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## 1. INTRODUCTION

The central question in this study is whether simultaneous satellite observations (projection to 2D) and aircraft observations (1D) of a cumulus cloud field (essentially 3D) in principle give rise to a similar cloud size distribution, and if not, what causes this disparity.

Starting point for this study is the paper by Rodts et al. (2003) in which it was shown that aircraft measurements over Florida (the SCMS field experiment) were strongly biased towards small clouds in comparison to LANDSAT satellite observations of the region. For both aircraft and satellite measurements Rodts et al. (2003) calculated the cloud size that dominated the cloud cover and found the resulting sizes to differ by an order of magnitude. The problem, however, is that the definition of a "cloud" differs between the two measurement methods, impeding strong conclusions to be drawn from the results.

To mitigate these experimental difficulties, in this study we resort to large-eddy simulations (LES) of cumulus cloud fields. We simulate both the satellite observations (vertical projection of the cloud field) and the aircraft observations (large series of line measurements) and determine in both cases the observed cloud size distribution, as well as the cloud size that dominates the cloud cover. The simulations reproduce the large disparity between the satellite/aircraft cloud size distributions as found in the observations. Since the "real" cloud size is known in the simulations, we find that the aircraft cloud statistics is indeed significantly biased to smaller cloud sizes, whereas the satellite cloud statistics are biased to larger sizes. We show that these properties can be attributed to the particular (fractal) geometry of cumulus clouds.

## 2. LES CASE DESCRIPTION AND METHODS

The cumulus cloud case we study with LES is BOMEX (write out), which has served for an LES intercomparison study (Siebesma et al., 2003), and is therefore very well documented. We used a uniform grid of  $256^2 \times 160$  cells, with a resolution of  $\Delta x = \Delta y = 25\text{m}$ , and  $\Delta z = 20\text{m}$ , covering a domain size of  $(6.4\text{km})^2$  in the horizontal, and  $3.2\text{km}$  in the vertical. The case was simulated for 8 hours, but in order to give the turbulent boundary layer and cumulus field ample time to develop, only the last 5 hours were used for the analysis.

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The satellite observations were mimicked by determined the vertical projection of the three-dimensional cloud field, implying an exact nadir position of the satellite. To this end, we first calculated the Liquid Water Path (LWP) of a column at position  $(x, y)$ :

$$\text{LWP}(x, y) = \int_{z_{\text{base}}}^{z_{\text{top}}} q_l(x, y, z) dz$$

where  $q_l(x, y, z)$  represents the liquid water mixing ratio of a cell at  $(x, y, z)$ . For the satellite a pixel is defined as cloudy if  $\text{LWP}(x, y) > 0$  and clear if  $\text{LWP}(x, y) = 0$ . Next, the individual clouds were identified in the satellite image by locating clusters of contiguous cloudy pixels in the  $x$  and  $y$  direction. Of each cloud the area  $A$  was calculated (=number of contiguous cloudy pixels times  $\Delta x \Delta y$ ), from which the size of the cloud was derived by

$$l_2 = \sqrt{A}$$

The subscript 2 refers to the two-dimensional nature of the satellite measurement methodology.

The aircraft observations were simulated by performing a large amount of line measurements, i.e. by "flying" horizontally through the three-dimensional cloud field at every height, ranging from cloud base to cloud top, both in the  $x$ -direction for all 256  $y$ -pixels, and in the  $y$ -direction for all 256  $x$ -pixels. A pixel at location  $(x, y, z)$  is defined as cloudy if  $q_l(x, y, z) > 0$  and clear if  $q_l(x, y, z) = 0$ . For the aircraft a cloud consists of contiguous cloudy pixels in the direction of flight. The size of the cloud was defined simply as the length of the cloud  $l_1$  (= number of contiguous cloudy pixels times  $\Delta x$ , respectively  $\Delta y$ ). The subscript 1 refers to the one-dimensional nature of the aircraft measurement methodology.

Obviously the aircraft flies a great many times through what the satellite calls the same cloud, but the aircraft does not have that information and will report a new cloud each time when it goes from clear to cloudy air. Note that cloud size  $l_1$  measured by the aircraft can be larger or smaller than the size determined by the satellite,  $l_2$ , depending on the position where the cloud was intersected.

Both the aircraft and satellite observations were carried out each minute during the last 5 hours to collect sufficient statistics.

## 3. CLOUD COVER DENSITIES

Once the individual clouds have been identified, one can determine the cloud number densities  $n_1(l_1)$  and  $n_2(l_2)$  for the aircraft and satellite data, respectively, where  $n(l)$

represents the number of clouds with a size in the interval  $l$  and  $l + dl$ . The total number of observed clouds  $N_1, N_2$  are the integrals of the respective cloud number densities

$$N_1 = \int_0^{\infty} n_1(l_1) dl_1, \quad N_2 = \int_0^{\infty} n_2(l_2) dl_2 \quad (1)$$

An important property of the cumulus field under study is of course the cloud cover  $\sigma$ , defined here as the ratio of cloudy points to the total number of points. The cloud cover  $\sigma$  can be decomposed analogous to the cloud number  $N$  in (1)

$$\sigma_1 = \int_0^{\infty} a_1(l_1) dl_1, \quad \sigma_2 = \int_0^{\infty} a_2(l_2) dl_2 \quad (2)$$

where  $a_1, a_2$  denotes the cloud cover density belonging to the aircraft and satellite data, respectively. For the aircraft the cloud cover density  $a_1(l_1)$  is related to the number density  $n_1(l_1)$  by

$$a_1(l_1) = \frac{1}{L} n_1(l_1) l_1, \quad (3)$$

with  $L$  the total flight length. For the satellite a comparable relation holds

$$a_2(l_2) = \frac{1}{S} n_2(l_2) l_2^2, \quad (4)$$

with  $S$  the total area covered by the image.

The cloud cover densities  $a_1, a_2$  provide information on the effect of cloud size on the cloud cover and combine the competing effects of cloud number and cloud size – small clouds are numerous but have little impact on the cloud cover because they are small, bigger clouds have a larger impact per cloud but occur less frequent.

#### 4. RESULTS

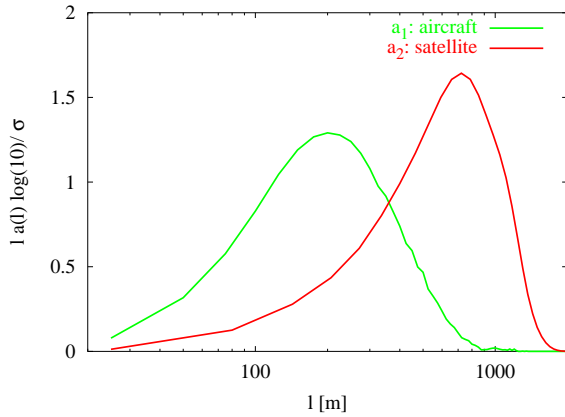


FIG. 1: satellite and aircraft cover densities

In Fig. 1 we have plotted the cloud cover densities  $a_1$  and  $a_2$ , determined according to equations (1-4). We have plotted the cloud size  $l = l_1, l_2$  on a log-scale and applied the following normalization

$$\int a(l) dl = \sigma \quad \rightarrow \quad \int \frac{l a(l) \log(10)}{\sigma} d \log_{10}(l) = 1,$$

to compensate for the logarithmic axis and to ensure that the area under both curves be equal to unity, which facilitates comparison. Fig. 1 shows the large disparity between the satellite and aircraft cloud cover densities. The peak of the densities are from a physics point of view most interesting since it reveals the cloud size that dominates the cloud cover (e.g. Neggers et al., 2003; Rodts et al., 2003). The peak represents an interesting balance between cloud size and cloud number. As argued above: Small clouds are most numerous but have a small effect on the cloud cover, large clouds have a big impact per cloud but there are only a few of them. Somewhere in between there is an optimum cloud size  $l_{\text{peak}}$ . Yet the aircraft and satellite measurements provide an entirely different estimate for  $l_{\text{peak}}$  – they differ by nearly one order of magnitude – even though they are probing the same cloud field. The large-eddy simulations thus clearly reproduce the significant disparity between the aircraft and satellite measurements as observed by Rodts et al. (2003).

While it is yet unclear what exactly causes the disparity, the results point at a generic – probably geometric – property of cumulus clouds, since the meteorological situation during BOMEX is rather different from the one during the SCMS case studied by Rodts et al. (2003).

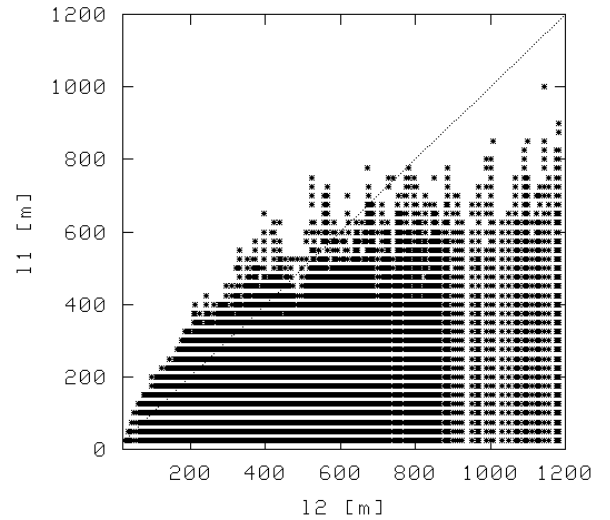


FIG. 2: Scatterplot of the aircraft measurements of cloud size  $l_1$  versus the corresponding satellite observation  $l_2$  of the same cloud. Mostly  $l_1 < l_2$ , though occasionally  $l_1 > l_2$ . The dotted line indicates  $l_1 = l_2$  for reference.

An interesting advantage of the LES data is that one can make a direct connection between the aircraft and satellite measurements, since it is possible to study for each individual cloud the relation between the size  $l_2$  as observed by the satellite and the size  $l_1$  as observed by the aircraft. Since the aircraft intersects the same cloud many times there will be many observations  $l_1$  corresponding to a single observations  $l_2$ . This is shown in

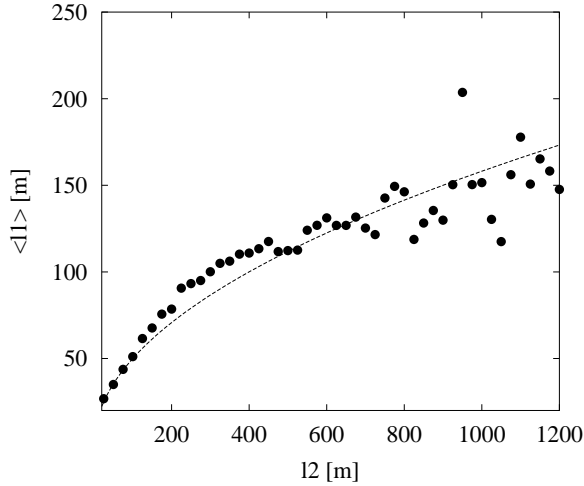


FIG. 3: Average cloud size estimate by the aircraft ( $\langle l_1 \rangle$ ) of clouds that have cloud size  $l_2$  according to the satellite measurement. The solid line is a fit to  $\sqrt{l_2}$ .

the scatter-plot Fig. 2. In the plot one observes that occasionally  $l_1$  is larger than  $l_2$  (points above the diagonal), but in the majority of the cases one has  $l_1 < l_2$ .

In Fig. 3 we show the average cloud size  $\langle l_1 \rangle$  estimated from the aircraft data for each cloud with a cloud size  $l_2$ , as determined from the satellite image. It reveals again the strong disparity between satellite and aircraft measurements. Interestingly, the factor  $l_2/\langle l_1 \rangle$  increases with cloud size, implying that the effect becomes much stronger for larger clouds. The data seem to follow a power-law  $\langle l_1 \rangle \sim l_2^D$ , with  $D \approx 0.5$ . The origin of this scaling law is not directly obvious. Below we will elaborate a, yet speculative, theory which could explain the exponent  $D = 1/2$ . To this end we must combine three elements.

1) A large-eddy simulation study by Siebesma and Jonker (2000) of BOMEX revealed that the surface area  $S$  of cumulus clouds follows the power-law

$$S(\lambda) \sim \lambda^{1+d} \quad (5)$$

where  $d = 4/3$  and where the linear cloud size  $\lambda$  was defined as the cubic root of the cloud volume  $V$ . This finding was shown to be in excellent agreement with the area-perimeter results of Lovejoy (1982) based on satellite images of real clouds (also  $d = 4/3$ ).

2) An interesting theorem from acoustic theory by Kosten (1961) states that if one randomly transects a volume, the average transection length  $l_1$  will amount to  $4V/S$ , where  $V$  is the body volume, and  $S$  its surface area. If we combine this theorem with the scaling law for clouds (5), we find

$$\langle l_1 \rangle \sim \lambda^{3-(1+d)} = \lambda^{2/3} \quad (6)$$

3) Finally we must find a relation between the projection based cloud-size  $l_2$ , as the satellite observes it,

and the “true” cloud size  $\lambda$ . The projected area of a cloud will increase for taller clouds, since there is more room for “excursions”. We therefore conjecture that  $l_2 \sim h^d$  on the one hand, and  $h \sim \lambda$  on the other hand. The latter scaling simply means that bigger clouds will also be taller clouds. So

$$l_2 \sim \lambda^d \quad (7)$$

If we combine (6) with (7) we obtain

$$\langle l_1 \rangle \sim l_2^{\frac{2-d}{d}}$$

So for  $d = 4/3$  one obtains

$$\langle l_1 \rangle \sim \sqrt{l_2} \quad (8)$$

which concludes the argument.

It is clear that several steps in the argument above need individual verification. The LES data form an excellent basis for these tests. This work is in progress. In any event, (8) shows how the specific geometric properties of cumulus clouds (surface-volume, area-perimeter relations) have an impact on the average intersection length. It shows how the dimensionality of the measurement method (1D, 2D or 3D) can influence the measured results.

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