4.15 MODELLING THE 1D STABLE BOUNDARY LAYER WITH AN $E - \ell$ TURBULENCE CLOSURE

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1. INTRODUCTION

In their studies of atmospheric boundary layer evolution, Weng and Taylor (2003) have evaluated several commonly used $1\frac{1}{2}$ order turbulence closure schemes. It has been shown that the simple $E - \ell$ turbulence closure, which uses a prognostic turbulent kinetic energy (TKE) equation together with diagnostic equations for turbulent length scales, performs well in most atmospheric conditions compared with the schemes which include prognostic equations for both the TKE and the turbulent length scales. To study the nocturnal stable boundary layer, this model has been run for 9 hours with specified initial wind, potential temperature and TKE profiles; and a specified cooling rate applied at the surface. Different runs are conducted for different cooling rates, geostrophic winds and surface roughness. The results are discussed and compared with other models, large eddy simulations and field data.

2. THE MODEL

The model is a 1-D horizontally homogeneous dry boundary layer. Both radiative flux divergence and moisture are neglected for simplicity. The turbulent fluxes are modelled through eddy diffusivity. Under stable condition, the diagnostic equations for the mixing and dissipation length scales (ℓ_m and ℓ_d) are

$$\frac{1}{\ell_m} = \frac{1}{\kappa(z+z_0)} + \frac{1}{\ell_0} + \frac{\beta}{\kappa L_O}, \quad (1)$$

$$\frac{1}{\ell_d} = \frac{1}{\kappa(z+z_0)} + \frac{1}{\ell_0} + \frac{\beta - 1}{\kappa L_O}, \quad (2)$$

where ℓ_0 is taken to be 100 m and $\beta = 4.8$. All the other symbols have their usual meanings. The model uses a stretched vertical coordinate (a log-linear transform) to ensure sufficient resolution near the surface and to resolve strong gradients. Equations are transformed into the new coordinate system before they are discretized into their finite difference equivalents. Flow variables are stored on a staggered grid, where mean variables (U, V and T) are at layer midpoints and turbulent quantities (E and turbulent fluxes) at the lower boundary level ($z = 0$) and z_t (the top of the computation domain which is set to 4000 m). To obtain a smooth solution, especially at the top part of the boundary layer, a total of 301 grid points are used. The numerical scheme employed for time integration is Crank-Nicolson. The resulting set of difference equations are solved using a block **LU** factorization algorithm. The surface boundary conditions used are a non-slip condition for velocity ($U = V = 0$), a specified cooling rate applied for potential temperature, and the assumption that production balances the dissipation of TKE ($P = \epsilon$). At the upper boundary, we specify $(U, V) = (U_g, V_g)$, $\Theta = \Theta_g$ (constant) and set the vertical derivatives of TKE, ϵ, shear stresses and other turbulent fluxes to zero.

3. RESULTS AND DISCUSSIONS

A total of forty-two simulations were conducted. The common parameters used in all the runs are $f = 1.39 \times 10^{-4}$ s^{-1} and $\Theta_0 = 263.5$ K. The initial conditions are $(U, V) = (U_g, V_g)$ for $z > 0$ m; $\Theta = 265$ K for $0 \le z \le 100$ m and increasing at 0.01 K m^{-1} to the domain top; $E = 0.4 \times (1 - z/250)^3$ m^{2}s^{-2} for $0 \le z \le 250$ m and a minimum value of 10^{-9} m^{2}s^{-2} for $z > 250$ m; and $\langle uw \rangle = 0.3E$. All the other parameters and a summary of all the runs can be found in Table 1. Note that Run A2 is the single-column GABLS model inter-comparison case (Cuxart et al., 2004).

Figure 1 shows the model predicted vertical profiles of wind speed ($\sqrt{U^2 + V^2}$), potential temperature (Θ), shear stress (τ) and kinematic heat flux ($\langle w\theta \rangle$) at different times after the transition from Run A2. The development of the nocturnal boundary layer appears reasonable — the supergeostrophic wind or nocturnal jet is apparent at low levels and the convex shape of the Θ profile is consistent with expectations for a turbulence-driven SBL.

Due to the net loss of heat to the ground and no compensating heat flux at the top, the boundary layer as whole must cool. A quasi-steady state or station-

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Table 1: Some parameters used in the simulations of SBLs. CR denoting the cooling rate.

Runs		CR $\left(\mathrm{K\ h^{-1}}\right)$	(U_g, V_g) $\left(\mathrm{m\ s^{-1}}\right)$	z_0 (m)
	1	0.125	$(8, 0)$	0.1
	2	0.25	$(8, 0)$	0.1
	3	0.5	$(8, 0)$	0.1
A	4	1.0	$(8, 0)$	0.1
	5	1.5	$(8, 0)$	0.1
	6	2.0	$(8, 0)$	0.1
	7	2.5	$(8, 0)$	0.1
	1	0.125	$(8, 0)$	0.01
	2	0.25	$(8, 0)$	0.01
	3	0.5	$(8, 0)$	0.01
B	4	1.0	$(8, 0)$	0.01
	5	1.5	$(8, 0)$	0.01
	6	2.0	$(8, 0)$	0.01
	7	2.5	$(8, 0)$	0.01
	1	0.125	$(10, 0)$	0.1
	2	0.25	$(10, 0)$	0.1
	3	0.5	$(10, 0)$	0.1
C	4	1.0	$(10, 0)$	0.1
	5	1.5	$(10, 0)$	0.1
	6	2.0	$(10, 0)$	0.1
	7	2.5	$(10, 0)$	0.1
	1	0.125	$(10, 0)$	0.01
	2	0.25	$(10, 0)$	0.01
	3	0.5	$(10, 0)$	0.01
D	4	1.0	$(10, 0)$	0.01
	5	1.5	$(10, 0)$	0.01
	6	2.0	$(10, 0)$	0.01
	7	2.5	$(10, 0)$	0.01
	1	0.125	$(5, 0)$	0.1
	2	0.25	$(5, 0)$	0.1
	3	0.5	$(5, 0)$	0.1
E	4	1.0	$(5, 0)$	0.1
	5	1.5	$(5, 0)$	0.1
	6	2.0	$(5, 0)$	0.1
	7	2.5	$(5, 0)$	0.1
	1	0.125	$(5, 0)$	0.01
	2	0.25	$(5, 0)$	0.01
	3	0.5	$(5, 0)$	0.01
F	4	1.0	$(5, 0)$	0.01
	5	1.5	$(5, 0)$	0.01
	6	2.0	$(5, 0)$	0.01
	7	2.5	$(5, 0)$	0.01

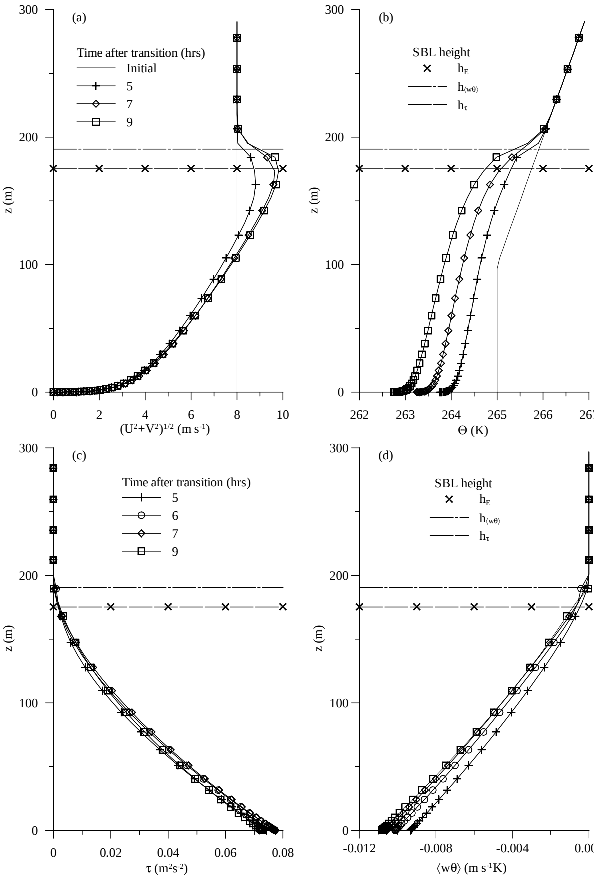

Figure 1: Vertical profiles of mean (a) wind speed, (b) potential temperature, (c) shear stress and (d) kinematic heat flux at three different times after the transition from Run A2.

ary stable boundary layer is achieved if the turbulent fluxes are independent of time (Nieuwstadt, 1984). We can say that after 7 hours of cooling, the boundary layer has reached its quasi-steady state, see Figure 1d.

One of the important parameters of the ABL modelling is the depth of the boundary layer, h. It is defined as the height where the selected flow variable falls to 5% of its surface value and divided by 0.95. This flow variable can be TKE, E or kinematic heat flux, $\langle w\theta\rangle$ or shear stress, τ. The calculated boundary layer height will be denoted as h_E, $h_{\langle w\theta\rangle}$ and h_τ respectively. Based on the 9th hour model results, h_E, $h_{\langle w\theta\rangle}$ and h_τ are shown in Figure 1. As can be seen that the maximum wind speed occurs around h_τ or h_E and above $h_{\langle w\theta\rangle}$ the turbulence almost diminishes; h_E is almost identical as h_τ and both are smaller than $h_{\langle w\theta\rangle}$ (this is true for all our model runs). Similar results were found in other models, see Cuxart et al. (2004).

For a quasi-steady-state SBL, Zilitinkevich (1972)

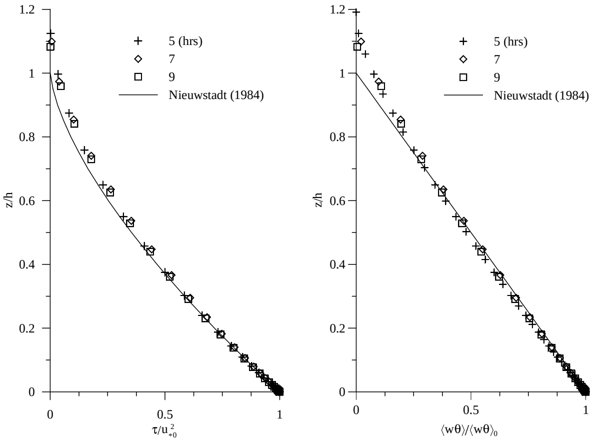

Figure 2: Turbulent fluxes from Run A2, normalized by their surface values as a function of non-dimensional height, z/h, are compared with Nieuwstadt's theoretical predictions. $h = h_\tau$ for model results.

suggested that the boundary layer height can be estimated from $h = c(u_* L_O/f)^{1/2}$, where c is a constant. From his local scaling model, Nieuwstadt (1985) found that $c^2 = \sqrt{3}\kappa Ri_f$. Freedman and Jacobson (2003) showed that $Ri_f \to 1/\beta$ from the Monin-Obukhov similarity theory. Taking the value used here, $\beta = 4.8$, we have $Ri_f \sim 0.21$. This leads to $c \approx 0.38$, slightly smaller than the value $c \sim 0.4$ given by Garratt (1982). From our model result of $u_* = 0.27$ m s^{-1} and $L_\text{o} = 122.5$ m, we have $h = 185$ m with $c = 0.38$. This compares well with $(h_E, h_{\langle w\theta\rangle}, h_\tau) = (175, 191, 175)$ m.

For the stationary stable boundary layer, Nieuwstadt (1984) suggested

$$\tau/u_*^2 = (1 - z/h)^{3/2} \tag{3}$$

$$\langle w\theta\rangle / \langle w\theta\rangle_0 = (1 - z/h) \tag{4}$$

Our model results together with the theoretical prediction of Nieuwstadt are shown in Figure 2. The agreement is very good except very close to the boundary layer top.

Figure 3 shows the time evolution of the surface friction velocity (u_{*0}), the surface kinematic heat flux ($\langle w\theta\rangle_0$) and the Obukhov length (L_o) from Run A. An initial increase in u_{*0} for a short period of time is due to the fact that the model adjusts itself after using specified initial profiles. There are fairly rapid decreases of u_{*0} within the first couple of hours after the transition, which then levels off, while initial decreases of $\langle w\theta\rangle_0$ last longer, specially for the relative large cooling rate cases. Note there are local maxima in $\langle w\theta\rangle_0$ for the cases with large cooling rates. Also the Obukhov length changes little after

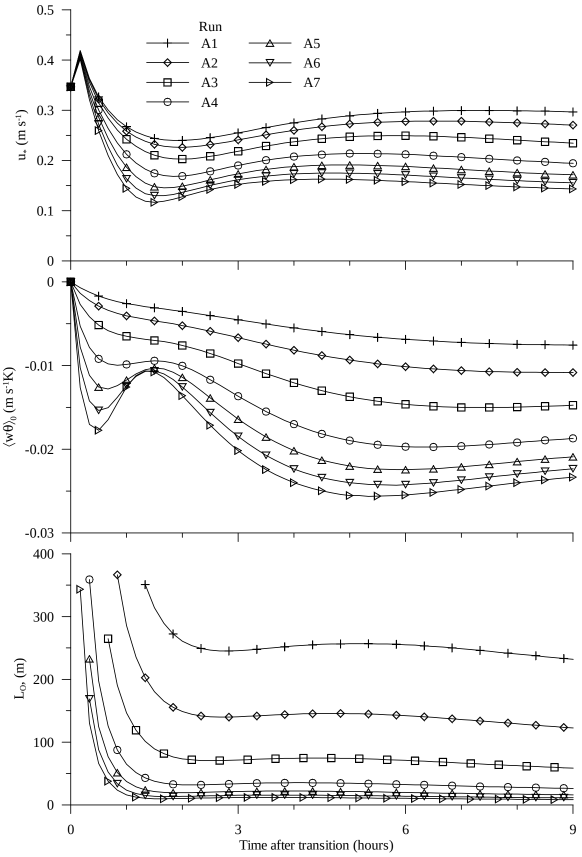

Figure 3: Evolution of the surface friction velocity, kinematic heat flux and Obukhov length under different cooling rates from Run A.

initial quick decreases despite continuous changes of u_* and $\langle w\theta\rangle$. Some of the useful parameters for Run A at hour 9 are listed in Table 2. As expected, the boundary layer depth h_τ, the friction velocity u_{*0} and the Obukhov length L_o decrease as the stability increases, while downward heat flux $-\langle w\theta\rangle$ and the surface mean wind angle α increase as the stability increases.

Table 2: Final values of some characteristics of simulated SBL for Run A

Run	u_* (m s^{-1})	$\langle w\theta\rangle$ (m s^{-1}K)	h_τ (m)	L_o (m)	α (degree)
A1	0.297	-0.0076	227.8	231.9	33.7
A2	0.270	-0.0108	175.2	122.5	35.7
A3	0.234	-0.0148	129.7	58.6	39.1
A4	0.194	-0.0187	84.4	26.3	43.5
A5	0.171	-0.0209	62.5	16.1	46.0
A6	0.155	-0.0224	49.9	11.3	47.7
A7	0.143	-0.0233	41.9	8.4	48.8

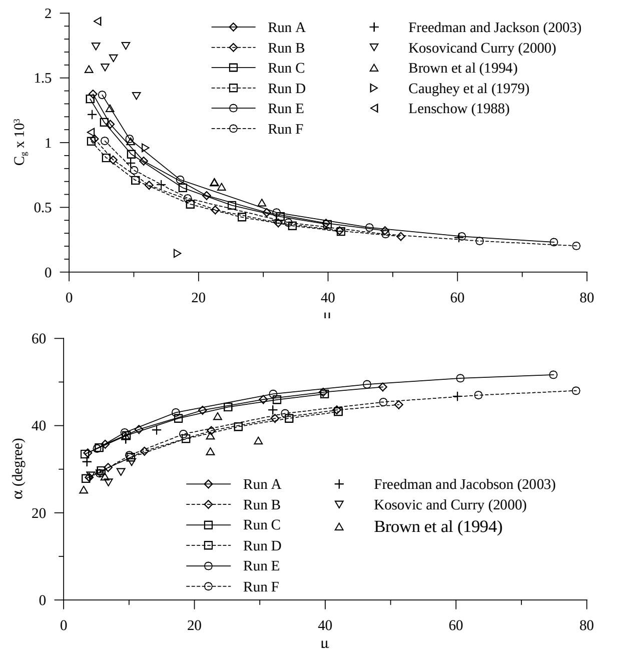

Figure 4: Variations of the geostrophic drag coefficient and surface mean wind angle with the stability.

The model predictions of geostrophic drag coefficient, $C_g = (u_*/|\mathbf{U_g}|)^2$, and the surface mean wind angle, α, plotted against the stability parameter, $\mu = \kappa u_*/fL_o$, are shown in Figure 4. Also shown are the model results of Freedman and Jacobson (2003), some LES results of Kosovic and Curry (2000) and Brown et al. (1994), field data of Caughey et al. (1979) and Lenschow et al. (1988). The scatter in LES (probably due to the different subgrid parameterizations and domain sizes used) and field data (possible effects of the terrain and synoptic scale system) make it difficulty to deduce the model accuracy. The broad tendency is that C_g (α) decreases (increases) with stability, as found in our model prediction and as well as in the data.

Our model results show good agreement with that of Freedman and Jacobson, who used a $E - \epsilon$ turbulence closure with a modified ϵ-equation through enforced consistency with Monin-Obukhov similarity theory. There is a little effect of $\mathbf{U_g}$ on C_g and α. However, the effect of z_0 on C_g and α is more pronounced. For small value of μ, there are difference in C_g. The larger the value of surface roughness, the larger C_g. As stability increases, the difference becomes small. At $\mu = 40$, it almost disappears. For a given stability, the large value of the surface roughness causes a large surface mean wind angle. The difference in α with two different surface roughness remains almost same (about 4 or 5 degrees) for all stability in our runs.

4. CONCLUDING REMARKS

A series of stable atmospheric boundary layers have been simulated with the PBL model of Weng and Taylor (2003). These stable boundary layers were developed by applying surface cooling to a specified initial state. These model results show good agreement with Nieuwstadt's theory, model results of Freedman and Jacobson, and to a lesser extent, with the LES model results of Brown et al. and Kosovic and Curry and field data of Caughey et al. and Lenschow et al. Future work will incorporate a soil model or land surface scheme coupled to the surface energy budget.

ACKNOWLEDGEMENTS

The authors thank Dr. Freedman for providing some of the data used in this paper. This research has been funded by grants from the Natural Science and Engineering Research Council of Canada.

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