w(t-\frac{\tau}{2})}{\rho}\frac{\partial p(t+\frac{\tau}{2})}{\partial z}=0. \qquad (11)$$

Adding (10) and (11) and ensemble averaging that result yields

$$\frac{1}{2}\frac{\partial C(t,\tau)}{\partial t}+F(t,\tau)+G(t,\tau)=0, \qquad (12)$$

where

$$C(t,\tau)=<w(t-\frac{\tau}{2})w(t+\frac{\tau}{2})>, \qquad (13)$$

$$F(t,\tau)=<w(t-\frac{\tau}{2})\frac{\partial w(t+\frac{\tau}{2})}{\partial \tau}-w(t+\frac{\tau}{2})\frac{\partial w(t-\frac{\tau}{2})}{\partial \tau}>, \qquad (14)$$

and

$$G(t,\tau) = < w(t+\frac{\tau}{2})\frac{\partial w^2(t-\frac{\tau}{2})}{\partial z} +$$

$$w(t-\frac{\tau}{2})\frac{\partial w^2(t+\frac{\tau}{2})}{\partial z} > + \qquad (15)$$

$$\frac{1}{\rho} < w(t+\frac{\tau}{2})\frac{\partial p(t-\frac{\tau}{2})}{\partial z} + w(t-\frac{\tau}{2})\frac{\partial p(t+\frac{\tau}{2})}{\partial z} > .$$

Equation (12) requires that the time dependence of the autocovariance be balanced by a variety of terms, viz. those defined by (14) and (15). Accordingly, we examined a sequence of CASES99 (Poulos et al. 2002) autocovariances for which we determined the mean consistent with (1). These are shown in Figure 2. The curves show 5-min averaged autocovariances of vertical velocity recorded at the 5-m level on the night of 18 October 1999 between 1:50 and 2:05 a.m. (UTC).

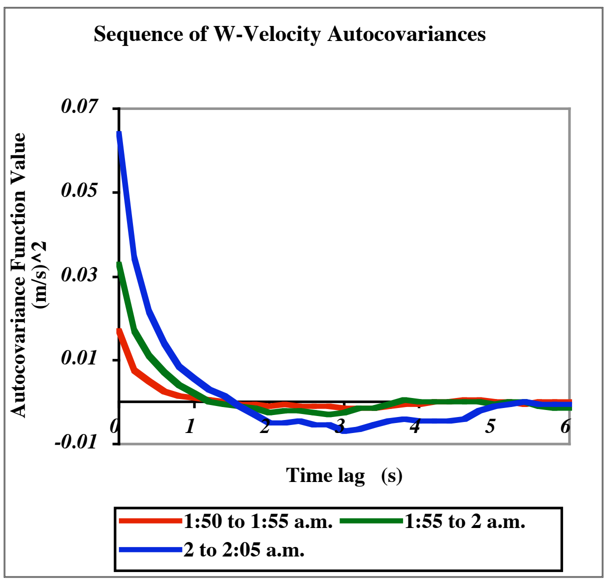

Figure 2. Sequence of three autocovariances for CASES99 data (vertical velocity only). These autocovariances were determined by block averaging over the indicated time intervals.

Cursory examination of these three curves establishes that there is time dependence in their shapes from one time block to the next and also in the variances (autocovariance values at zero lag). One might even be tempted to conclude that when properly normalized these curves could be made to collapse into a single curve (not shown), resulting in a contention of self-similarity. However, all autocovariances that cross the abscissa only once are self-similar to some degree, regardless of the vertical level z at which they are determined (suggesting homogeneity). Thus, any self-similarity in such analyses is spurious because the method by which it emerges is biased toward that result (cf. Treviño and Andreas 1996). In short, self-similarity observed in Reynolds averages of turbulence is an artifact of the averaging process and not a property of the flow. This result likewise biases interpretations of the terms described by (14) and (15).

An added caveat is that Reynolds averaging produces curves similar to $C_{ww}(\tau)$ for longitudinal and transverse velocities as well. This similarity creates the impression that $C_{uu}(\tau)$ and $C_{vv}(\tau)$ are like those of the blue line in Figure 1. Generalization to isotropy of the underlying flow thus seems warranted but is completely in error for boundary-layer turbulence.

Lastly, we invoke the effects of the zero integral scale constraint by first noting that the expressions (13) –(15) are all *even* functions of the lag and then integrating (12) in τ from $-\infty$ to $+\infty$ to produce

$$\frac{\partial(\sigma^2\Lambda)}{\partial t} + 2\int_0^\infty F(t,\tau)d\tau + 2\int_0^\infty G(t,\tau)d\tau = 0, \qquad (16)$$

where $\int_{-\infty}^{\infty} C(t,\tau)d\tau = 2\sigma^2\Lambda$. The result (16) stipulates that the time evolutions of the variance and the integral scale are intertwined and defined by terms of extreme complexity. As long as we are using ensemble means to represent each of the terms in (12), (16) is legitimate. However, when we estimate all terms in (12) by time averaging over a sequence of stationary time blocks, then all terms in (16) become zero (cf. (3)-(5)). That is, when we use the same interval ΔT to estimate both means and variances, there is no dynamics. This result, again, is due to the 'suspect' τ-functional forms of all the integrands in (16).

In other words, setting Λ=0 in (16), as would be the case when the turbulence is stationary over each block, destroys not only its own time evolution but that of all the other terms, including the variance. In short, we cannot have a time evolution in the variance without a time evolution in the integral scale. Thus, approximating the time evolution of the variance by block averaging, even when it's applicable, does not depict the true dynamical behavior of the flow—there is no concomitant integral scale variation. Imposing zero integral scale on (16) is equivalent to setting to zero the energy level of each time block. Block averaging is thus unsuccessful at approximating how the variance changes with respect to time, as required by the Navier-Stokes equation.

4. DISCUSSION

Dynamics is critical for questions of nonstationary turbulence and it is natural to begin with simple dynamics that we can understand. However, attempts to approximate by block averaging the time evolution of the large-scale features of turbulence in terms of superposition of the features of each block requires that the randomness of each block be compatible with the randomness of the turbulence as a whole. An integral scale computed to be identically zero for each block leads, as deduced above, to a trivial time evolution for the turbulence record as a whole. Block averaging is motivated by the lure of easy computation and the belief that nonstationary turbulence can be approximated as a concatenation of stationary segments. The manner, though, in which it is implemented in practice, viz. constant-width segments and the same segments for both mean and variance computation, is inappropriate, even when coupling it with what we believe to be 'correct' physics. Formulation of correct physics must be preceded by a correct mathematical description of the behavior. The principles quantifying the behavior of turbulence are the principles of probability, whereas the principles of Reynolds averaging are the principles of certainty.

What this means, of course, is that if we were to analyze several hours of data over successive, say, 5-min time blocks and for the entire data set obtain the variance sequence $\sigma(t_0)$, $\sigma(t_0+5\text{-min})$, $\sigma(t_0+10\text{-min})$,…, where t_0 is some initial time, the fact that we've used traditional block averaging to obtain each of these variances automatically invalidates not only their magnitudes but their time evolution as well, *even if the raw data were found to be stationary over each block*. The requisite sequence is the sequence of pairs $\{\sigma(t_0),\Lambda(t_0)\}$, $\{\sigma(t_0+5\text{-min}),\Lambda(t_0+5\text{-min})\}$, $\{\sigma(t_0+10\text{-min}),\Lambda(t_0+10\text{-min})\}$,… Such a sequence is more characteristic of turbulence dynamical behavior than is the sequence $\{\sigma(t_0),0\}$, $\{\sigma(t_0+5\text{-min}),0\}$, $\{\sigma(t_0+10\text{-min}),0\}$,… Since both the variance and the integral scale emerge from the same function, viz. the autocovariance, the fact that the integral scale is zero casts doubt on the validity of the related magnitude of the variance. Averaging raw data over a succession of 5-min intervals is, in and of itself, tenuous but acceptable. Averaging the random part over the *same* 5-min intervals is what generates the incompatibility.

Even allowing the magnitude of ΔT to vary from one block to the next, the so-called *variable interval time averaging* (VITA) method (Gupta 1996), does not avoid the zero integral scale feature. Zero integral scale emerges from using the same time interval to estimate both mean and variance of stationary turbulence. Since for stationary turbulence Reynolds averaging produces autocovariances with zero integral scale, it is untenable. For nonstationary turbulence it is self-defeating. We cannot obtain a meaningful description of any turbulence behavior if we invoke the same window to estimate both the mean and the variance. Each requires estimation over its own time scale.

5. CONCLUDING REMARKS

de Waele et al. (2002) state that applying the standard definition of the integral scale "leads to results that are difficult to interpret." Actually, the results are not difficult to interpret if we are willing to accept the fact that the principles of traditional Reynolds averaging are flawed. In particular, using the same ΔT to obtain mean and variance estimates will, as shown above, always lead to autocovariances that have zero integral scale.

de Waele et al. (2002) suggest using a truncated autocovariance, a procedure consistent with the principle (Papoulis 1965, p. 330) that defining a safe averaging interval for the mean requires knowledge of the autocovariance while defining a safe averaging interval for the autocovariance requires knowledge of fourth-order moments. de Waele et al. (2002) caution, though, that their approach is a "highly arbitrary…subjective business." Sreenivasan et al. (1978) also report that knowing the higher-order moments is necessary for establishing record length requirements.

We, on the other hand, suggest the *time-dependent memory method* (TDMM) (Treviño and Andreas 2000). TDMM is a patented real time algorithm (U. S. Patent No. 6,442,506) that uses turbulence properties, viz. the numerical *accuracy* of the measurements and a turbulence time scale designated the *memory*, to define the optimal time-dependent averaging intervals for mean and variance estimates respectively. These intervals are typically not the same. Thus, consistent with theory, it allows for independent time variations in both variance and integral scale and is compatible with the fact that we cannot obtain robust estimates of nonstationary turbulence statistics using constant-width averaging intervals.

Consistent with inhomogeneity, TDMM also provides for potentially different averaging scales for different directions. It is also devoid of any "arbitrary" or "subjective" components. In particular, it avoids the perpetual zero integral scale feature and accordingly produces viable time evolutions for both the variance and the integral scale. A utilitarian description of TDMM is 'how to window nonstationary turbulence'.

TDMM generates averages consistent with *truths* about turbulence rather than with *agreements* among the community of researchers on how to analyze and discuss it. It is 'economic' in the sense that it uses the optimum amount of raw data to estimate mean and variance, and removes the user from the window-width decision-making process. It also allows for realistic and objective judgments to be made about the statistical structure of the turbulence being analyzed; particularly, the presence or non-presence of self-similarity. Last, it does not narrow the possibilities of applying the theoretical results to practical scenarios. Block averaging doesn't produce acceptable estimates of autocovariances under any circumstances, even for stationary data; TDMM does.

TDMM answers the question: If we want to average mean and variance over different time intervals how do we do it in an optimal way? The results reported here answer the questions: Why would we want to do that in the first place? What's wrong with Reynolds averaging? What do we get from TDMM that we don't get from block averaging? The present results are compatible with the fact that mean and variance represent different degrees of freedom and should therefore each be averaged over their own time scales (cf. Papoulis 1965, p.330).

In closing, we add that the community sometimes uses a quantity proportional to $z/\overline{U}$ as a measure of integral scale, ostensibly as a means of eluding the perils of the Reynolds averaging. There are two problems with this formulation. First, it does not eliminate the flaw in question. That is, for stationary turbulence, autocovariances with zero net area under the curve still manifest. In other words, the problem at hand is not how we define integral scale; the problem is how we average turbulence time series. As long as we continue to use the same interval ΔT to estimate both mean and variance, the zero integral scale conflict with dynamical theory will persist.

Second, when the turbulence is nonstationary, the averaging operation invoked becomes a contentious issue. Specifically, for a specified time t, the averaging operators

$$\overline{U}_R(t,\Delta T) = \frac{1}{\Delta T}\int_t^{t+\Delta T} U(\zeta)d\zeta \quad (17)$$

$$\overline{U}_C(t,\Delta T) = \frac{1}{\Delta T}\int_{t-\Delta T/2}^{t+\Delta T/2} U(\zeta)d\zeta \quad (18)$$

$$\overline{U}_L(t,\Delta T) = \frac{1}{\Delta T}\int_{t-\Delta T}^{t} U(\zeta)d\zeta \quad (19)$$

provide unequal estimates of the ensemble mean at time t. In stationary turbulence they produce roughly equivalent results (within statistical scatter). But, in nonstationary turbulence, a decision has to be made (by the researcher) as to which of the three averaging operations produces the most 'representative' estimate of the true mean at time t. Finnigan et al. (2003) state that "the particular averaging operation that is applied to the instantaneous flow field determines what part of the velocity will be treated as 'mean flow' and what as turbulence." We add here that the particular user-defined window width also determines what part of the velocity will be treated as mean flow and what as turbulence. It is obvious that, in general, $<\overline{U}_L(t,\Delta T)> \neq <\overline{U}_C(t,\Delta T)> \neq <\overline{U}_R(t,\Delta T)>$ and moreover that the autocovariance or spectrum of $u_L(t,\Delta T) = U(t) - \overline{U}_L(t,\Delta T)$, nor its time evolution, does not equal that of $u_C(t,\Delta T) = U(t) - \overline{U}_C(t,\Delta T)$ or of $u_R(t,\Delta T) = U(t) - \overline{U}_R(t,\Delta T)$.

We must always be cognizant of the fact that when using any algorithm to extract statistical information from turbulence, the information extracted is a convolution of the properties of the turbulence and the properties of the algorithm. In effect, every experimentalist faces a similar question: What effect does the instrument used to measure a phenomenon have on the measurements themselves? Just as an object appears different under normal light, X-rays light, and ultraviolet light, "what we observe is not nature itself but nature exposed to our method of questioning" (Heisenberg 1958, p. 58).

6. ACKNOWLEDGMENTS

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