7.15 A coarse-grain, quasi-normal model of stably stratified turbulent flows and its implementation in $K-\epsilon$ modeling of atmospheric ABLs

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A coarse-grain, quasi-normal model of turbulence is applied to flows with stable stratification. The model's parameters are calculated based upon a self-consistent recursive procedure of small-scale modes elimination starting at the Kolmogorov scale k_d. The model includes both vertical and horizontal eddy viscosities and diffusivities and, thus, explicitly recognizes the anisotropy introduced by stable stratification. There are significant differences in the behavior of these turbulent exchange coefficients with increasing stratification. Generally, the vertical coefficients are suppressed while their horizontal counterparts are enhanced. The model accounts for the combined effect of turbulence and internal waves on the exchange coefficients. When the process of scale elimination is extended up to the turbulence macroscale, the model yields the eddy viscosities and eddy diffusivities in the RANS format. These parameters are utilized in a $K-\epsilon$ model of a stably stratified ABL. The use of the spectral model makes it possible to circumvent the problematics of the Reynolds stress closure models. The dependence of the coefficients of the dissipation equation on the flow characteristics is explored and tested in the limiting case of the structural equilibrium. The new $K-\epsilon$ model is validated in simulations of the atmospheric stable boundary layer (SBL) over sea ice.

I. THE SPECTRAL MODEL

The spectral closure theory is developed for a fully three-dimensional, incompressible, turbulent flow field with imposed homogeneous vertical stable temperature gradient; the flow is governed by the momentum, temperature and continuity equations in Boussinesq approximation,

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\nabla)\mathbf{u} + \alpha g \theta \hat{e}_3 = \nu_0 \nabla^2 \mathbf{u} - \frac{1}{\rho}\nabla P + \mathbf{f^0}, \quad (1)$$

$$\frac{\partial \theta}{\partial t} + (\mathbf{u}\nabla)\theta + \frac{d\Theta}{dz}u_3 = \kappa_0 \nabla^2 \theta, \quad (2)$$

$$\nabla \mathbf{u} = 0, \quad (3)$$

where $\mathbf{u}$ and θ are the fluctuating velocity and potential temperature; P is the pressure, ρ is the constant reference density, ν_0 and κ_0 are the molecular viscosity and diffusivity, respectively, α is the thermal expansion coefficient, g is the acceleration due to gravity directed downwards, $\frac{d\Theta}{dz}$ is the mean potential temperature gradient, and $\mathbf{f}^0$ represents a large-scale external energy source customarily used in spectral theories of turbulence; it maintains turbulence in statistically steady state and may originate from large-scale shear instabilities. Due to strong nonlinear interactions, the external forcing excites all Fourier modes down to the dissipative scale k_d. The modes exert random agitation upon each other which manifests as a stochastic *modal* forcing $\mathbf{f}$. This forcing is used to replace the non-linear equations (1), (2) by modal stochastic equations

$$u_i(\mathbf{k},\omega) = G_{ij}(\mathbf{k},\omega) f_j(\mathbf{k},\omega), \quad (4)$$

$$\theta(\mathbf{k},\omega) = -\frac{d\Theta}{dz}u_3(\mathbf{k},\omega)G_\theta(\mathbf{k},\omega), \quad (5)$$

also known as the Langevin equations. Here, $G_{ij}(\mathbf{k},\omega)$ and $G_\theta(\mathbf{k},\omega)$ are the velocity and the temperature Green functions, respectively. They include terms accounting for the damping of a given mode by all other modes due to nonlinear interactions. Eventually, these terms are associated with k-dependent viscosities and diffusivities [1, 2]. The replacement of the fully nonlinear Navier-Stokes equations by the Langevin equations represents a *mapping* of the original flow field onto a quasi-Gaussian field $\mathbf{f}(\mathbf{k},\omega)$ under the constraints of incompressibility and conservation of the modal energy flux. In the case of neutral stratification, this approach recovers some basic features of isotropic homogeneous turbulence including the Kolmogorov spectrum [1].

The eddy damping parameters are calculated using a systematic algorithm of successive averaging over small scales of velocity and temperature which yields a system of four coupled ODEs for turbulent viscosities and diffusivities,

$$\frac{d}{dk}(\nu_h, \nu_z, \kappa_h, \kappa_z) = -\frac{\epsilon}{k^5}R_{1,2,3,4}(\nu_h, \nu_z, \kappa_h, \kappa_z), \quad (6)$$

where ϵ is the rate of the viscous dissipation; ν_h and ν_z are the horizontal and the vertical eddy viscosities, respectively; κ_h and κ_z are the horizontal and the vertical

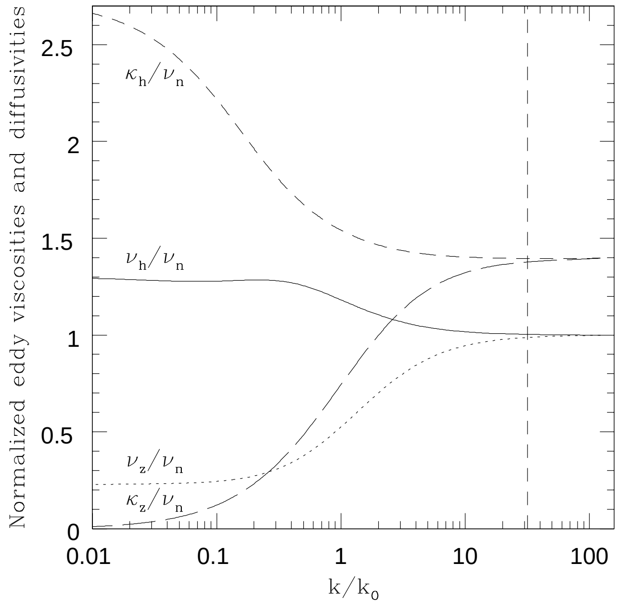

FIG. 1: Normalized horizontal and vertical eddy viscosities and diffusivities as functions of k/k_O. The vertical dashed line corresponds to k_t, the threshold of internal wave generation in presence of turbulence [3].

eddy diffusivities, respectively, and R_1 through R_4 are algebraic expressions. The procedure takes account of the combined effect of turbulence and internal waves. The computation starts at the Kolmogorov scale k_d where the initial values of the vertical and the horizontal viscosities and diffusivities are equal to their respective molecular values ν_0 and κ_0 and is continued to an arbitrary wave number $k < k_d$. The system (6) can be solved numerically. Solutions obtained for the nondimensional variables ν_h/ν_n, ν_z/ν_n, κ_h/ν_n and κ_z/ν_n are presented in Fig. 1 as functions of the ratio k/k_O, where $k_O = (N^3/\epsilon)^{1/2}$ is the Ozmidov wave number and ν_n is the eddy viscosity for neutral stratification.

Partial scale elimination up to the wave number k_c can be used as a subgrid-scale (SGS) parameterization for LES with k_c associated with the grid resolution. Then, the ratio k_c/k_O can be viewed as a measure of the deviation of the SGS viscosities and diffusivities from their values under neutral stratification. In many SGS parameterizations in use today, the effect of stable stratification is not taken into account. Figure 1 indicates that for $k_c/k_O \gg 1$, such an approximation is, indeed, satisfactory. However, for $k_c/k_O = O(1)$, the effect of stable stratification cannot be neglected. The need to improve SGS representation for LES of strongly stratified flows was emphasized in the recent GABLS report [4]. The present theory provides a self-consistent framework to address this need.

II. IMPLEMENTATION OF THE SPECTRAL RESULTS IN $K-\epsilon$ MODELING

If the process of small scales elimination is extended to the largest scales of the system, i.e., the integral length scale, k_L^{-1}, then a Reynolds average scheme, or a RANS model would emerge. In a two-equation format, RANS models employ prognostic equations for turbulent kinetic energy, K, and the rate of dissipation, ϵ, or, alternatively, the turbulence macroscale while the nondimensional stability functions that enter expressions for the eddy viscosity and eddy diffusivity are usually obtained from the second-moment closure models. We have used ν_z and κ_z from Fig. 1 to develop a $K-\epsilon$ model based upon the spectral theory rather than the Reynolds stress closure. In simulations of SBLs, it was found necessary to generalize the formulation of the ϵ-equation given in [5] to include the effect of stratification in addition to the rotation,

$$C_1 = C_1^0 + C_f Ro_*^{-1} - C_N Fr_*^{-1}, \tag{7}$$

$$C_1 \geq C_1^0, \tag{8}$$

where $Ro_* = u_*/|f|L$, $Fr_* = u_*/NL$, u_* is the friction velocity, f is the Coriolis parameter, C_1^0 is the standard coefficient equal to 1.44, $L = 0.16K^{3/2}/\epsilon$ is the turbulence macroscale used in the $K-\epsilon$ modeling, $C_f = 111$ and $C_N = 0.55$ are empirical constants. The new $K-\epsilon$ model has been tested in simulations of ABL over sea ice and compared with LES of the Beaufort Arctic Storms Experiment (BASE) [6] and the data from the Surface Heat Budget in the Arctic program (SHEBA). The results of the simulations with the new $K-\epsilon$ model are shown in Figs. 2-4; generally, they are in good agreement with both LES and the data. Some discrepancy in the case of strong stratification is attributed to the effect of stable stratification on the SGS parameterization that was not accounted for in the LES.

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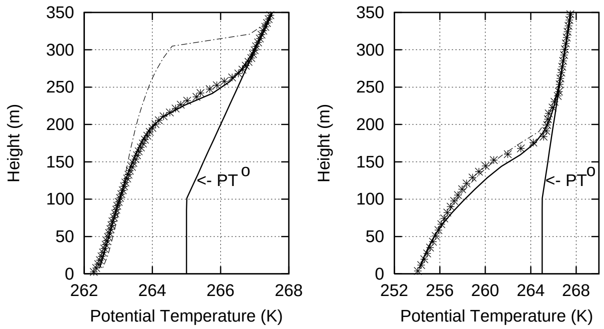

FIG. 2: Vertical profiles of mean potential temperature for the cases of moderate (left panel) and strong (right panel) stable stratification simulated with the new (solid line) and the standard (dashed-dotted line) $K-\epsilon$ models. The LES results [6] are shown by asterisks. The initial PT profiles are shown by straight solid lines.

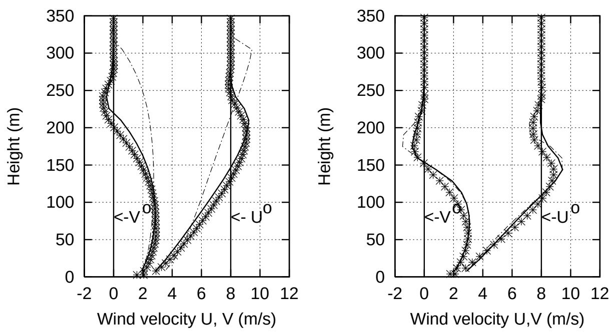

FIG. 3: Vertical profiles of mean horizontal wind components, U and V, simulated with the new and the standard $K-\epsilon$ models. The order of the panels and the description of the lines and asterisks are the same as in Fig. 2.

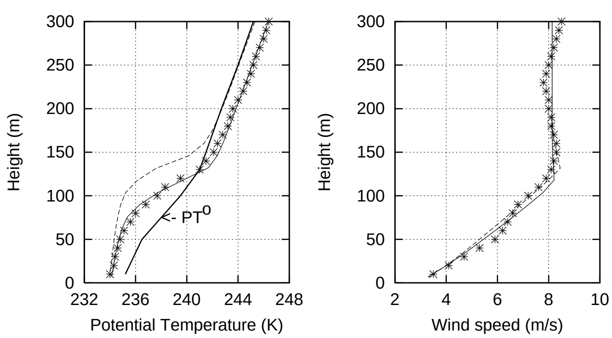

FIG. 4: Vertical profiles of potential temperature (left panel) and wind speed (right panel) in SHEBA experiment. The observational data is represented by asterisks, the simulations are shown by the thin solid lines.