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AN EXAMINATION OF SEVERAL PARAMETRIZATIONS OF MIXING LENGTHS IN A STABLE BOUNDARY LAYER: THE GABLS CASE

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1. INTRODUCTION

Turbulent kinetic energy (TKE) schemes depend heavily on the parametrization of turbulence length scales to describe eddy diffusion coefficients for momentum and scalar mixing. This is especially true for shallow, stably stratified boundary layers. Recently, GABLS (GEWEX Atmospheric Boundary Layer Study) focused on the representation of the stable atmospheric boundary layer with an intercomparison of single column models (SCM) and large-eddy simulation (LES) models for a shear-driven, clear-sky stable case over the Arctic (Cuxart *et al.* 2004; Beare *et al.* 2004).

The study focuses on the examination of several popular parametrizations of turbulent length scales to simulate the GABLS case in a SCM. These formulations include: 1) a local Richardson number scheme (hereafter LR) used in the Canadian operational model, 2) the Bougeault-Lacarrere (1989) scheme (hereafter BL), 3) a length scale proposed by Therry and Lacarrere (1983; hereafter TL), and 4) the original and a modified version of the Lenderink and Holtslag (2004) formulation (hereafter LHO and LHM, respectively). These length scales are tested against a LES of the GABLS case.

2. MIXING LENGTH SCALES

TKE turbulence closures require parametrizations of three conceptually different length scales. Two are the mixing lengths used in the parametrization of the diffusion coefficients for momentum and scalar fields (heat, moisture):

$$K_\chi = c_\chi l_\chi e^{1/2} = l'_\chi e^{1/2} \tag{1}$$

where c_χ are constants, e is the TKE and l_χ are the mixing lengths. The third, l_ϵ, is the length scale that appears in the parametrization of the dissipation term in the TKE evolution equation,

$$\epsilon = c_\epsilon \frac{e^{3/2}}{l_\epsilon} = \frac{e^{3/2}}{l'_\epsilon} \tag{2}$$

where c_ϵ is another constant. The values of the constants c_m, c_h and c_ϵ may vary considerably between formulations. Therefore, in order to be able to compare different schemes, equivalent or "normalized" length scales l'_χ and l'_ϵ are defined in (1) and (2) that include those constants.

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2.1 Local Ri-dependent scheme (LR)

From Belair *et al.* (1999), RPN has a parametrization directly dependent on the local gradient Richardson number, Ri, namely:

$$l_m = \frac{\lambda_n}{\phi_m} \tag{3}$$

where the neutral mixing length, λ_n, is given by

$$\lambda_n = \min[kz,\, \lambda_0] \tag{4}$$

$\lambda_0 = 200$m and the stability function, ϕ_m, in stable conditions ($Ri > 0$) is given by

$$\phi_m = (1 + 12\, Ri) \tag{5}$$

The values of the constants are $c_m = c_h = 0.516$ and $c_\epsilon = 0.14$.

2.2 Bougeault and Lacarrere scheme (BL)

In unstable boundary layers significant transport by eddies on the scale of the boundary layer depth mean that the mixing properties depend not so much on local stability (via Ri, for example) but on bulk properties of the layer. Thus, the method of BL calculates the potential displacements of parcels released upwards, l_{up}, and downwards, l_{down}, from a given level:

$$\int_z^{z+l_{up}} \beta(\theta_v(z') - \theta_v(z))\, dz' = e(z)$$
$$\int_{z-l_{down}}^{z} \beta(\theta_v(z) - \theta_v(z'))\, dz' = e(z) \tag{6}$$

where β is the buoyancy parameter and θ_v is the virtual potential temperature.

In the current RPN formulation, l_{up} and l_{down} are then combined as follows to give

$$l_m = \min[l_{up},\, l_{down}] \tag{7}$$

In addition, l_ϵ is made proportional to l_m but dependent also on the stability, as described in section 3. Finally, the values of the constants c_m, c_h and c_ϵ are the same as in LR.

2.3 Therry and Lacarrere scheme (TL)

The length scale proposed by TL takes into account the various local stability conditions. For locally stable stratification it is given by

$$\frac{1}{l_m} = \frac{1}{kz} + \frac{d_1}{h} + \frac{d_5}{l_s} \tag{8}$$

where h is the height of the boundary layer

$$l_s = \frac{e^{1/2}}{N} \tag{9}$$

and $N^2 = \beta d\theta_v/dz$ is the Brunt-Vaisala frequency, $d_1 = 15$ and $d_5 = 1/0.36$ $(= 2.78)$ as in Duynkerke and Driedonks (1987); note that this value of d_5 is quite similar to the original value used by Therry and Lacarrere $(= 3)$. Here again, the values of the constants c_m, c_h and c_ϵ are taken as in LR.

2.4 Original Lenderink and Holtslag scheme (LHO)

This method is motivated by that of Bougeault and Lacarrere but attempts also to include the observed dependence of the mixing length in unstable boundary layers on the degree of instability present. Note that for a given boundary layer depth, the mixing lengths of Bougeault and Lacarrere will be practically identical from near-neutral to free-convective. Instead, LHO calculate upwards and downwards integrals of a function of Ri:

$$\begin{aligned} l_{up} &= \int_{z_{bot}}^{z} F(Ri)\, dz' \\ l_{down} &= \int_{z}^{z_{top}} F(Ri)\, dz' \end{aligned} \tag{10}$$

which are combined into a single 'integral' length scale as

$$\frac{1}{l_{int}} = \frac{1}{l_{up}} + \frac{1}{l_{down}} \tag{11}$$

In order to remove an unwanted influence of l_{down} in stable boundary layers, LHO introduced an additional pragmatic condition on l_{down}, namely that $l_{down} > 75e^{-z/500}$. In addition, LHO define a mixing length appropriate for stable conditions, l_s, and a minimum length scale, l_{min}, as:

$$l_s = c_s^{m,h}\frac{e^{1/2}}{N}, \qquad \frac{1}{l_{min}} = \frac{1}{akz/2} + \frac{1}{l_\infty} \tag{12}$$

where $l_\infty =$ 75m, c_s^h is a constant (=0.2) and $c_s^m = c_s^h(1+2Ri)$.

These length scales are finally combined using

$$\frac{1}{(l_{m,h})^2} = \frac{1}{l_{int}^2 + l_{min}^2} + \frac{1}{l_s^2} \tag{13}$$

and $l_\epsilon = l_m$. LHO include all empirical constants in the mixing lengths themselves. They also use a different value for c_ϵ $(= 0.071)$.

2.5 Modifications to Lenderink and Holtslag scheme (LHM)

Some revisions to the original formulation of Lenderink and Holtslag have been found to give improved agreement with LES results of an idealized equilibrium stable boundary layer with a specified surface heat flux of -15 Wm^{-2}. The revised stable coefficients in LHM for each length scale are

$$\begin{aligned} c_s^\epsilon &= 0.1(1+5Ri) \\ c_s^m &= 0.25c_s^m|_{LHO} = 0.05(1+2Ri) \\ c_s^h &= 0.36c_s^h|_{LHO} = 0.072 \end{aligned} \tag{14}$$

where the subscript LHO denotes the original LH formulation in (12) and c_s^ϵ is the coefficient to be used for l_ϵ. These revised coefficients give

$$\begin{aligned} Pr &= 0.7(1+2Ri) \\ \frac{l_\epsilon}{l_m} &= 2\frac{1+5Ri}{1+2Ri} \end{aligned} \tag{15}$$

where $Pr = K_m/K_h$ is the Prandtl number.

3. OTHER LENGTH SCALES

Within the framework used here (i.e., with the constants used in (1) and (2)), the LR formulation of RPN assumes the length scales for momentum mixing and dissipation are equal. In the parametrization of LHO (and LHM), they also set $l_\epsilon = l_m$. Even with the different proportionality constants this is still approximately true since the factor of a half in c_ϵ is matched by the absence of c_m (=0.516) in their definition of K_m. For the BL and TL schemes, the dissipation length is made proportional to l_m and a function of the flux Richardson number, Rf $(= Ri/Pr)$:

$$\frac{l_\epsilon}{l_m} = \frac{1-Rf}{1-2Rf} \tag{16}$$

Thus, the two are equal at neutrality and for stable conditions, $l_\epsilon > l_m$, with l_ϵ/l_m increasing to 3 at $Rf = 0.4$, where it is capped.

Regarding the Prandtl number, the various constants in LHO imply that $Pr > 1$ in stable conditions (via l_s) while for LHM $Pr > 1$ for $Ri > 0.22$. The Prandtl number in LR and BL formulations under stable conditions is given by $Pr = 0.85$, implying that $l_h > l_m$. The TL formulation specifies $Pr = 0.74$ in stable conditions.

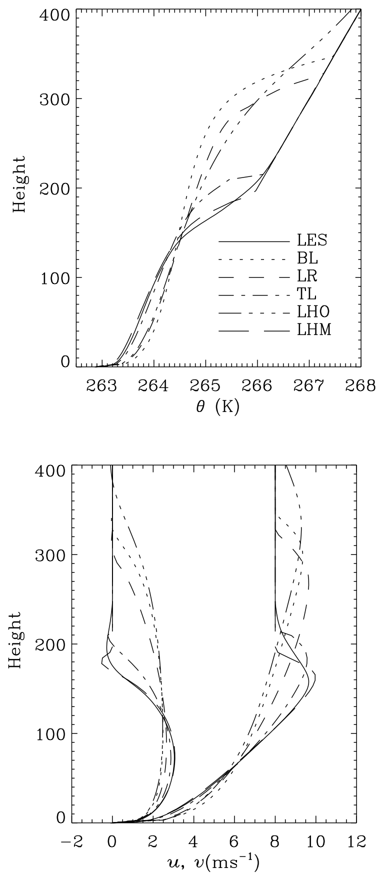

Figure 1: Profiles of potential temperature θ (K) (upper panel) and wind components (lower panel) derived from the stable LES (solid line) and as parametrized by the different schemes.

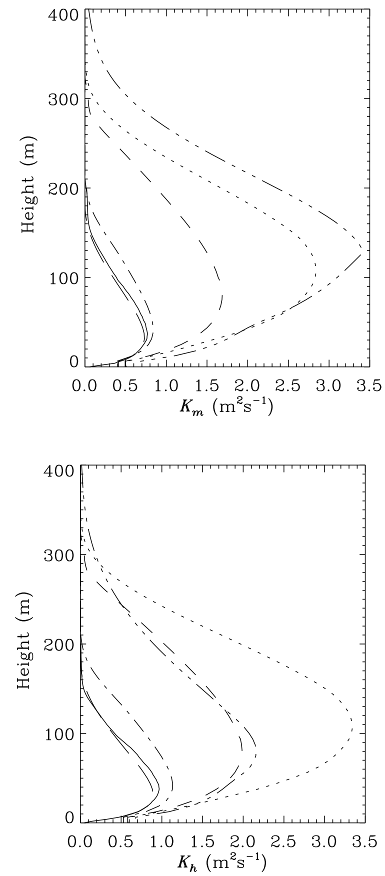

Figure 2: Turbulent diffusion coefficients K_m for momentum (upper panel) and K_h for heat (lower panel) derived from the stable LES (solid line) and as parametrized by the different schemes. Other lines are as in Fig. 1.

4. RESULTS

The various parametrizations of the turbulence length scales have been used to simulate the GABLS case with a SCM. The features of the case are described in Cuxart *et al.* (2004) together with the methodology used for the simulation. The results obtained with several LES models for the GABLS case are discussed by Beare *et al.* (2004). LES results are used here to provide detailed datasets of turbulence statistics from which diagnoses of the above length scales can be obtained to test parametrizations. In particular, the LES results from the Met Office model at 2m resolution are used as a reference to assess the parametrizations.

Figure 1 shows that under the influence of surface cooling, the LES stable boundary layer reaches a quasi-steady state characterized by a stable layer reaching about 150 m capped by a stronger temperature inversion extending to about 200 m. The LR and BL parametrizations clearly result in a too deep stable layer reaching more than 300 m, being too warm at lower levels and too cold aloft. The LHO formulation gives an even deeper extension of the boundary layer top due to too much mixing at upper levels, as will be seen below. In contrast, the TL and LHM schemes give reasonable agreement with the LES structure. It should be noted that the small differences between TL and LHM are within the variance observed between the various LES models themselves. Similar conclusions apply to the wind structure with too much momentum mixing for the LR, BL and LHO schemes at upper levels.

Figure 2 indicates that for the LES, the diffusion coefficients K_m and K_h have maximum values of about $0.8m^2s^{-1}$ and $1.0m^2s^{-1}$, respectively, at a height of 40m. Considering the spread in the LES results, the TL and LHM schemes both seem quite realistic while the rest of the schemes give much larger values, in particular the LHO scheme for K_m and the BL scheme for K_h. This enhanced mixing in the upper inversion explains most of the overestimed development of the stable boundary layer in Fig. 1. From the LES, Pr varies between 0.8 in the lower part of the boundary layer (and

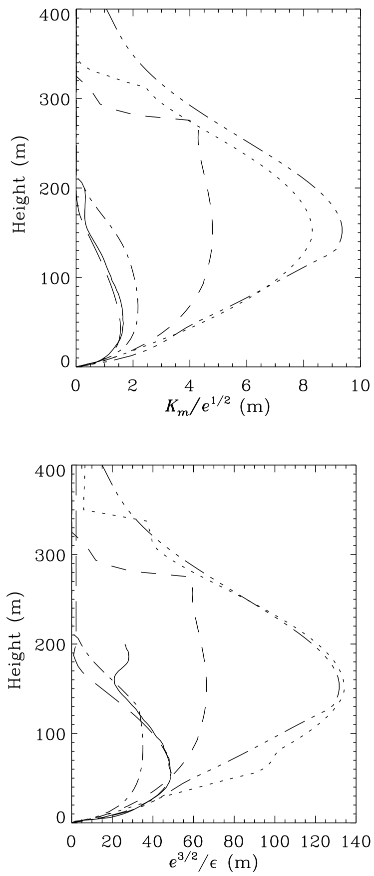

Figure 3: Normalized mixing lengths l'_m for momentum (upper panel) and dissipation lengths l'_ϵ (lower panel) derived from the stable LES (solid line) and as parametrized by the different schemes. Other lines are as in Fig. 1.

so agreeing with RPN formulation that $Pr = 0.85$) but increases to about 2 at the top (at $z_\mathrm{h} \sim 150\mathrm{m}$). At the same time, Ri increases roughly linearly from near zero but positive at the surface to around 0.4 at z_h, where both LHO and LHM give better agreement ($Pr = 1 + 2Ri$).

In Fig. 3, the LES indicates that the mixing length l'_m increases linearly near the surface, reaches a maximum value of only 1.5m and decreases to tiny values above the inversion. The TL and LHO schemes are close to the LES, while the other schemes give much larger values, reaching almost 10m for the LHO parametrization. Figure 3 also shows the dissipation length l'_ϵ from the LES and the various parametrizations. The LES maximum value reaches about 50m, a value much larger than the corresponding l'_m value. This is well captured by the LHM scheme and slightly underestimated by TL. The LR parametrization is found to be too large and to extend far too high. This situation is even worse with the BL and LHO schemes.

5. CONCLUSIONS

Length scales appropriate for use in TKE closures have been examined and compared with LES diagnoses for the GABLS case. In general, neither of the two parametrizations in the current RPN physics package perform very well: both the Ri and the Bougeault-Lacarrere scheme significantly overestimate the length scales in the stable boundary layer. This results in a too deep stable boundary layer. The original scheme of Lenderink and Holtslag also yields much too large values for the mixing length scales. In contrast, the scheme of TL and a modified version of the Lenderink and Holtslag scheme are found to perform remarkably well when compared to the reference LES results.

It should be pointed out that these are highly idealized equilibrium simulations and experience at the Met Office and elsewhere is that good agreement in these idealized conditions does not necessarily imply good forecast skill. Examination of the impact of those formulations on mesoscale forecasts with the Canadian GEM model are underway and will be presented at the Symposium.

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