P7.6 COMPARISON AMONG FIRST, SECOND AND THIRD ORDER CBL MODELS

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1 INTRODUCTION

The pure convective boundary layer (CBL) occurs in the atmosphere typically during windless sunny days. It consists of a well mixed convective region, a capping inversion and stably stratified air aloft. The turbulent kinetic energy (TKE) flux makes the stable fluid to be entrained into the mixed layer producing a negative heat flux that depends on the surface heat flux, the initial and the boundary conditions. Turbulent flows are often modeled with a Reynolds stress approach. In the CBL the characteristics of the turbulence are mainly non-local, thus the prognostic equations of all second-order moments (SOMs) with realistic third order moments (TOMs) are required. This kind of models is computationally too expensive to be implemented in operational models. Therefore lower order models are usually preferred. Turbulence closure models can be simplified by invoking: downgradient approximation of the TOMs, isotropy of TKE, downgradient approximation of temperature flux and the parameterization of the mixing length through a diagnostic formulation. Here these successive simplifications are applied in this order. If the first three simplifications are applied, the model is called E - ϵ. When the last is also applied it is an E-ℓ. This two last models, of the first order, are simple enough to be used in the operational models. The aim is to understand the conseguences of these simplifications. Moreover the formulations of the mixing length and the ϵ equation in the first order models are modified in order to try to reproduce the results of the third order model.

2 THE MODEL EQUATIONS

The dynamical equations for the first and second order moments are (Canuto (1994), hereafter C94),

$$\frac{\partial\Theta}{\partial t} = -\frac{\partial}{\partial z}\overline{w\theta} \quad (1)$$

$$\frac{\partial}{\partial t}\overline{w\theta} + \frac{\partial}{\partial z}\overline{w^2\theta} = \beta\overline{w^2} + \lambda\overline{\theta^2} - \Pi_3^\theta \quad (2)$$

$$\frac{\partial\overline{\theta^2}}{\partial t} + \frac{\partial}{\partial z}\overline{w\theta^2} = 2\beta\overline{w\theta} - 2\epsilon_\theta \quad (3)$$

$$\frac{\partial\overline{w^2}}{\partial t} + \frac{\partial}{\partial z}\overline{w^3} = 2\lambda\overline{w\theta} - \Pi_{33} - \frac{2}{3}\epsilon \quad (4)$$

$$\frac{\partial\overline{u^2}}{\partial t} + \frac{1}{2}\frac{\partial}{\partial z}\left(\overline{wq^2} - \overline{w^3}\right) = \frac{1}{2}\Pi_{33} - \frac{2}{3}\epsilon \quad (5)$$

where Θ is the mean potential temperature, $\beta = -\partial\Theta/\partial z$, $\lambda = g\alpha$, g is the acceleration of gravity and α is the volume expansion coefficient and $\overline{q^2}$ is twice the TKE ($\overline{q^2} = \overline{w^2} + 2\overline{u^2}$). The pressure correlation terms are modeled with the following expressions:

$$\Pi_3^\theta = 2c_6\tau_p^{-1} + c_7\lambda\overline{\theta^2} \quad (6)$$

$$\Pi_{33} = 2c_4\tau_p^{-1}\left(\overline{w^2} - \frac{\overline{q^2}}{3}\right) + \frac{4}{3}c_5\lambda\overline{w\theta} + 2\frac{\partial}{\partial z}\overline{pw} \quad (7)$$

The return-to-isotropy time scale is modeled as:

$$\tau_p^{-1} = \tau(1 - C_w N^2\tau^2)^{-1} \quad (8)$$

where τ is the turbulence time scale ($\tau = \overline{q^2}/\epsilon$), N^2 is the Brunt-Väisälä frequency ($N^2 = -g\alpha\beta$) and the last term of (7) is $\overline{pw} = -a\overline{wq^2}$. The dissipation of TKE is modeled as:

$$\frac{\partial\epsilon}{\partial t} + \frac{\partial}{\partial z}\overline{w\epsilon} = 2a_1\tau^{-1}\lambda\overline{w\theta} - a_2\epsilon\tau^{-1} + a_3N\tau^{-1}\overline{w^2} \quad (9)$$

with

$$\overline{w\epsilon} = -\frac{3}{2}A_0\tau(\overline{w^2} + A_0\tau\lambda\overline{w\theta})\frac{\partial\epsilon}{\partial z}. \quad (10)$$

Finally,

$$\epsilon_\theta = 2c_2\tau^{-1}\overline{\theta^2}. \quad (11)$$

Prognostic Third order moments (PT) The TOMs dynamical equation, taken from C94 and not re-written here, are based on the quasi-normal approximation of the fourth order moments.

3 SIMPLIFICATIONS

Model Third order moments (MT) In the stationary case the TOMs dynamical equation become a linear system of equations that can be inverted resulting in an analytic model for the TOMs. For the TOMs model it is possible to refer, another time, to C94. The same closure constants will be used also in the models with the other simplifications. Different parameterizations of the TOMS was tested too: the Zeman Lumley

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(as reported in C94) TOMs, that are similar to C94 ones but neglect some of the dependecies on the SOMs gradients, and Canuto (2001) TOMs, that avoid the quasi-normal approximation of the fourth order moments.

Downgradient Third order moments (DT) In C94 the TOMs are linear combinations of all the SOMs gradients; downgradient approximation of a TOM, that is the flux of a SOM, can be achieved by neglecting its dependency on all the other SOMs gradients.

Isotropy of TKE (IT) It is possible to derive the equation for $\overline{q^2}$ from eqs. (4) and (5). The result is

$$\frac{\partial \overline{q^2}}{\partial t} + \frac{\partial}{\partial z}\overline{wq^2} = 2\lambda\overline{w\theta} - 2\epsilon. \qquad (12)$$

This is used instead of the eqs. for $\overline{w^2}$ and $\overline{u^2}$, that are now defined as $\overline{w^2} = \overline{u^2} = \overline{q^2}/3$. These values are used for the TOMs computation without any changing in the TOMs model.

Downgradient Heat flux (DH) Downgradient approximation for the temperature flux can be achieved from the analytic solutions of the system composed by eqs. (2) and (3) after neglecting their non-stationary and non-local terms. The result is

$$\overline{w\theta} = \frac{\tau_p \overline{w^2}}{2c_6 - \frac{1-c_7}{2c_2}\tau\tau_p\lambda\beta} \cdot \beta \qquad (13)$$

$$\overline{\theta^2} = \frac{\tau\overline{w\theta}}{2c_2} \cdot \beta \qquad (14)$$

diagnostic Mixing Length (ML) The use of the prognostic equation for the TKE dissipation can be avoided by prescribing an analytical mixing length, that is linked to the TKE and to the TKE dissipation by the Kolmogorov relation

$$\ell = (c_E\overline{q^2}/2)^{3/2}/\epsilon \qquad (15)$$

Here a non-local formulation in the mixed layer and the Deardorff formula in the stable region like those in Cuxart (2000) are used

$$\ell = c_\ell(l_{up}l_{down})^{\frac{1}{2}} \; z < z_i \qquad (16)$$

$$\ell = \left(-\frac{\overline{q^2}}{\lambda\beta}\right)^{\frac{1}{2}} \; z > z_i \qquad (17)$$

where l_{up} and l_{down} are the distance between the level in which the mixing length is computed and the ground or the inversion (z_i) respectively, and c_ℓ ($\simeq 0.5$) is an adjustable parameter.

NUMERICAL SOLUTIONS AND RESULTS

Here Deardorff (1985) experiment, that is a classical CBL case study, is simulated in order to check the model

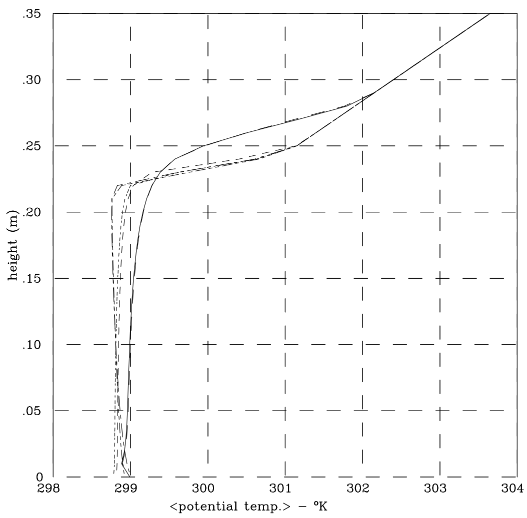

Figure 1: Temperature profiles after 600 sec. Continuous line: PT; dashed lines (from bigger to smaller segments): CT, DT, IT; dash-dotted line: DH (E-ϵ model); dash-dot-dotted line: ML (E-ℓ model).

reliability trough the comparison with the results of C94. This is useful also because C94 showed that the results of the model that here is called MT are in good agreement with the LES ones. For the actual implementation and boundary condition it is possible to refer to the same paper. There are differencies in the specification of the lower boundary conditions, because here any computational level below ground is used, and probably in the numerical implementation, that can explain the differencies that are found between this and C94's results.

Effects of the Simplifications

Fig.1 shows that the forecasted temperature at the lower boundary is comparable in all the models, while differencies are found mainly in the upper mixed layer and in the entrainment region: the models can be splitted into two classes, PT and MT give almost the same results, because the system is quasy-stationary, while all the others, based on the DT simplification, produce a shallower mixed layer, with lower temperature values at the top, and more stable layer aloft. In the models with DH simplification the mixed layer stratification is unstable, in order to mantain the upward heat flux. From fig.2 it comes out that the models with the DT simplification underestimate the magnitude of the inversion and produce a shallower entrainment region respect to the PT and MT models. This different heat budget explains the differences in the mixed layer temperature shown in fig.1. The difference in the intensity of the entraining process is explained by fig.3, that shows that the class of models with the DT simplification un-

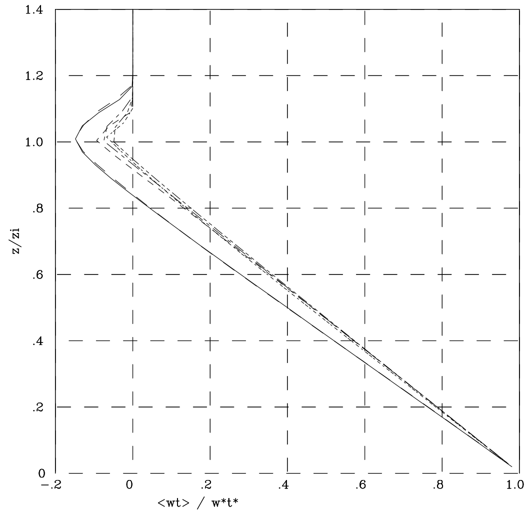

Figure 2: Same as fig.1, but for the normalized temperature flux.

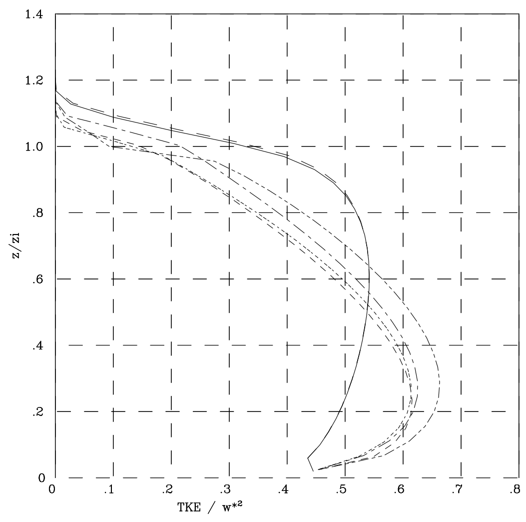

Figure 3: Same as fig.1, but for normalized TKE.

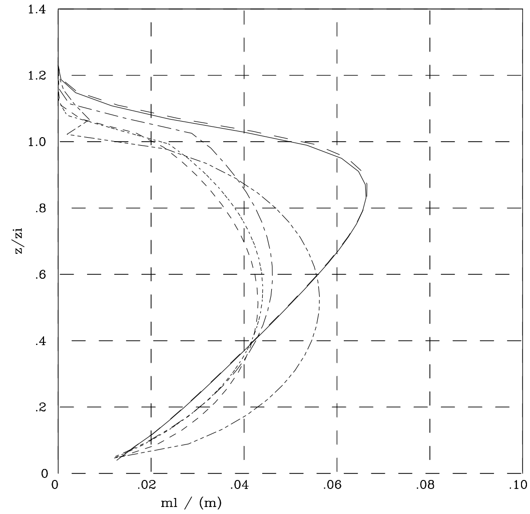

Figure 4: Same as fig.1, but for the equivalent mixing length, (obtained using the Kolmogorov relation (15)).

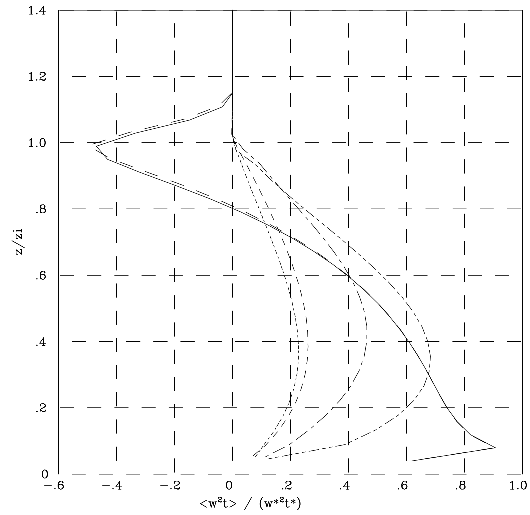

Figure 5: Same as fig.1, but for the normalized temperature flux of the flux.

derestimate the TKE in the entrainment region while overestimate it in the mixed layer. Fig.4 shows that DT and MT models produce a maximum of equivalent mixing length located an upper portion of the mixed layer. This corresponds to a maximum of $\overline{u^2}$ that represents the strong horizontal mixing due to the eddies splattering effect as they reach the inversion. Fig.5 shows that DT and MT models produce, in the entraining region, a negative peak of temperature flux of the flux that is missed by the models with DT simplification, in which this variable is always positive. In this latter models the resulting TKE flux (not shown) is negative in the lower part of the mixed layer, while it is always

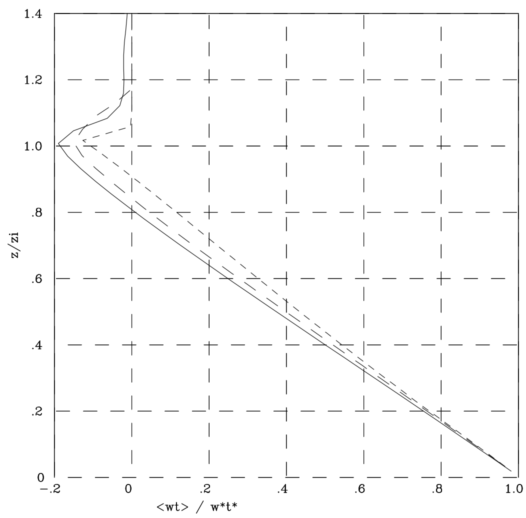

Figure 6: Normalized temperature flux profiles after 600 sec. TOMs by Zeman Lumley (as reported in C94), continuos line; C94 and Canuto (2001), dashed lines with bigger and smaller segments, respectively.

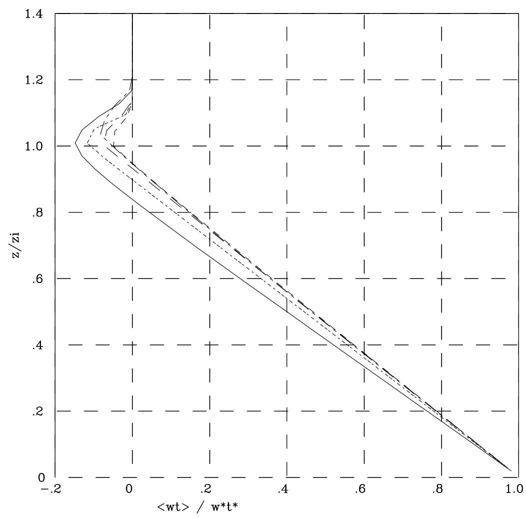

Figure 7: Normalized temperature flux profiles after 600 sec. Continuous line: PT; dashed lines (from bigger to smaller segments): E-ϵ, E-ℓ, E-ℓ'; dash-dotted line: E-ϵ'; (explanations in the text).

positive in the DT and MT models.

Comparison among TOMs models

Different parameterizations of the TOMS was tested too (fig.6). Here it is not intended to give a full description of this results but only to stress the fact that the model can be also very much sensitive of the different TOMs parameterizations. Even bigger difference can be found in the other fields, not presented here.

Trying to set the ML and the ϵ Equation

It is possible to try to recover the full model fenomenology with the simpler first order ones by setting the ML or the ϵ equation till the resulting equivalent mixing length profiles (see fig.4) are close to PT one. This can be done, in the first case, by defining the mixing length, below the inversion, as $\ell' = c_\ell (l_{up} l^3_{down})^{1/4}$, while, for the ϵ equation, by simply reducing its turbulent trasport (i.e. multiplying the Zeman coefficient A_0 by 0.55) (ϵ'). In fig.7 it is shown the results for the temperature flux profile. There is an improving, but the temperature profile (not shown here) is almost uneffected.

4 CONCLUSIONS

From the third order CBL model successive simplifications are applied till the model is formally equivalent to the E-ϵ and the E-ℓ models and their results are compared. The most critic simplification is the downgradient approximation of the third order moments that cause the underestimation of the magnitude and the of height of the inversion. All the models that make use of this approximation give similar results. The results are also very much sensitive to different TOMs parameterizations. There is no simple way to reproduce the third order model fenomenology with first order models by acting on the ML or the ϵ equation.

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