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A new mixing length formulation for the parameterization of dry convection: implementation and evaluation in the COAMPS mesoscale model

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1. INTRODUCTION

The entrainment at the top of the planetary boundary layer (PBL) is a fundamental aspect of the dynamics of the dry convective boundary layer. A realistic parameterization of the entrainment and of the growth of the PBL in atmospheric models has been a major challenge in boundary layer research. It is well known that large-scale and mesoscale models have serious deficiencies in representing the development of the dry convective PBL (e.g. Ayotte et al. 1996; Beljaars and Betts 1993).

In Teixeira and Cheinet (2004) (hereafter TC04) a simple mixing length formulation for the eddy-diffusivity parameterization of dry convection was proposed, in order to realistically represent the PBL evolution. The new formulation relates the mixing length (l) to the square root of the turbulent kinetic energy (e) and a time-scale (τ): $l = \tau\sqrt{e}$. Two different ways of determining the time-scale were analyzed in TC04: (i) calculated as proportional to the ratio between the boundary layer height (h) and the convective velocity scale (w_*), $\tau \propto h/w_*$; or (ii) taken as a constant, equal to the typical mean eddy turnover-time in a dry convective PBL, $\tau = 600s$. The simulation of dry atmospheric convection events showed that the new formulation reproduces in a realistic way the top entrainment and the overall PBL evolution. Although the approach of assuming a constant time-scale produced slightly worse results than the more physical one, it still showed a surprising robustness in its sensitivity to a spectrum of differing surface fluxes and tropospheric lapse-rates.

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This new formulation has been generalized successfully for cloud-topped boundary layers, both in stratocumulus and cumulus cases (Cheinet and Teixeira 2003), in the context of one-dimensional (1D) models.

In this paper we test this new mixing length formulation using the US Navy Coupled Ocean-Atmosphere Mesoscale Prediction System (COAMPS[TM]) that is briefly described in section 2. The new formulation is introduced in section 3. The observations and the mesoscale model results are analyzed in section 4. A discussion using 1D simulations is presented in section 5 and some conclusions in section 6.

2. COAMPS

COAMPS (Hodur 1997) is a mesoscale model with a finite-difference approximation to the fully compressible, non-hydrostatic equations. COAMPS can be used as an analysis-nowcast and short-term forecast (up to 72 hours) tool, applicable for any given region on Earth. COAMPS includes a full atmospheric data assimilation system with data quality control, analysis, initialization, and non-hydrostatic atmospheric model components, coupled with a hydrostatic ocean circulation model. COAMPS uses a terrain-following vertical coordinate and can be integrated on a system of nested grids that enables the highest resolution to be focused over a specific region of interest.

The boundary layer and turbulence parameterization uses a prognostic equation for the turbulent kinetic energy (TKE) based on Mellor and Yamada (1982). The surface fluxes are computed based on Louis et al. (1982) and the radiation parameterization follows Harshvardhan et al. (1987). The moist convection processes are parameterized following the approach of Kain and Fritsch

(1993) and the cloud microphysics processes are parameterized based on Rutledge and Hobbs (1983). The boundary conditions are from the Navy Operational Global Atmospheric Prediction System – NOGAPS (Hogan and Rosmond 1991).

3. MIXING LENGTH

The boundary layer parameterization in COAMPS is based on the eddy-diffusivity closure with a prognostic equation for TKE. The eddy-diffusivity coefficients and the TKE dissipation are parameterized as follows:

$$K_\theta = K_q = K_e = S_{\theta,q,e} l_h \sqrt{e} \qquad (1)$$

$$K_u = K_v = S_m l_m \sqrt{e} \qquad (2)$$

$$\varepsilon = C_\varepsilon \frac{e^{3/2}}{l_\varepsilon} \qquad (3)$$

where θ is the potential temperature, e is the TKE, q is the water vapor mixing ratio, u and v are the horizontal wind components, ε is the TKE dissipation, l_h is the mixing length for potential temperature, water vapor and TKE and l_m is the momentum mixing length. In the control version of COAMPS $S_{\theta,q,m}$ are functions of the Richardson number (Chen et al. 2003), S_e is a constant and the different mixing lengths are equal to a master length scale ($l_h = l_m = l$), with l being calculated using Blackadar's formulation (Blackadar 1962), hereafter B62,

$$\frac{1}{l} = \frac{1}{kz} + \frac{1}{\lambda} \qquad (4)$$

where k is the von Karman constant, and the length λ is calculated as

$$\lambda = \alpha \frac{\int zedz}{\int edz} \qquad (5)$$

The value of α is often taken as constant: *α=0.1* as used in Yamada and Mellor (1975) or *α=0.2* as suggested by Moeng and Wyngaard (1989) (note that in these two studies the TKE is replaced by $\sqrt{2e}$ inside the integrals). In COAMPS, *α=0.1* for stable and neutral boundary layers, and has a stability correction for the unstable PBL (Chen et al. 2003). In a new version of COAMPS, the new formulation for the mixing length proposed in TC04 is used for potential temperature, water vapor mixing ratio and TKE. In this new formulation the mixing length is proportional to the square root of the TKE multiplied by a time scale:

$$l_h = \tau\sqrt{e} \qquad (6)$$

where τ is the time-scale.

For convective situations (positive surface buoyancy flux) we use in this study a constant time-scale equal to 600 s that produced realistic results in TC04. For stable situations we combine TC04 with Deardorff (1976) by determining the time scale as $\tau = \min(600, 0.76/N)$ where N is the Brunt-Vaisala frequency. Furthermore, $S_{\theta,q,e,m} = 0.5$ and $C_\varepsilon = 0.16$.

Close to the surface the mixing length is a linear function of height, and the actual formulation used in the model is:

$$l_h = \tau\sqrt{e} + (kz - \tau\sqrt{e})e^{-z/\mu} \qquad (7)$$

where $\mu = 100$ *m* is a crude approximation for the height of the surface layer. The exponential interpolating function (7) is used, instead of the approach of B62, in order to be able to represent the influence of the large eddies close to the surface in a convective PBL.

Since the B62 mixing length formulation produces realistic neutral boundary layers and has been successfully used for a number of years (e.g. Louis et al. 1982; ECMWF 2000), we use it for the momentum mixing length with $\lambda = 150$ *m* (e.g. ECMWF 2000). In principle, there is no *a priori* physical reason to assume that the mixing lengths for momentum and heat must be the same. Also, 1D simulations using the new formulation as the mixing length for momentum, produced mixed-layer wind values that were too low when compared to observations (not shown).

We assume that the TKE dissipation can be divided in two terms, one related to the production of TKE due to shear and the other due to buoyancy, which leads to a dissipation length that is a combination of the heat and momentum mixing lengths:

$$\frac{1}{l_\varepsilon} = \frac{1}{l_h} + \frac{1}{l_m}. \qquad (8)$$

Note that in this particular version of the model, stability corrections to the surface layer mixing length, based on Monin-Obukhov similarity, are not being taken into account. Sensitivity experiments for dry convection situations have shown that these corrections do not seem to have a significant impact on the results.

4. COAMPS SIMULATIONS

The climate impact of changes in land use (CICLUS) field experiment was performed between October 1997 and September 1999. It includes two years of continuous surface observations in 16 automatic weather stations, installed at the Dejebe Valley, Alentejo, South Portugal. Between 16 and 31 July 1998, an intensive observation period was performed, consisting of radiosondes (at latitude 38.53 N and longitude 7.88 W), some tethered balloon ascents, continuous sodar operation, and near surface turbulence measures with an ultra-sound turbulence sensor (eddy correlation system).

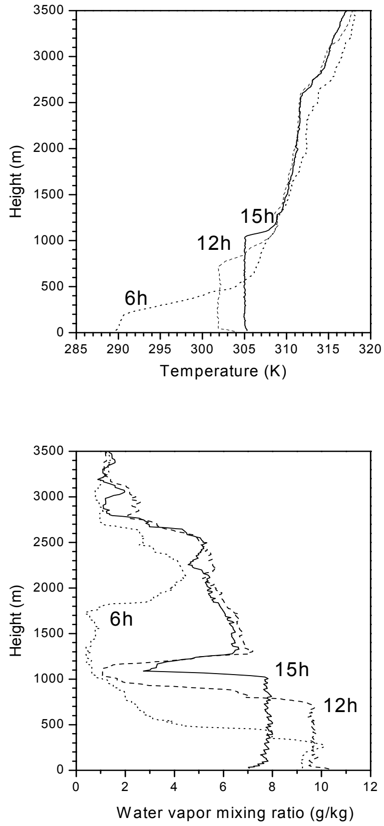

Fig.1 - Observed (a) potential temperature (K) and (b) water vapor mixing ratio (g kg^{-1}) profiles in Evora (38.53 N, 7.88 W) on 24 July 1998 at 6, 12 and 15 UTC.

On 24 and 25 July 1998, two days with a clear-sky situation, radiosonde observations were performed every 3 hours, providing a detailed picture of the boundary layer evolution. In figures 1a and 1b, the observed potential temperature and water vapor mixing ratio are plotted at 6, 12 and 15 UTC (same local time), 24 July 1998. As expected, the PBL height increases throughout the day, reaching its maximum at 15 UTC. During this time, the PBL develops from a stable boundary layer into a well-mixed PBL, toped by a sharp inversion, typical of dry convective situations.

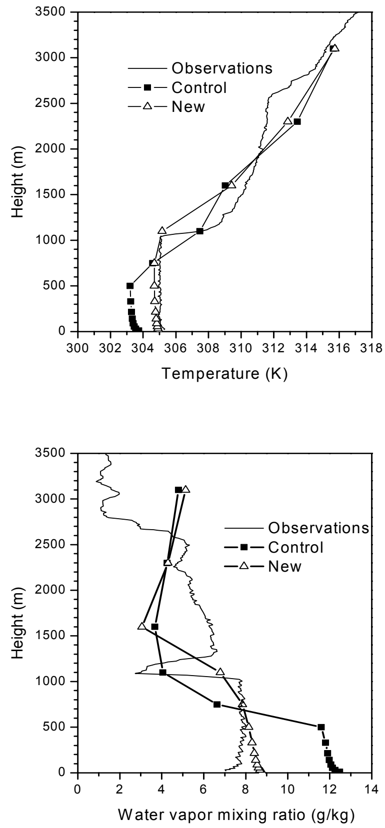

Fig.2 – Profiles of (a) potential temperature (K) and (b) water vapor mixing ratio (gkg^{-1}) at 15 UTC, from the observations, the current COAMPS parameterization (control) and the new parameterization (new). See text for details.

For this particular simulation, the atmospheric component of the COAMPS model was configured in a three-dimensional mode over an area around point [38.53 N, 7.88 W] in a Lambert conformal projection with the standard parallels being 30° and 45° N. In this application COAMPS uses 30 vertical levels and 3 horizontal domains. The outer grid has 45 km horizontal resolution and uses 45 grid points in each horizontal direction. Nest 1 has 15 km resolution with 49×49 grid points. Nest 2 has 5 km resolution with 85×85 grid points in both horizontal directions. The initial and boundary conditions for the simulation are taken from NOGAPS. Two 24 hour COAMPS forecasts were produced starting from July 24th 1998 at 00 UTC: (i) a control version (CTRL) with the standard mixing length and (ii) a new version (NEW) with the new mixing length formulation. The observations were taken at latitude 38.53 N and longitude 7.88 W, and the COAMPS model results were obtained in the nearest grid point, at latitude 38.529 N and longitude 7.904 W.

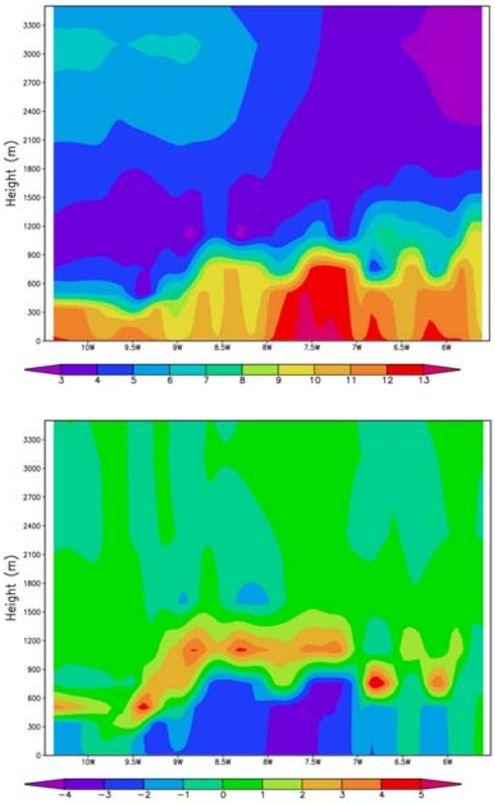

Fig. 3 – Cross-section at latitude 38.53° N and at 15 UTC of the water vapor mixing ratio (gkg^{-1}) (a) control simulation and (b) differences between the new and the control versions of COAMPS.

Figure 2a shows the potential temperature from the observations and the two model versions at 15 UTC. It is clear that for this situation the current COAMPS parameterization is unable to realistically represent the boundary layer height and mean potential temperature: the control experiment is almost two degrees too cold compared to the observations, and the PBL height is around 500 to 600 m, which is about half of the observed height. With the new formulation the simulation is strikingly better. Both the mean PBL potential temperature and the PBL height are very close to the observations, showing that the new mixing length formulation is able to produce a realistic entrainment and PBL growth. These results confirm and generalize the findings of TC04 that were obtained in the context of 1D model simulations.

In fig. 2b, the same is shown but for the water vapor mixing ratio. Again the control version produces a PBL that is not realistic: the model PBL top is too low, leading to a value of the water vapor mixing ratio that is about 4 g kg^{-1} too high. The new formulation leads to values of the mixing ratio that are quite close to the observations.

The evolution of the boundary layer was analyzed in detail. The profiles of potential temperature and water vapor for the CTRL and NEW experiments at 6, 12 and 15 UTC (not shown) confirm that the new formulation produces more entrainment than the control version, leading to a deeper and more realistic boundary layer.

Fig. 3a shows a cross-section of the water vapor mixing ratio at latitude 38.529 N, for the CTRL experiment at 15 UTC. This cross section starts offshore in the west and crosses the south of Portugal and Spain, showing a deeper boundary layer over land. Figure 3b shows the differences in water vapor mixing ratio between the NEW and CTRL experiments. As expected, the new formulation produces deeper boundary layers, leading to higher values of the mixing ratio closer to the top (above the CTRL PBL height) and lower values closer to the surface, due to a more realistic vertical redistribution of the water vapor mixing ratio.

5.ONE-DIMENSIONAL SIMULATIONS

In order to further investigate the role of entrainment in the improved representation of the convective PBL using the new mixing length, we use a simple 1D model and compare its results to large eddy simulation (LES) model results.

5.1 ONE-DIMENSIONAL MODEL

The 1D boundary layer model used in the present study has prognostic equations for the mean potential temperature and the TKE. Under horizontally homogeneous conditions, assuming a zero mean vertical velocity and with no diabatic forcing, the energy conservation equation is

$$\frac{\partial \theta}{\partial t} = -\frac{\partial}{\partial z}\left(\overline{w'\theta'}\right). \qquad (9)$$

In the absence of wind and moisture, the prognostic equation for TKE is (e.g. Stull, 1989)

$$\frac{\partial e}{\partial t} = -\frac{\partial}{\partial z}\left(\overline{w'e} + \frac{\overline{w'p'}}{\rho_0}\right) + \frac{g}{\theta_0}\overline{w'\theta'} - \varepsilon \quad (10)$$

where ε represents the TKE dissipation.

The parameterization of the turbulent terms uses the eddy-diffusivity approach (eq. 1-3) with $S_e = S_\theta = 0.5$ and $C_\varepsilon = 0.16$, and assumes $l_\varepsilon = l/2.5$, following Therry and Lacarrère (1983).
Several different mixing length formulations are tested using the 1D model: (i) the new

assumption where the mixing length is diagnosed as a function of TKE (with $\tau = 600s$) and (ii) the classic formulation of B62 (originally used in COAMPS) with differing methods of calculating the asymptotic value λ. The first two options use eq. (5) to calculate λ with α=0.1 as in COAMPS or with α=0.4, but without the stability correction. The reason we ignore the stability correction is to make the comparison straightforward and more general, since stability corrections may be different from model to model. In any case, the impact of the stability corrections can be represented in our simulations by increasing α or λ. A third option that was analyzed is to have λ=150 m, as in the ECMWF model (ECMWF 2000). It should be noted, however, that the ECMWF model does not use this formulation for dry convective boundary layer situations.

5.2 RESULTS

As a case study we use the dry convection intercomparison case from Nieuwstadt et al. (1992) where the surface heat flux is imposed as 0.06 K m s^{-1}. The surface TKE is imposed as zero and at the upper boundary (z = 3 km) the fluxes of both variables are set to zero. The spatial discretization of the equations uses a finite difference method, and the time discretization is performed using a fixed stability coefficient method (Teixeira 1999). This method can be simply described as a semi-lagrangian equivalent for the diffusion equation, and has been shown to provide results that are more stable and accurate than the implicit method as is typically used. The vertical resolution for the 1D model is 20 m and the time step is 60 s.

The results from the 1D model are compared with results from a three-dimensional large eddy simulation (LES) model. The resolution of LES models is usually such that the large eddies, which are responsible for most of the mixing within the convective PBL, are well resolved. In this test case the LES model uses a resolution of 20 m in the vertical and 78.125 m in the horizontal in a domain of (64×64×200) points. This particular LES model has been used in many boundary layer convection studies, such as Siebesma and Cuijpers (1995). In particular it has been used in some recent studies of the dry convective boundary layer (Siebesma and Teixeira 2000; Soares et al. 2004).

The potential temperature profile for the different formulations of the mixing length is shown in Fig. 6a together with the LES results after 8 hours of simulation (hourly mean). The new formulation simulates the boundary layer properties quite well with a realistic PBL height and a well mixed profile. The formulations with α=0.1 and λ=150 m clearly show some major problems: the entrainment is unrealistically small and there is little mixing close to the surface, leading to a highly unstable layer. The version with α=0.4 shows a slightly larger entrainment and exhibits a somewhat more realistic PBL evolution.

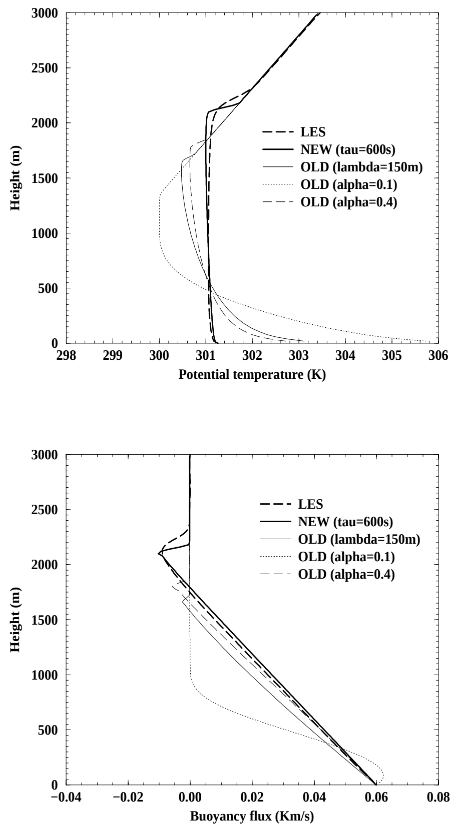

Fig. 4 – Profiles at hour 8 of the simulation (hourly means) of (a) potential temperature and (b) buoyancy flux from: LES, the new mixing length formulation and three versions of the old formulation.

It can be argued that just using larger values for λ may lead to a situation where the PBL growth is realistic. In fact, other common versions of the B62 formulation use values of λ=450 m (as in NOGAPS) or proportional to the PBL height. The results with λ=450 m are similar to the ones obtained α=0.4, and indeed using the PBL height for λ does improve the results. However, it can be shown in the framework of this simple 1D model that, whatever the value of λ may be, the PBL never grows deep enough. The best results are achieved when λ=10000 m, but even then the PBL growth is too weak and the lower part of the PBL is still too unstable. It is interesting to note that for values of λ > 10000 m the results virtually do not change. Values of this magnitude are physically unrealistic and not justifiable, and may also lead to unrealistically large values of the diffusivity coefficient above the PBL.

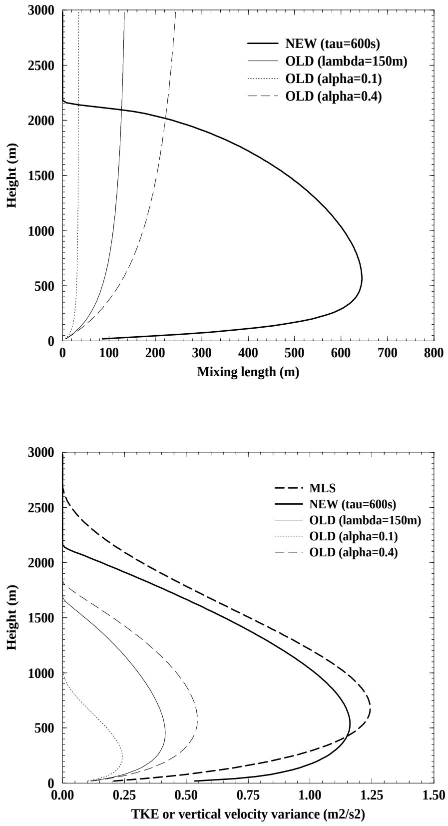

Fig. 5 - Profiles at hour 8 of (a) the mixing length and (b) the turbulent kinetic energy from: the new mixing length formulation and three versions of the old formulation (also in (b) is the profile of vertical velocity variance from mixed-layer scaling, MLS).

Similar results can be seen when analyzing Fig. 6b, where the corresponding evolution of the buoyancy flux profile is shown. The new formulation produces a realistic linear buoyancy flux profile with the correct amount of entrainment. The version of the old model with α=0.1 exhibits unrealistic fluxes, with no clear linear flux or entrainment. The other two versions indicate more realistic profiles of buoyancy flux, but still insufficient entrainment, as previously discussed.

The different mixing length profiles are shown in fig. 7a. The new formulation leads to a much larger mixing length in the boundary layer that decreases naturally to a very small value above the PBL. The B62 formulations are all rather similar except in the magnitude of the mixing length. As expected, they all increase with height and are not able to distinguish between the PBL and the atmosphere above. These results clearly confirm that the traditional B62 formulation was not originally developed for convective boundary layers and that the new formulation provides a rather natural and simple way of representing the convective boundary layer mixing length.

The profiles of TKE from the different versions of the model are shown in fig. 7b along with the vertical velocity variance from mixed-layer scaling (Stull 1989) using the LES PBL height. It should be noted that the model TKE can be compared directly with mixed-layer vertical velocity variance because, in general, it can be assumed (e.g. Therry and Lacarrère 1983) that $e/\overline{w'w'} = 2.5 l_\varepsilon / l$. In our model this leads to $e = \overline{w'w'}$ since we assume $l = 2.5 l_\varepsilon$. Fig. 7b shows that the TKE values produced by the new formulation are quite comparable with the results based on mixed-layer scaling. In fact, the results from the new formulation are within the range of uncertainty provided by previous studies (e.g. Garratt 1992; Stull 1989). On the other hand, the three versions of the old formulation clearly underestimate the TKE, which again shows that these versions are not capable of generating enough convective boundary layer mixing

6. SUMMARY

A new physically based mixing length formulation for the eddy-diffusivity parameterization was tested in COAMPS, in the simulation of a dry convective boundary layer observed during a field experiment in Portugal. The current COAMPS formulation produces boundary layers that are too shallow due to a lack of entrainment. As a consequence, the PBL is too cold and moist when compared to the observations.

The new formulation directly relates the mixing length to a time scale and the square root of the turbulent kinetic energy. This formulation, previously found to compare well with large eddy simulation model results, dramatically improves the simulation of the dry convective boundary layer in a mesoscale model. The evolution of the vertical structures of both potential temperature and water vapor mixing ratio is much more realistic, with the new formulation producing boundary layers that are deeper, warmer and dryer than the current formulation. This implies a better representation of the dry boundary layer development process in general, and of the top entrainment in particular.

A data assimilation experiment showed that these results are significant and that the new formulation reduces the humidity biases in COAMPS. One-dimensional simulations showed that compared to traditional methods of calculating the mixing length (B62 formulations), the new formulation produces a more realistic top entrainment and vertical mixing in general. They also support the idea that it is actually not possible for B62 formulations to reproduce LES results for the dry convective PBL, however large the value of λ may be.

These results overall suggest that this new simple parameterization could have a positive impact in the performance of numerical weather prediction models, with little or no additional computational cost.

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