**6.12** ESTIMATION OF MOMENTUM FLUX IN THE ABL FROM REMOTE SENSING DATA AND UNIVERSAL FUNCTIONS OF FLUX RICHARDSON NUMBER

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**Abstract**

A method for the estimation of profiles of momentum flux and turbulent kinetic energy (TKE) in the atmospheric boundary layer (ABL) from ground based remote sensing data is suggested. This method can be used in sodar or profiler radar applications. Two universal functions of Rf used in this method are estimated using data taken from 99 m meteorological tower. Analytical approximation of these functions is constructed. The shape of one of them is obtained from basic equations and dimensionality.

## 1. INTRODUCTION

During last three decades the Monin-Obukhov (MO) stability parameter *z/L* is widely used as a measure of atmospheric stratification. Many researchers have determined universal functions of this parameter, (e.g. Pahlow et al., 2001). These functions can be used in principle to derive some turbulent parameters from measured others (e.g. friction velocity from vertical velocity variance). Although there is some of agreement about analytical approximations of these functions, they are not widely used for such kind of applications. The disadvantage of the MO similarity theory is that it implies the logarithmic wind profile. Though it is the case for the surface layer of stationary flow, the MO universal functions can hardly be applied for ABL studies.

The main task of this study is to derive the profiles of turbulence parameters in the ABL from remote sensing data on boundary layer height, and profiles of mean wind $U(z)$ and vertical velocity variance $\sigma_w^2$. It was shown (Kouznetsov et al., 2003) that under neutral stratification above the surface layer the ratio between the specific momentum flux mf = $\langle uw \rangle$ and $\sigma_w^2$ is a constant estimated to be equal to 0.7; the corresponding value for turbulent kinetic energy $b/\sigma_w^2$ = 3.4. For generalization to non-neutral stratification, the idea is to replace these constants by some functions of stratification.

The flux Richardson number is chosen as a stratification parameter since it has very clear and simple physical meaning and does not imply any assumptions on the flow except stationarity and horizontal homogeneity. The $\sigma_w$ is chosen as characteristic turbulent velocity scale ("analogue" of MO friction velocity $u_*$) because many modern wind profilers (acoustic or radio) are able to measure it.

In the section 2 the method for remote estimation of momentum flux and TKE from profiler measurements is suggested. The universal functions of Rf used in this method are defined. In the section 3 these empirical functions are estimated basing on the experimental data and their analytical approximation is constructed. Some features of these functions can be obtained from basic equations. This point is discussed in the section 4.

## 2. REMOTE ESTIMATION OF MOMENTUM FLUX

The flux Richardson number is usually defined as:

$$\mathrm{Rf} = \frac{g}{\Theta} \frac{\langle w\theta \rangle}{\langle uw \rangle \, \partial U/\partial z}, \qquad (1)$$

where $g$ – gravity, $\Theta$ – absolute temperature, and $\langle w\theta \rangle$ is the temperature flux; Rf is the ratio of buoyancy and shear production of turbulent kinetic energy.

The flux Richardson number can be used as a parameter of universal functions in a similar way as z/L is used in MO theory. If the function

$$\mathrm{F_{mf}(Rf)} = \frac{\langle uw \rangle}{\sigma_w^2} \qquad (2)$$

is known, the data on $\sigma_w^2$, wind profile and temperature flux can be used to estimate the momentum flux by means of solving (2) as an equation with respect to $\langle uw \rangle$.

Common sodars and radar profilers are unable to measure the temperature flux, however, the shape of $\mathrm{F_{mf}(Rf)}$ (see below) is such that the estimate of momentum flux does not require precise

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data on temperature flux. Many methods to estimate the sensible heat flux using sodar data were described in literature (see e.g. Fiocco et al., 1986). The use of a simple linear function equal to surface heat flux at $z = 0$ and zero at $z = z_i$ leads to reasonable results (Kouznetsov and Beyrich, 2004). The TKE can be estimated then using the empirical function

$$F_{TKE}(Rf) = \frac{b}{\sigma_w^2} \tag{3}$$

## 3. EMPIRICAL UNIVERSAL FUNCTIONS

The experimental measurement of universal function was performed using the data obtained during LINEX-2000 experiment (Engelbart et al., 2000).

### 3.1 *Measuring Site and Equipment*

The measurements were carried out in August and September 2000 at the boundary layer field site (in German: Grenzschichtmessfeld=GM) Falkenberg (Neisser et al., 2002) of the Meteorological Observatory Lindenberg (MOL) of the German Meteorological Service (Deutscher Wetterdienst, DWD). At GM Falkenberg, a 99 m meteorological tower is in continuous operation. The tower is equipped for standard measurements of wind speed, temperature and relative humidity at the 10 m, 20 m, 40 m, 60 m, 80 m and 98 m levels. Wind speed measurements are performed using Thies wind transmitter cup anemometers. Sensors are mounted at each of the given heights at three booms roughly pointing into the N, S, and W directions, respectively. Depending on wind direction it is thus possible always to choose data from an anemometer which is not severely affected from the flow around the tower. During LINEX-2000, turbulence measurements were carried out using METEK USA-1 ultrasonic anemometer-thermometers at the 50 m and 90 m levels of the tower. Raw data of three wind components and of sonic temperature at 15 Hz resolution were recorded. Afterwards they were averaged by 30-minute intervals.

### 3.2 *Data Treatment*

The universal functions $F_{mf}(Rf)$ and $F_{TKE}(Rf)$ were estimated from a one-month data set of half-hour averaged wind speed from cup anemometers (80 and 98 m levels) and turbulence parameters (TKE, $\langle uw \rangle$, $\langle w\theta \rangle$ and $\sigma_w^2$) from a sonic anemometer installed at the 90 m tower level. The data were selected by the stationarity. For the processing only those data with deviations of 10 minutes averaged wind speed from 30 minute averaged values not exceeding 0.5 ms$^{-1}$ were taken. Then for each data point the value of flux Richardson number was calculated. The points with Rf exceeding 1 were dropped as erroneous. (In this case the turbulence fades and both numerator and denominator in (1) tend to zero.)

The remaining data were separated by classes with different Rf . The classes were chosen to provide about 50 data points for each class. Then for each class the linear least square regression with zero bias was calculated. The slope of regression was taken as a value of universal function in the middle of the class. The resulting data are presented in Figures 1 and 2.

In order to have an idea on the accuracy of each data point the averaged values of momentum flux (TKE), the mean-square deviation of initial data points (30 min average) from regression line, and their ratio are also presented. For calculations the following approximations can be used:

$$F_{mf}(Rf) = \begin{cases} 0.2Rf^{0.4} - 0.7, & Rf < 0; \\ Rf - 0.7, & 0 < Rf < 0.3. \end{cases} \tag{4}$$

$$F_{TKE}(Rf) = \frac{3}{2}\left(0.52 - 0.08\frac{1 + 2Rf}{1 - Rf}\right). \tag{5}$$

The points with Rf exceeding $\sim 0.3$ apparently contain large error due to small values of fluxes and variances of wind components when close to the critical Rf . The formula (5) is obtained in the next section.

## 4. ON REDISTRIBUTION OF KINETIC ENERGY

Some information on the shape of $F_{TKE}(Rf)$ can be extracted from TKE components budget equations in conjugation with dimensionality consideration. The budgets of TKE components for stationary conditions can be written as follows (see e.g. Businger, 1982):

$$\begin{aligned} \frac{1}{2}\frac{\partial\sigma_u^2}{\partial t} &= -\langle uw \rangle \frac{\partial U}{\partial z} + \left\langle p\frac{\partial u}{\partial x} \right\rangle - \frac{\varepsilon}{3} = 0, \\ \frac{1}{2}\frac{\partial\sigma_v^2}{\partial t} &= + \left\langle p\frac{\partial v}{\partial y} \right\rangle - \frac{\varepsilon}{3} = 0, \\ \frac{1}{2}\frac{\partial\sigma_w^2}{\partial t} &= \frac{g}{\Theta}\langle w\theta \rangle + \left\langle p\frac{\partial w}{\partial z} \right\rangle - \frac{\varepsilon}{3} = 0. \end{aligned} \tag{6}$$

The TKE dissipation rate $\varepsilon$ is assumed to be equally distributed between the components since the dissipation occurs at small scales, where the turbulence may be considered as isotropic. Using the definition (1) the terms describing redistribution of energy by

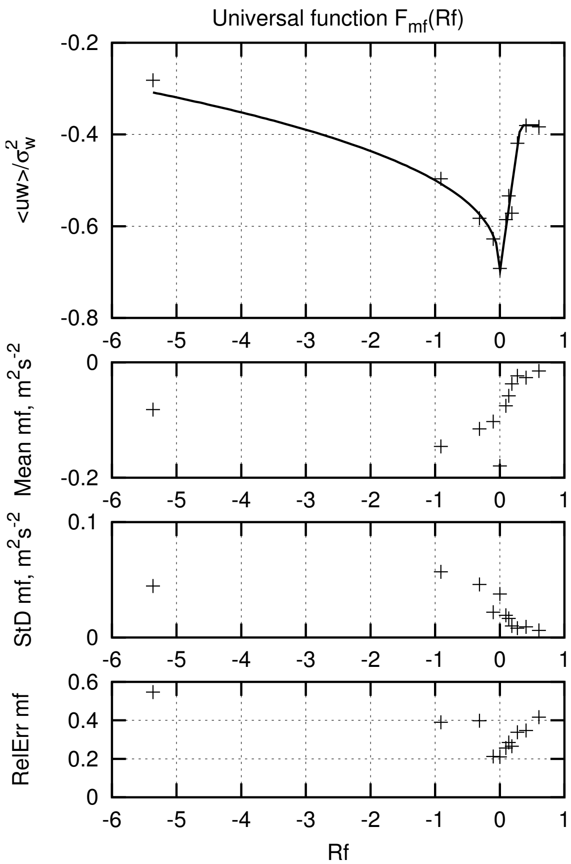


FIG. 1: $F_{mf}$(Rf) and statistical parameters of data used to calculate it.

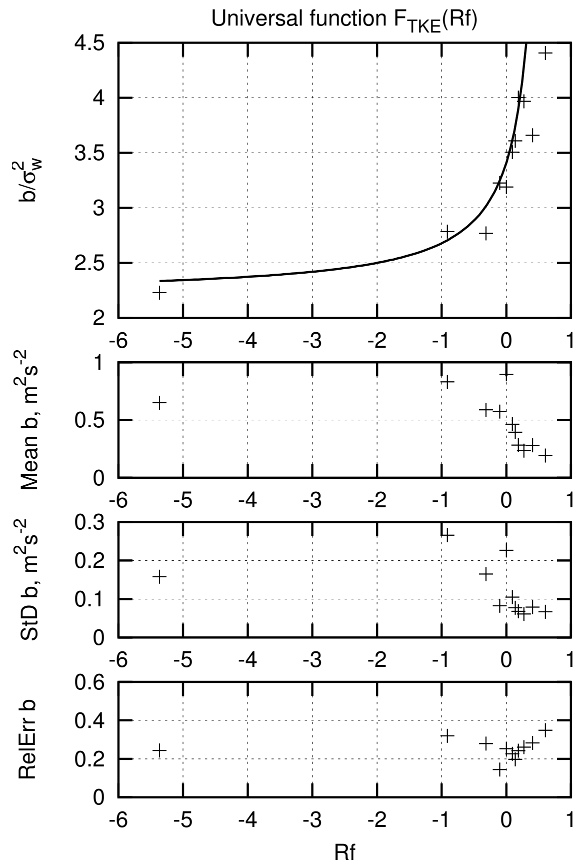


FIG. 2: Same as in Fig. 1 for $F_{TKE}$(Rf).

pressure fluctuations $p$ (divided by air density) can be expressed through $\varepsilon$ and Rf :

$$R_u \equiv \left\langle p \frac{\partial u}{\partial x} \right\rangle = -\frac{\varepsilon}{3}\frac{\mathrm{Rf}+2}{\mathrm{Rf}-1},$$
$$R_v \equiv \left\langle p \frac{\partial v}{\partial y} \right\rangle = \frac{\varepsilon}{3}, \tag{7}$$
$$R_w \equiv \left\langle p \frac{\partial w}{\partial z} \right\rangle = \frac{\varepsilon}{3}\frac{1+2\mathrm{Rf}}{\mathrm{Rf}-1}.$$

The current of energy between each two modes can be expressed as the energy difference between these modes multiplied by the efficiency of energy transfer. This efficiency is assumed to be the same for all modes. The energy difference is simply the difference between variances of corresponding wind components. The efficiency can be found from dimensionality consideration.

$$R_u = A\frac{b^{1/2}}{l}(b - \frac{3}{2}\sigma_u^2),$$
$$R_v = A\frac{b^{1/2}}{l}(b - \frac{3}{2}\sigma_v^2), \tag{8}$$
$$R_w = A\frac{b^{1/2}}{l}(b - \frac{3}{2}\sigma_w^2);$$

where $b$ is TKE, $l$ is some length scale[1] and $A$ is some universal constant. The energy dissipation $\varepsilon$ is usually expressed as (see e.g. Tennekes, 1982):

$$\varepsilon = \frac{b^{3/2}}{Cl}. \tag{9}$$

Here $C$ is also some constant. All stratification dependence is included into $l$. Substituting this expression into the last equations of (7) and (8) one can obtain the following expression for $F_{TKE}$(Rf):

$$\mathrm{F_{TKE}}(\mathrm{Rf}) \equiv \frac{b}{\sigma_w^2} = \frac{3}{2}\left(A - \frac{1}{3C}\frac{1+2\mathrm{Rf}}{1-\mathrm{Rf}}\right)^{-1}. \tag{10}$$

[1]proportional to the Prandtl scale under neutral stratification

Kouznetsov and Beyrich (2004) have shown using carefully selected data for neutral stratification that $F_{TKE}(0) = 3.4$. This leads to $\frac{1}{3C} = A - 0.44$. Choosing $A = 0.08$ we obtain the formula (5), that gives a good agreement with experimental data (see Figure 2).

## 5. CONCLUSIONS

In a few words the idea of the method can be expressed as follows. Basing on sodar (radar) and surface data one estimates the profiles of $\partial U/\partial z$, $\langle w\theta\rangle$ and $\sigma_w^2$. Then using analytical approximation of $F_{mf}(Rf)$ (4) and the definition of Rf (1) one solves (2) as an equation with respect to $\langle uw\rangle$ for each level of initial profiles. The branch of the function $F_{mf}(Rf)$ can be chosen basing on the sign of the surface temperature flux. After that one can obtain the profile of TKE using (3) and definition (1).

The method was shown to provide reasonable results with sodar data obtained during LINEX-2000 experiment by means of comparison with *in situ* measurements of momentum flux (Kouznetsov and Beyrich, 2004). However it was not yet tested at altitudes others then 90 meters.

The analytical approximations presented in section 3.2 are to be tested with larger (more accurate, better processed) data set. In ideal case it would be useful to calculate them from initial (about 30 minute averaged) data points. This requires a comprehensive analysis of errors of all the measurements and of stationarity of flow instead of a simple procedure used in this study.

The equations and constants obtained in section 4 allow one to construct also the functions $\sigma_u^2/\sigma_w^2$ and $\sigma_v^2/\sigma_w^2$. The comparison of these functions with experimental data would provide a basis for judgment about the correctness of approach used. This is also a subject for further studies.


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