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Scaling laws for shallow moist convection

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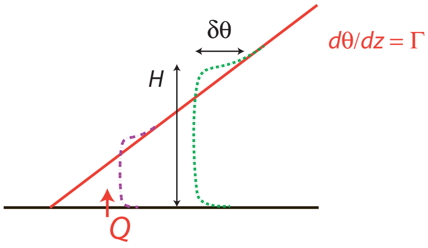

Figure 1: Idealized cartoon of the development of a convective boundary layer

1. INTRODUCTION

Large-eddy simulation (LES) is used to explore the similarity structure of, and scaling theories for, shallow moist cumuliform convection. In the past, studies of this convective regime have been based on forcings derived from field data, such as taken from BOMEX (Siebesma et al., 2003) or ATEX (Stevens et al., 2001); so doing imparts a variety of scales onto the problem which in principle should arise spontaneously from a more simply configured problem. For instance, the dry convective boundary layer is often observed to have a thermal structure similar to what is shown in Fig. 1. When simulating such a flow one frequently starts from a set of initial data consisting of a well mixed profile separated from the free troposphere, whose lapse rate is uniform with height, by a jump condition in temperature (Sullivan et al., 1998). However, the similarity structure of the dry convective boundary layer is more readily evident by imposing a constant heat flux on a layer whose lapse rate is initially uniform. From the perspective of LES this more simplified problem has five ostensible parameters: a buoyancy factor g/θ; the model grid scale Δ; the lapse rate Γ; the surface buoyancy flux Q; and time t. This yields a problem in two non-dimensional numbers, $\Pi_1 = (\Gamma t^2 g/\theta_0)$ and $\Pi_2 = Qt/(\Gamma\Delta^2)$, for which similarity solutions can be sought. The premise of LES is Reynolds number similarity in which case any explicit dependence on the grid, i.e., Π_2 vanishes. Similarity in Π_1 (which effectively measures the ratio of convective to gravity wave timescales) can be evaluated by exploring the extent to which the non-dimensional depth

$$\alpha(\Pi_1) \equiv H\left(\frac{\Gamma}{2Qt}\right)^{1/2} \qquad (1)$$

is constant, where H is the depth of the convecting layer at some time t. Because Π_1 increases with t^2 similarity in Π_1 at large t is not an unreasonable expectation. Simulations bear this out, and also show that an inversion (or overshooting) layer emerges spontaneously with a strength $\Delta\theta$ which scales with $\sqrt{2Qt\Gamma}$ and a depth which scales with h. From this perspective initializing simulations with an initial mixed layer depth h and inversion jump $\Delta\theta$ inconsistent with the chosen forcing effectively introduces an additional, and largely unnecessary parameter into the problem.

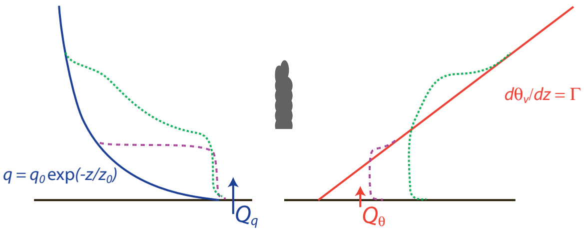

Figure 2: Idealized cartoon of the development of a convective boundary layer

This motivates us to ask how best to explore the structure of the trade-wind boundary layer. The framework we have developed draws heavily on insights taken from a study of the dry convective boundary layer, and is illustrated in Fig. 4. Here the moisture profile is chosen to be $q = q_0 \exp -z/z_0$ where z_0 is a moisture scale height. With respect to surface forcing our interest in maritime regimes encourages us to formulate the problem by adjusting the sea surface temperatures with time in a fashion which maintains a constant surface buoyancy flux

$$Q = \frac{g}{\theta_0}\,\overline{w'\theta'}\Big|_{\rm srf} + g(R_v/R_d - 1)\,\overline{w'q_t'}\Big|_{\rm srf}. \qquad (2)$$

Where surface fluxes are obtained from the bulk-aerodynamic expression with a specified exchange velocity, specified surface properties, and the values at the lowest resolved atmospheric level in the simulation. Thus, except for implicitly specifying a surface exchange velocity, the mean wind is zero, as would be expected

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Q [Wm^{-2}]	Γ [Km^{-1}]	z_0 [m]
5	6	1500
15	”	”
25	”	”
40	”	”
25	4	”
”	8	”
”	6	500
”	6	1000
”	6	1500

Table 1: Base suite of simulations. Here Q measures the buoyancy flux in equivalent heat flux units, i.e., $Q = \rho c_p \overline{w'\theta_v'}$. All of the base simulations are carried out on a grid whose dimensions are 96 × 96 × 131 where the horizontal mesh is fixed with a 75m spacing and the vertical mesh is stretched from 5m near the surface through a depth of 5km.

for free convection. From this perspective, q_0 and z_0 add two additional parameters to the problem as does the ratio between the moist and dry adiabatic lapse rates.

2. SIMULATIONS

We conduct a number of simulations, as indicated in Table 1, to explore the parameter space of this problem. Additional sensitivity studies starting with the base ($Q = 25$ W m^{-2}, $\Gamma = 6$ K m^{-1}, $z_0 = 1500$ m) simulation but doubling the size of the horizontal mesh, refining the grid for the same domain size, or exploring the sensitivity to the chosen value of q_0 are also explored. Simulations are performed using the UCLA large-eddy code (Stevens et al., 1999) and are run for between 24 and 36 hours of simulated time with an ≈ 1 s timestep. Initially a dry convective boundary layer develops in accord with the similarity concepts developed above. After deepening to the lifting condensation level a cloud layer spontaneously develops (e.g., Fig. 3), with many of the characteristics of the trade-wind layer: A well mixed subcloud layer, a transition layer wherein a region of CIN is evident, a cloud layer containing CAPE and an inversion layer of finite depth. The cloud layer does not grow self similarly, but may approach such a regime as the depth of the CAPE containing layer becomes much larger than the depth of the CIN containing layer—although in this limit the assumption of no precipitation will be violated. Preliminary analysis also suggests that the amount of CIN remains constant in time, while the CAPE increases. Noteworthy is that the cloud layer develops through a cooling and moistening of the free atmosphere.

3. ANALYSIS

As would be expected the depth of the layer, as measured by the height of the maximum θ or q_t gradient, initially grows with $t^{1/2}$. After the cloud layer develops the boundary layer grows linearly in time. The ballistic growth of the cloud layer can be understood through an evaporative cooling mechanism, wherein liquid water is transported by clouds into the inversion layer, where it is detrains and evaporates (Betts, 1973; Riehl, 1954, cf.). If the liquid-water flux scales with cloud depth, following energetic arguments akin to those for the dry convective boundary layer, one would expect growth rate proportional to t, in accord with the simulations. Analysis of the liquid water flux profiles (not shown) also support this argument.

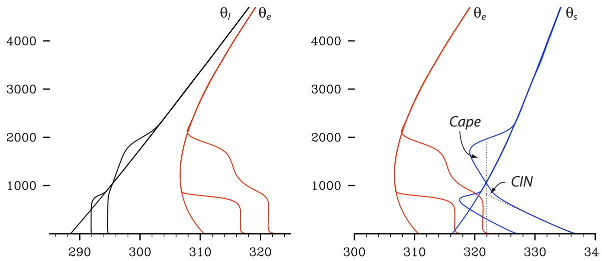

Figure 3: Development of boundary layer depth for trade-wind simulations. Profiles (at the initial time, a time just before the development of the cloud layer, and at a time late in the simulation) are shown in $\{\theta_l, \theta_e\}$ and $\{\theta_e, \theta_{e,s}\}$ space.

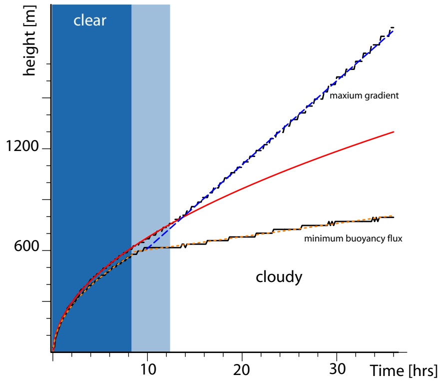

Figure 4: Development of boundary layer depth for trade-wind simulations. Red and orange lines are best fit lines to cloud free growth regime (shaded blue) of the maximum gradient layer (red) and minimum buoyancy flux layer (orange). Blue dashed line is fit to maximum gradient height for steady cloud regime (no shading). Light blue shading illustrates regime where cloud fraction is increasing.

Analysis of a variety of simulations shows that the growth rate scales well with M_{cb}/Γ, where M_{cb} is the cloud base mass flux and Γ is the non-dimensional lapse rate of the free troposphere. M_{cb} scales with $w_* B$ with w_* being the convective velocity scale for the subcloud layer and B the bowen ratio. This resultant scaling $dz/dt \propto w_* B/\Gamma$ is shown in Fig. 5 for the full suite of simulations. The scaling of M_{cb} with w_* was much earlier suggested on the basis of observations by (Nicholls and LeMone, 1980).

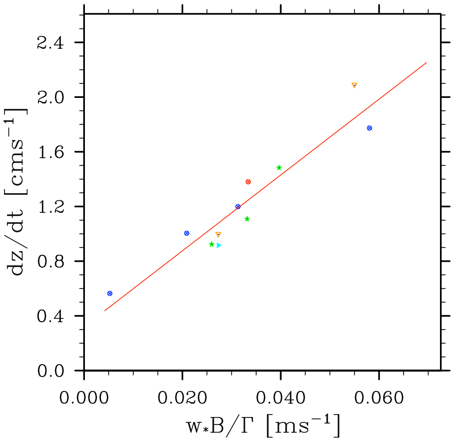

Figure 5: Growth rate scaled by $w_* B/\gamma$.

4. SUMMARY

There has been a resurgence of interest in the structure of shallow non-precipitating cumuliform cloud layers. Here we advocate a simple framework which allows their study as a function of a minimal number of parameters. Self similar growth regimes are sought, but are not readily apparent. Nonetheless, some interesting phenomena emerge, ranging from the observed structure of trade-cumulus like layers, to a variety of scaling laws. In particular the cloud base mass flux is found to scale well with $w_* B$ where the bowen ratio, B, also scales with cloud fractions at cloud base. The growth of the cloud layer scales with $w_* B/\Gamma$ following energetic arguments akin to those for the growth of the dry convective boundary layer. The cloud layer is found to grow balistically, as compared to the diffusive growth for the dry CBL under a fixed surface buoyancy flux. This ballistic growth is explained through a growth mechanism which emphasizes liquid water transport)and evaporation) into the inversion layer, rather than entrainment.

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