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## 1. Introduction

Climate models, numerical weather prediction (NWP) models, and atmospheric dispersion models often rely on parameterizations of planetary boundary layer height. In the case of a stable boundary layer, errors in boundary layer height estimation can result in gross errors in boundary-layer evolution and in prediction of turbulent mixing within the boundary layer.

Ideally, observations would be used to determine the dependence of boundary layer height on parameters governing the flow in a boundary layer. However, observations of stably-stratified atmospheric boundary layers (ABLs) under idealized conditions which provide sufficient information about ABLs evolution under a wide range of conditions are either unattainable or rare. We therefore use controlled numerical experiments by performing numerical simulations of homogeneous, quasi-steady, stably-stratified ABLs which commonly occur in wintertime over sea-ice in polar regions. A similar approach was previously used by Kosović and Curry (2000).

Correct parameterizations or approximations of the height  $h$  of the stable boundary layer (SBL) is often critical for accurate modeling of pollution dispersion and for weather and climate modeling. Starting with Zilitinkevich (1972), a number of different parameterizations have been proposed and evaluated; they are summarized in Zilitinkevich et al. (2002). Most parameterizations rely on quantities which can be evaluated at the surface, like the friction velocity  $u_*$ , the Coriolis frequency  $f$ , and the Monin-Obukhov length  $L$ . Pollard et al. (1973) were the first to suggest that  $N$ , the Brunt-Väisälä frequency of the free flow above the stable boundary layer would affect the height of the boundary layer. This suggestion was first followed by Kitaigorodskii and Joffre (1988) and more recently by Zilitinkevich et al. (2002).

We use large-eddy simulations (LES) of moderately stable boundary layers to characterize the effects of various physical processes on stable boundary layer (SBL) height. The SBL height is assumed to be a function of surface friction velocity, geostrophic wind, Monin-Obukhov length, and the strength of the temperature inversion atop the stable boundary layer. The strength of temperature inversion determines the frequency of gravity waves that are induced when turbulent boundary layer disturbs the overlying inversion.

Using LES, we show that gravity waves atop a stable boundary layer do affect the height of the stable boundary layer, and the domain size of an LES of a SBL must be

sufficient to resolve those gravity waves and their effects on an evolving stable boundary layer.

Due to the buoyant destruction of turbulent kinetic energy, the characteristic length-scale of turbulent eddies in SBLs is smaller than in convective or neutral ABLs. Previous attempts to use LES to resolve SBL flows have therefore focused on smaller computational domains, resulting in smaller grid-cell sizes for the same number of grid points. Few large-eddy simulations have been able to both resolve the turbulence within the SBL and to incorporate a domain large enough to include the effect of gravity waves in the free-atmosphere above the boundary layer Saiki et al. (2000). In our LES study, we have used domain sizes which are by a factor of four or more larger than those used in previous LES studies of SBLs, and thus we are able to explore various parameterizations of  $h$ , including those that account for the effects of stratification in the free atmosphere above the ABL.

## 2. Large Eddy Simulations

The large-eddy simulations were carried out with the CU LES of Kosović and Curry (2000). The traditional resolved momentum conservation equations are solved, *cf.* Moeng (1984), but with a subgrid-scale model that accounts for the backscatter of energy and isotropy due to shear Kosović (1997). The algorithm is coded in Fortran 77 and parallelized using Message Passing Interface (MPI). The code was executed on a Compaq cluster at Lawrence Livermore National Laboratory.

We performed a series of medium-resolution runs using  $160^3$  grid points. We carried out one high-resolution,  $96^3$  LES over a small-domain where the sides of domain box were 600m in the stream-wise direction 500m, in the cross-stream direction, and 400m in the vertical direction. The small domain simulation is similar to those previously reported by Andren (1995). The initial conditions, surface cooling rate, and the inversion strength for these simulations were based on the Beaufort Sea Arctic Stratus Experiment (BASE) case of 1 October 1994 from flight 7, *cf.* Kosović and Curry (2000). This case is also used for the GEWEX Atmospheric Boundary Layer Study (GABLS) - large-eddy simulation intercomparison project, results of which are reported in this volume (preprint 4.1).

In the baseline, high-resolution simulation, named wavesvhr, the latitude was 73 deg N, the geostrophic wind was set to  $8 \text{ m s}^{-1}$ , the surface cooling rate was  $0.25 \text{ K h}^{-1}$ , the overlying inversion strength was  $0.01 \text{ K m}^{-1}$ , and the surface roughness was 0.1m. For the ensuing simulations, we varied the geostrophic wind ( $5 \text{ m s}^{-1}$  (waves05gw) and  $11 \text{ m s}^{-1}$  (waves11gw)) and the latitude (22 degrees N (waves22la) and 45 degrees

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N (waves451a) ). These runs are summarized in Table 1. In previous small-domain runs (600 m × 500 m × 400 m), which are summarized in Kosović and Curry (2000), other parameters such as cooling rate, inversion strength, and surface roughness were varied, but the strength of the geostrophic wind appeared to influence  $h$  more than these other parameters.

### 3. Stable Boundary-Layer Height

In the past, various working definitions for the height  $h$  of the SBL have been proposed (Wyngaard, 1975; Caughey et al., 1979; Derbyshire, 1990). While Andreas et al. (2000) suggested that the height of the jet core (or the height of the lowest maximum in wind speed) should be used as  $h$ , LES carried out by Kosović and Curry (2000) indicated that under ideal conditions three definitions of boundary layer height,  $h$ , are equivalent: the height where the (surface-originated) turbulent stress vanishes, the height of the base of the capping temperature inversion, and the height of the inversion wind maximum (Banta et al., 2002) (sometimes called low-level jet). Here, we adopt a definition based on the height at which surface-based turbulent stresses vanishes, which can be calculated as the height at which  $(u'w'^2 + v'w'^2)^{1/2}$  falls to five percent of the surface value  $u_*$ , divided by 0.95 (e.g., Wyngaard (1975); Brost and Wyngaard (1978); Zilitinkevich and Mironov (1996)). Note that this definition does not hold in the event that surface-based turbulence does not dominate the boundary layer as in the case of a so-called "top-down" boundary layer (Mahrt, 1999).

A definition based on the level at which heat flux disappears has been proposed (Caughey et al., 1979; Derbyshire, 1990). However, heat flux near the top of SBL is often dominated by the effect of breaking internal waves and therefore vanishing heat flux does not clearly delineate the boundary of the turbulent layer.

Equation (30) in Zilitinkevich and Mironov (1996) gives a relationship between different mechanisms governing the height of an SBL as

$$1 = \left( \frac{fh}{C_n u_*} \right)^2 + \frac{h}{C_s L} + \frac{NH}{C_i u_*} + \frac{h|f|^{1/2}}{C_{sr}(u_* L)^{1/2}} + \frac{h|Nf|^{1/2}}{C_{ir} u_*}, \quad (1)$$

where  $C_n$ ,  $C_s$ ,  $C_i$ ,  $C_{sr}$ , and  $C_{ir}$  are empirical constants. In Kosović and Curry (2000), a simplification of Zilitinkevich and Mironov (1996) was proposed, so that

$$\left( \frac{fh}{C_n u_*} \right)^2 + \frac{h}{C_s L} + \frac{h|f|^{1/2}}{C_{sr}(u_* L)^{1/2}} = 1, \quad (2)$$

where the best-fit empirical constants  $C_n$ ,  $C_s$ , and  $C_{sr}$ , based on the simulations presented in Kosović and Curry (2000), were found to be  $C_n^2 = 0.002$ ,  $C_s = 2/3$ , and  $C_{sr} = 2/27$ . (Note that equation (2) does not appear correctly in the text of Kosović and Curry (2000); it should consist of the terms of equation (30) in Zilitinkevich and Mironov (1996) which do not involve  $N$ .)

As noted above, Pollard et al. (1973) suggested that the height of the stable boundary layer should be a function of the strength of the inversion atop a stable boundary layer. Most recently, Zilitinkevich et al. (2002) proposed that the equilibrium height  $h_E$  of a SBL can be expressed as

$$h_E = C_R \frac{u_*}{f} \left\{ 1 + \frac{C_R^2 u_* (1 + C_{UN} Fi)}{C_S^2 f L} \right\}^{-1/2}, \quad (3)$$

where  $Fi = LN/u_*$  and the constants  $C_R$ ,  $C_{UN}$ , and  $C_S$  are tentatively defined by Zilitinkevich et al. (2002) to be, respectively,  $C_R = 0.4$ ,  $C_{UN} = 0.25$ , and  $C_S = 0.74$ , based on three field studies. Note that this equation is intended to describe the height of a stable boundary that has reached equilibrium, and is not expected to describe a transitional SBL.

Equation (3) can more appropriately be expressed in terms of non-dimensional parameters such as Monin-Kazanski parameter

$$\mu = \frac{u_*}{fL}, \quad (4)$$

a stability parameter  $h/L$ , and the dimensionless imposed stability parameter  $N/|f|$  (Zilitinkevich and Esau, 2003). This simplifies equation (3) to

$$1 = A\mu^2 \left( \frac{L}{h} \right)^2 + B\mu + C \frac{N}{f}. \quad (5)$$

Below, we will use the array of LES to determine appropriate values of the empirical constants  $A$ ,  $B$ , and  $C$  and compare them to the experimental values found by Zilitinkevich et al. (2002).

These five methods (height of the maximum wind speed, height where turbulent stresses disappear, height where heat flux disappears, the expressions from Kosović and Curry (2000), and the expression from Zilitinkevich et al. (2002)) for identifying the height of the stable boundary layer are compared for the model simulations described in previous section. Note that only the last two prognostic equations would have utility for ABL parameterizations in numerical models where surface conditions and perhaps the strength of the overlying inversion are known, while other quantities must be predicted.

In Figure 1 the non-dimensional SBL heights from a high-resolution, small-domain simulation are shown. In this case, the domain was too small to include the effects of gravity waves that develop above the SBL the boundary layer. The SBL height parameterization proposed by Kosović and Curry (2000) appears to match the height defined by turbulent shear stress better than that of Zilitinkevich et al. However, results obtained with the domain size sufficiently large to resolve gravity waves presented in Figure 2 show that the parameterization proposed by Zilitinkevich et al. (2002) gives a better prediction of the SBL boundary layer height.

Table 1: Large-Eddy Simulations

Name	Resolution	Dimensions (x,y,z) [m]	x,y grid size [m]	z grid size [m]	$U_g$ [ $\text{m s}^{-1}$ ]	Latitude [deg] N
jethr	$96^3$	(600,500,400)	6.25	5.2	4.2	73
wavesvhr	$160^3$	(2400,2400,1200)	15.0	7.5	8.0	73
waves11gw	$160^3$	(2400,2400,1200)	15.0	7.5	11.0	73
waves05gw	$160^3$	(2400,2400,1200)	15.0	7.5	5.0	73
waves45la	$160^3$	(2400,2400,1200)	15.0	7.5	8.0	45
waves22la	$160^3$	(2400,2400,1200)	15.0	7.5	8.0	22

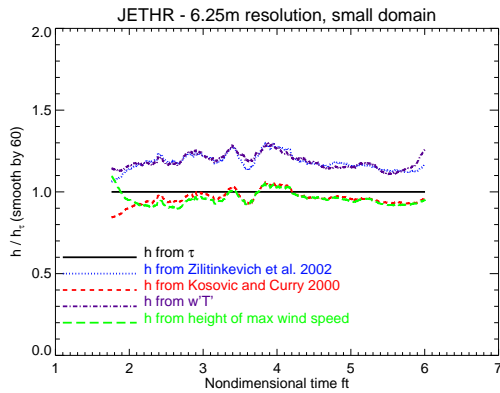


Figure 1: Comparison of various methods for calculating  $h$ , compared with  $h_\tau$  for the jethr simulation. Note that the horizontal domain for this run is too small to resolve gravity waves above the boundary layer, and thus the Zilitinkevich et al. (2002) parameterization is expected to perform poorly.

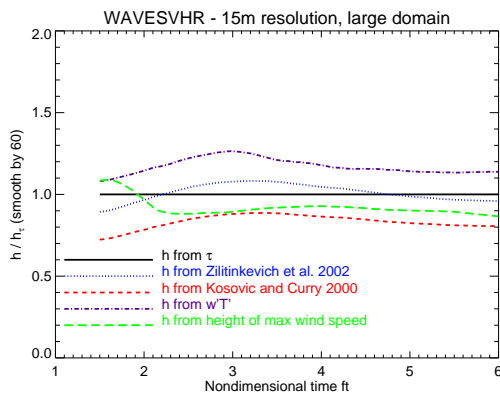


Figure 2: As in Figure 1 but for simulation wavesvhr, with a domain sufficiently large to resolve gravity waves above the stable boundary layer. The Zilitinkevich et al. (2002) method approaches the correct  $h$  after some relaxation time.

#### 4. Summary

Recent work has shown that the height of the stable boundary layer (SBL) is influenced by the presence of gravity waves in the free atmosphere above the boundary layer (Zilitinkevich et al., 2002). We demonstrate these effects using large-eddy simulations of moderately stable boundary layers that persist over long periods of time such as those observed over the ice in the Arctic. Only the parameterization that incorporates such influence calculates stable boundary layer heights that agree with the height as calculated from turbulent stresses.

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