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A GAUSSIAN DISPERSION/PLUME MODEL EXPLICITLY ACCOUNTING FOR WIND SHEAR

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1. INTRODUCTION

Classical plume and puff dispersion models are derived from differential equations that assume that homogeneous turbulence alone is responsible for dispersing pollutants away from the plume centerline downwind of point sources. However, observed dispersive behavior of atmospheric plumes cannot be explained in terms of turbulent diffusion alone, requiring empirical adjustment of horizontal and vertical dispersion parameters (σ) used in regulatory plume models.

In this paper, a more general Gaussian dispersion plume model is derived by considering the effects of shearing motions on plume dispersion. In section 2, the mathematical formulation of a steady-state plume emitted into an environment containing wind shear is presented. General features of this sheared plume are described in Section 3, showing more consistency with observed plume dispersion. Section 4 specifically addresses the effects of shear on horizontal "size" or dispersion of pollution plumes. Observations of typical shear magnitudes derived from profiler wind measurements are briefly presented in Section 5. Such measurements would be required to fully explain observed pollution plumes.

2. DERIVATION: PLUME WITH SHEAR

The initial transport and dispersion of pollutants in plumes downwind of point sources in the atmosphere can be mathematically quantified using a steady-state three-dimensional advection-diffusion equation:

$$u\frac{\partial c}{\partial x} + v\frac{\partial c}{\partial y} = K_h\frac{\partial^2 c}{\partial y^2} + K_z\frac{\partial^2 c}{\partial z^2} \qquad (1)$$

where c is the pollutant concentration, K_h and K_z are the horizontal and vertical turbulent diffusion coefficients, and u and v are the wind speeds parallel to and perpendicular to the mean wind. Classical Gaussian plume models used in regulatory applications for the past 40 years represent one solution to this equation under conditions when there are no mean motions perpendicular to the average wind (v=0). Analytical solutions to Eq. (1) exist for some simple configurations of v. For example, If v varies linearly with height (z)

$$v = z\frac{\partial v}{\partial z} \qquad (2)$$

and the shear perpendicular to the mean ($\partial v/\partial z$) is constant, the following analytical solution exists for a point source emitted at x=y=z=0:

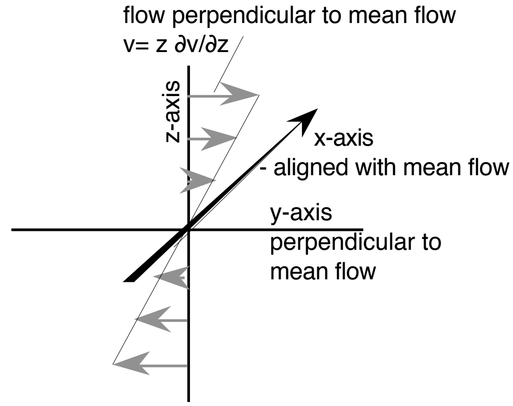

Fig. 1. Schematic of coordinate system adopted for this derivation showing mean flow (directed into page here) along x-axis and shearing motions perpendicular to mean wind moving right and left above and below the plume centerline.

$$c = \frac{Q}{2\pi u\sigma_z\sigma_y\sqrt{1+s^2/12}}\exp\left[\frac{-y^2}{2\sigma_y^2\left(1+s^2/12\right)} + \frac{-z^2\left(1+s^2/3\right)}{2\sigma_z^2\left(1+s^2/12\right)} + \frac{yz}{2\sigma_y\sigma_z}\left(\frac{s}{1+s^2/12}\right)\right] \qquad (3)$$

where Q is the emission rate (mass s^{-1}). Lateral and vertical turbulent dispersions are given as

$$\sigma_y = \sqrt{\frac{2K_h x}{u}}\ ,\ \sigma_z = \sqrt{\frac{2K_z x}{u}}\ , \qquad (4)$$

and s is a nondimensional shear factor

$$s = \frac{\partial v}{\partial z}\frac{x}{u}\frac{\sigma_z}{\sigma_y} = \frac{\partial v}{\partial z}\frac{x}{u}\sqrt{\frac{K_z}{K_h}} \qquad (5)$$

Cursory analysis of Eq. (3) shows that it reverts to the classical Gaussian plume formulation when shear is neglected. When ∂v/∂z=0, s=0 and the third "yz" term in the exponential factor of Eq. (3) drops out. Fig. 1 shows schematically the coordinate system and configuration of winds used for this derivation.

The solution presented here was derived in Konopka (1995) to describe the cross-section of a stationary plume segment in a sheared environment. Here the "time" variable used in their formulation is substituted with the distance downwind of the point source divided by the mean wind speed transporting the plume (t=x/u).

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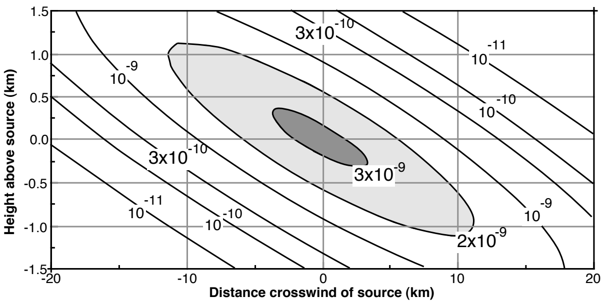

Fig. 2. Vertical "slice" through plume perpendicular to the mean wind showing tilted axis of normalized concentration (C/Q) 40 km downwind of release.

3. CHARACTERISTICS OF SHEARED PLUME

Fig. 2 shows a vertical "slice" of a "curtain" of concentrations perpendicular to the mean wind through a plume 40 km downwind of its emission point calculated using Eq. (3). For this plume, typical mid-day boundary layer wind speed (7 m s^{-1}), diffusivities (K_h=K_z=125 m^2 s^{-1}) and shear (-3 m s^{-1} km^{-1}) are specified. Winds perpendicular to the mean flow (which in this case flows out of the page) below the centerline of the plume displace the plume to the right in this view, while winds above the centerline displace the plume to the left. The vertical scale of Fig. 2 is greatly exaggerated, and the "tilt" of the plume is only about 10% from horizontal. For most observed plumes in the atmosphere, a "tilt" of this magnitude would be indistinguishable from a horizontal plume, however, it is clear that the shearing motions have significantly enhanced the horizontal plume dispersion.

Fig. 3 shows concentration vs. crosswind distance 1000 m below the plume centerline at 3 distances

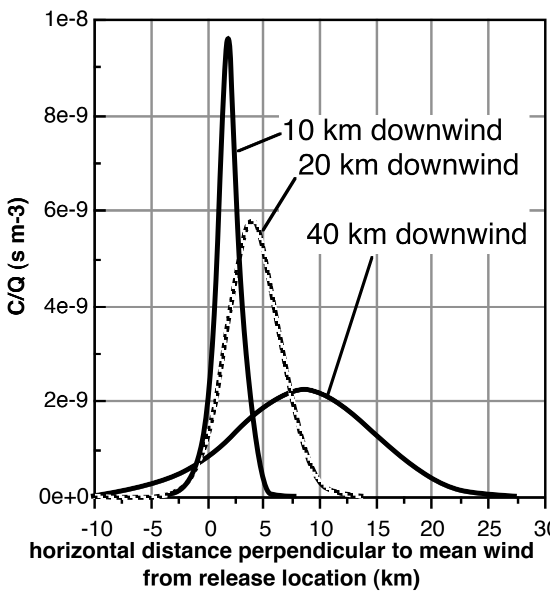

Fig. 3. Normalized concentration 1000 m below release altitude vs. crosswind distance for several distances downwind of release point.

downwind of the emission point for the conditions described above. As one moves downwind, the highest concentrations encountered in the cross-wind direction are laterally displaced from the plume centerline in the direction of the shearing winds that carry the plume away from its centerline. Numerous field observations of plumes have shown that plume centerlines are often laterally displaced from measured wind directions (e. g. Shannon 1981), and these results suggest that shearing motions, which are difficult to measure, are probably influencing those measured plumes.

4. HORIZONTAL DISPERSION WITH SHEAR

Visual inspection of the terms in Eq. (3) shows that the horizontal dispersion is enhanced relative to purely turbulent diffusion, and this "effective" horizontal dispersion (σ_y^{eff}) can be represented by:

$$\sigma_y^{eff} = \sigma_y\sqrt{1+s^2/12} = \sqrt{\frac{2Kx}{u}}\sqrt{1+\frac{1}{12}\left(\frac{\partial v}{\partial z}\frac{x}{u}\frac{\sigma_z}{\sigma_y}\right)^2} \quad ,(6)$$

where σ_y is dispersion resulting from pure diffusion.

Fig. 4 shows the effective horizontal dispersion quantified in Eq. (6) as a function of distance downwind of a point source. Here a wind speed of 10 m s^{-1} is used, shear in ranges from 0 – 10 m $s^{-1}km^{-1}$, and diffusion coefficients range from nominal stable night values

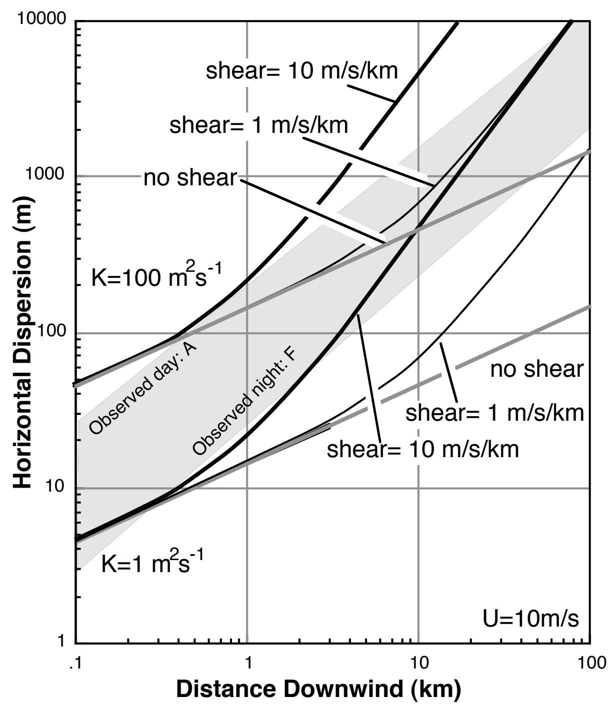

Fig. 4. Effective horizontal dispersion as a function of distance downwind of a point source. Wind speed =10 m s^{-1}, shear in range of 0 – 10 m $s^{-1}km^{-1}$, diffusion coefficients range from nominal stable night values (1 m^2s^{-1}) to typical unstable daytime (100 m^2s^{-1}). Gray area denotes range of observed horizontal plume dispersion from standard Gaussian plume empirical formulations for daytime "class A" (greatest) to night "class F" (lowest) dispersion.

(1 m^2s^{-1}) to typical unstable daytime (100 m^2s^{-1}) conditions. The gray area on Fig. 4 denotes the range of observed horizontal plume dispersion used in standard Gaussian plume empirical formulations for daytime "class A" (greatest) to night "class F" (lowest) dispersion.

Fig. 4 shows that near the source, shear is not important, and plume dispersion is governed by purely turbulent processes, thus yielding $\sigma \sim x^{0.5}$.

At larger distances downwind, growth rates that spread pollutants with distance of $x^{1.5}$ power result from the fact that the plume is growing vertically by a turbulent process ($\sim x^{0.5}$), but this vertical growth exposes the plume to shearing motions that grow linearly with distance ($\sigma \sim \partial v/\partial z \bullet x/u \sim x^1$). These two effects are essentially multiplicative, yielding plumes that grow in proportion do the 3/2 power of distance from release or emission point. In the limit of shear-dominated dispersion, the shear term in Eq. (6) involving $\partial v/\partial z$ is >>1, and plumes grow in proportion to the 1.5 power of the downwind distance.

Fig. 4 shows that the range of observed plume sizes predicted by Eq. (3) encompasses the size of observed plumes and the power-law vs. distance relationships observed in atmospheric plumes. Fig. 4 also shows that for distances beyond about 1 km downwind of a point source, shearing motions will under many conditions dominate the horizontal dispersion process relative to the dispersion caused by turbulence alone. For distances beyond 1-2 km downwind, even small amounts of shear are shown to enhance plume horizontal size by factors of 5-10 relative to plumes growing without shear.

Another note of interest is that under stable, night conditions, the only way the theory derived here can match observed dispersion 1-10 km downwind of the release point is for there to be considerable shear present under night conditions, which is consistent with many observations. At night, with diffusion coefficients of ~1 m^2s^{-1}, in the downwind range 1-10 km, plumes only grow to 15-40 m in width in environments without shear. Observations under "class-F" stability conditions are considerably greater than this (30-300 m), suggesting that relatively high shear (~10 m $s^{-1}km^{-1}$) must occur in order to obtain such large horizontal spread.

5. TYPICAL SHEAR PROFILES IN PBL

Fig. 5 shows a vertical profile of winds measured in the lowest 1000 m above the local terrain using the NOAA profiler over Schenectady, NY at 1 PM local time on 23 Oct 2003. At the time of this sounding, the turbulent atmospheric mixed layer was higher than the height of wind measurements shown in this figure. Winds plotted here are defined relative to the MEAN wind in the entire layer, decomposed into deviations parallel to and perpendicular to the mean. Average winds were blowing at 8.5 m/s from a northwesterly direction (312˚). Fig. 5 shows that winds perpendicular to the mean flow in this layer vary in an approximately linear fashion with height for these typical conditions, thus justifying a key assumption of this plume model derivation (Eq. 2).

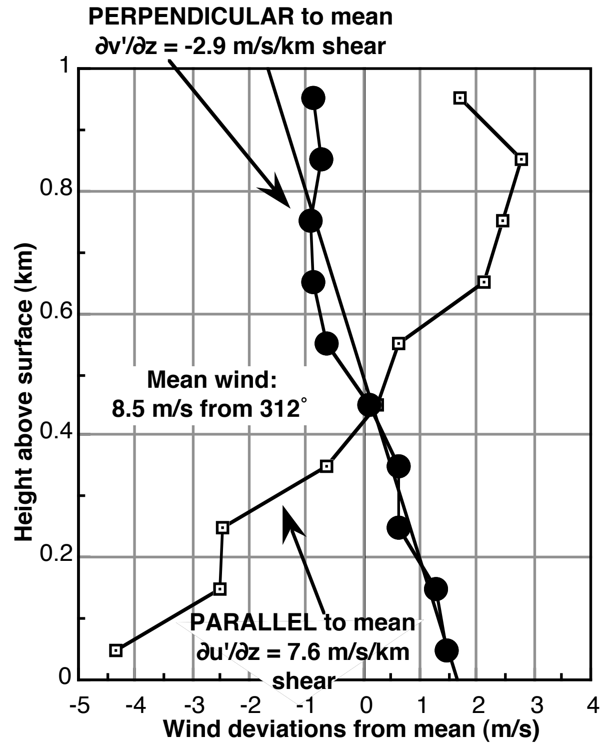

Fig. 5. Variation of wind with height in lowest 1000m above surface over Schenectady, NY at 1PM local time on 23 Oct 2003. Winds are decomposed into deviations parallel to and perpendicular to the mean wind averaged over the lowest 1000m of 8.5 m/s blowing from 312°.

6. CONCLUSIONS

A mathematical solution to the steady-state advection-diffusion equation for point sources of pollution emitted into an environment containing turbulence and wind shear perpendicular to the mean flow is derived. It is found that shearing motions perpendicular to the mean flow significantly enhance horizontal plume dispersion in a manner consistent with observations. According to the ideal mathematical derivation, shearing effects lead to plume dispersions (σ) increasing with powers of downwind distance (x) ranging from $\sigma \sim x^{0.5}$ to $\sigma \sim x^{1.5}$, depending on the distance from the release location, and the relative magnitudes of turbulence and shear in the flow.

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