# 11.11 A GENERALIZED PLANAR FIT METHOD FOR SONIC ANEMOMETER TILT CORRECTION 

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#### Abstract

Previous methods of sonic anemometer instrument tilt correction have assumed the mean vertical wind component, $\bar{w}$, to vanish. Analogous to the planar fit method, we have established a Fourier fit method of instrument tilt correction which can be used in complex canopies and does not require the mean vertical wind to be zero. A brief analysis of the effect of each method on the mean winds and the flux calculations is done.


## 1. INTRODUCTION

The presence of instrument tilt has long been recognized as a source of error in micrometeorological flux calculations (Wilczak, et al. 2001, and references therein). Traditional methods of tilt correction have only been interested in calculating fluxes in an along-wind coordinate system, and therefore rotate the three dimensional wind vector such that $\bar{v}=\bar{w}=0$. With these methods, no attempt is made at determining if a non-zero $\bar{w}$ is present. Within urban settings as well as forest canopies, $\bar{w}$ is expected to be non-zero and possibly even significant in magnitude.

Sonic anemometers are able to measure very low wind speeds very accurately compared to either cup or propeller anemometers, but the mean vertical wind, $\bar{w}$, is still too small even for sonic anemometers to measure. Instrument tilt is the major limiting factor. A tilt of only $1^{\circ}$ can alter the measured $\bar{w}$ by more than $0.08 \mathrm{~ms}^{-1}$ with a $5 \mathrm{~ms}^{-1}$ mean horizontal wind. Even in situations where $\bar{w}$ is non-zero, it may be smaller than $0.08 \mathrm{~ms}^{-1}$.

## 2. DATA

The Army Research Laboratory (ARL) deployed an array of sonic anemometers mounted on five

[^0]towers in Oklahoma City, Oklahoma, during the Joint Urban 2003 field campaign, a cooperative undertaking to study turbulent transport and dispersion in the atmospheric boundary layer within an urban environment. The towers were located in a variety of locations to sample both industrial (urban) and semi-rural (suburban) conditions. Data from two of these towers will be used for this analysis: one suburban location with relatively few local obstructions and one industrial location with many buildings and some trees surrounding the tower.

Both towers had sonic anemometers at 10 meter and 5 meter elevations. Both the 10 m sonics were mounted above the tower, so little influence is expected from the tower for these instruments. The 5 m instruments were mounted due south of the tower.

## 3. TRADITIONAL METHODS

One of the most used methods of tilt correction rotates the 3D wind vector such that $\bar{v}=\bar{w}=0$ for each averaging segment before calculating the fluxes (Kaimal and Finnigan, 1994). Another method adds a third rotation which sets $\overline{v^{\prime} w^{\prime}}=0$ for each segment. Since $\bar{w}$ is set to zero for each segment of data, all directional and diurnal information about the mean vertical wind is lost.

The planar fit tilt correction method outlined by Wilczak, et al., (2001) can preserve information about small differences in $\bar{w}$ for an instrument located over flat terrain with no large obstructions nearby. With this method, the data are visualized as a plane in the three-dimensional $[\bar{u}, \bar{v}, \bar{w}]$ space. A best-fit plane is calculated, and coordinate rotation angles, $\alpha$ and $\beta$, and an offset term are then determined that will rotate the data so that the best-fit plane is now the $\bar{w}=0$ plane, thereby setting the average of all the $\bar{w}$ to zero over the duration of the experiment. The angle $\alpha$ is the rotation about the $y$-axis and $\beta$ is the rotation about the $x$-axis. An additional
rotation is required to align the mean wind vector with the $x$-axis. After the rotation, the average of all the tilt-corrected $\bar{w}$ values is zero, so each averaging segment could have a non-zero $\bar{w}$ value. The presence of a large obstruction nearby, such as the tower on which the instrument is mounted, will cause the wind data to no longer be planar, making this method inappropriate.

If there are a limited number of local obstructions and the data from the unobstructed directions sample a sufficiently broad region of $[\bar{u}, \bar{v}, \bar{w}]$ space, a modified planar fit may be performed using only the unobstructed data. This will allow accurate determination of the rotation parameters by eliminating the biases introduced by local obstructions. The resulting tilt-correction parameters are then applied to all the data, thereby correcting for instrument tilt and also preserving any directional dependence in $\bar{w}$.

## 4. VERTICAL WIND ANGLE

In another approach to the same problem, instrument tilt will produce a net apparent vertical wind angle, $\quad \delta=\tan ^{-1}\left(\bar{w} / \sqrt{\bar{u}^{2}+\bar{v}^{2}}\right)$, which varies with wind direction even over flat terrain where $\bar{w}$, and therefore $\delta$, would otherwise be zero (Paw U et al., 2000; Mahrt et al., 2000). The magnitude of the vertical wind angle due to instrument tilt varies sinusoidally with wind direction, and the larger the instrument tilt, the larger the amplitude of the sinusoid. The phase of the sinusoid depends on the relative direction of the instrument tilt. This sinusoid has exactly one period over the entire 360 degrees of wind direction.

Local obstructions will alter the vertical wind angle with respect to the sinusoid resulting from instrument tilt. When there are obstructions in all directions, the sinusoid due to instrument tilt is completely obscured.

Any series of data can be expressed in terms of a Fourier series: a sum of sines and cosines. When the vertical wind angle data are Fourier decomposed, the amplitude and phase of the fundamental Fourier frequency is directly related to the instrument tilt. The higher order frequencies are the result of the local obstructions.

The fundamental Fourier frequency could also include other influences in addition to instrument
tilt. If obstructions in opposite directions have opposite effects on $\delta$, this will create a contribution to the Fourier fundamental. Since the sum of two sinusoids of the same frequency is another sinusoid of the same frequency, these additional effects cannot be separated from instrument tilt using only the wind data. It is necessary to know the tower site to be aware of such possibilities.

## 5. FOURIER FIT METHOD

To obtain the Fourier series, evenly spaced vertical wind angle data with respect to wind direction is required. This is accomplished by separating the data into evenly sized bins ( $15^{\circ}$ for this analysis) based on the mean horizontal wind direction and obtaining separate averages of the mean vertical winds and mean horizontal winds for each bin. Then a series of mean vertical wind angles can be computed which are evenly spaced in wind direction.

Once the evenly spaced data series is constructed, the terms of the Fourier series are calculated as

$$
\begin{align*}
& A_{j}=\frac{2}{n} \sum_{t=0}^{n-1} x_{t} \cos \omega_{j} t  \tag{1}\\
& B_{j}=\frac{2}{n} \sum_{t=0}^{n-1} x_{t} \sin \omega_{j} t
\end{align*}
$$

where $x_{t}$ are the data, $t$ is the index number of the $t$ th data value, n is the total number of data points and $\omega_{j}=2 \pi j / n$ is the $j$ th Fourier frequency (Bloomfield 1976). The Fourier fundamental is of the form

$$
\begin{equation*}
\delta_{\text {Fourier }}=R \cos (\pi(\theta+\phi) / 180) \tag{2}
\end{equation*}
$$

where $R=\sqrt{A_{1}^{2}+B_{1}^{2}}$ is the amplitude of the fundamental, $\phi=\tan ^{-1}\left(-B_{1} / A_{1}\right)$ is the phase of the fundamental in degrees and $\theta$ is the wind direction in degrees.

The parameters $R$ and $\phi$ can be related to the rotation angles $\alpha$ and $\beta$ through the relationships

$$
\begin{align*}
& \alpha=\tan ^{-1}(-R \sin \phi) \\
& \beta=\tan ^{-1}(-R \cos \phi) \tag{3}
\end{align*}
$$

assuming the small angle relationships. The magnitude of the offset term is somewhat arbitrary
with this method. The offset term can be set to either force the average of all $\bar{w}$ to be zero or to set the average $\bar{w}$ from a specific wind direction to be zero.

Tab. 1 Rotation angles in degrees for each sonic anemometer as calculated from both the planar fit method and the Fourier fit method.

|  | Planar fit |  | Fourier fit |  |
| :--- | :---: | :---: | :---: | :---: |
| Instrument | $\alpha$ | $\beta$ | $\alpha$ | $\beta$ |
| 10m suburban | 2.3 | -2.7 | 1.6 | -2.7 |
| 5m suburban | 1.3 | -0.1 | 0.8 | -0.2 |
| 10m urban | -1.2 | -1.4 | 0.1 | 1.2 |
| 5m urban | -3.0 | -2.0 | -1.1 | 1.3 |

## 6. EFFECT ON MEAN VERTICAL WIND

For the 10 meter instrument at the suburban tower, there are no obstructions, and the planar fit method is straightforward (Fig. 1). The Fourier fit tilt-correction parameters agree well with the planar fit tilt-correction parameters (Tab. 1) with the amplitude of the Fourier fit being slightly smaller. After tilt correction, the vertical wind angles are much closer to zero, but some directional dependence remains (Fig. 1). These variations may be due to local obstructions or subtle variations in the terrain.

The 5 meter instrument at the suburban tower not only is influenced by the tower to the north, but also by a mobile trailer to the SSW and a bus parking area to the NW. A modified planar fit using wind data only from wind directions $45^{\circ}-200^{\circ}$ and $220^{\circ}-255^{\circ}$ has parameters that roughly agree with the Fourier fit using all the data (Tab. 1). The amplitude of the Fourier fit sinusoid is smaller than the amplitude from the planar fit, but the phases are nearly identical.

At the urban tower, the planar fit and Fourier fit derived parameters do not agree (Fig. 2). For both instruments, the amplitude of the planar fit is much larger than the amplitude from the Fourier fit, implying a much larger instrument tilt: $1.8^{\circ}$ total as compared to $1.2^{\circ}$ for the 10 m instrument and $3.6^{\circ}$ compared to $1.7^{\circ}$ for the 5 m instrument. The phases disagree as well, which means that the methods disagree on the direction of the instrument tilt: over $140^{\circ}$ difference for the 10 m
instrument and about $90^{\circ}$ difference for the 5 m instrument.


Fig. 1 10m sonic at the suburban tower. Circles are the vertical wind angles (in radians) as a function of wind direction calculated from the raw data. The curve is the sinusoid defined by the best fit plane. The red stars are the vertical wind angles after tilt correction.

The urban tower site is in a parking lot which occupies the entire city block. The streets are on a grid aligned with the cardinal directions. Across the streets from the parking lot, about 35-45 meters from the tower, are one and two story buildings to the S and W (3-6 meters tall), a 7.5 meter tall wall to the E , and large trees to the N . A 10-12 meter tall tree grows in the parking lot, 15 meters SW of the tower.

The two tilt correction methods have very different effects on the mean vertical wind. The planar fit tilt correction (Fig 3, x-data) results in net upwelling for winds from the SW and SE, a net downward motion for winds from the N and NE , and a very large downward motion for winds from the W and NW. The Fourier fit tilt correction (Fig 3., *-data) results in net downward motions for winds from the $\mathrm{E}, \mathrm{S}$ and W and net upward motion for winds from the SW and N. This corresponds to a net upward motion when trees are upwind and a net downward motion when rigid objects like buildings and walls are upwind. The Fourier fit tiltcorrected winds correlate better with the local obstructions than the planar fit winds.


Fig. 2 Comparison of planar fit tilt correction (solid line) to Fourier fit correction (dashed line) for the 5 m sonic at the urban tower. The two methods disagree significantly in both the amount and direction of the instrument tilt. The effect of surrounding buildings and a large tree to the SW make the planar fit method inappropriate.


Fig. 3 Urban tower 5m sonic anemometer data. Comparison of vertical wind angles for the raw data (circles), the Fourier fit tilt corrected data (stars) and the planar fit tilt corrected data (x) (lines added to aid the eye). The exaggerated wind angles for the planar fit data in the NW quadrant do not correlate with the layout of the local surface obstructions.

## 7. EFFECT ON FLUX CALCULATIONS

For data from the suburban tower, calculated values for $\overline{u^{\prime} w^{\prime}}, \overline{v^{\prime} w^{\prime}}$, and $\overline{w^{\prime} T^{\prime}}$ are all similar to each other no matter which tilt correction method was used. All the tilt-corrected fluxes differed significantly from the fluxes calculated from the raw data.

At the urban tower, calculated fluxes were similar for the raw data and the Fourier fit tilt-corrected data. Fluxes calculated from the planar fit tiltcorrected data are probably not reliable based on observation of the tilt corrected vertical winds (Fig. 3). Flux values calculated using the traditional tilt correction (set $\bar{v}=\bar{w}=0$ for each flux calculation) agree with each other whether starting with the raw data or the Fourier fit tilt corrected data.

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