2.11 EVALUATION of a NEW PBL PARAMETERIZATION WITH EMPHASIS IN SURFACE FLUXES

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1. Introduction

The parameterization of PBL processes in a general circulation model of the atmosphere (AGCM) provides the exchanges of momentum, heat and mass between the atmosphere and the underlying surface. These fields are of crucial importance for simulating the air-sea interaction in coupled Atmosphere-Ocean models. The PBL parameterization also provide boundary layer cloudiness, which strongly influences the surface radiative fluxes and hence the predicted SSTs in coupled Atmosphere-Ocean simulations (Ma et al. 1996, Mechoso et al. 2000).

In this paper, we describe a new PBL parameterization that has been recently implemented and tested in the UCLA AGCM. As in other versions of this AGCM, a variable-depth with a coordinate surface at the PBL-top is embedded in the GCM (Suarez et al. 1983). Such a framework facilitates the explicit representation of processes concentrated near the PBL top and the prediction of PBL clouds. The new parameterization introduces multiple layers between the PBL top and the earth surface, thus allowing for vertical shears and deviations from well-mixed profiles within the PBL. The PBL processes are then formulated following a hybrid approach, in which the effects of large-scale eddies and small-scale eddies are formulated separately. For the large-scale eddies, a relatively well-mixed vertical structure is assumed for conservative thermodynamic variables and a bulk approach is applied to the properties vertically averaged over the entire PBL so that the formulation is non-local. This formulation includes the prediction of the bulk turbulence kinetic energy (TKE). The effects of small-scale eddies are considered through a K-closure type diffusive process. The surface fluxes are determined from an aerodynamic formula, in which both the square root of the bulk TKE and the mean large-scale PBL velocity are used to determine the velocity scale. With this formulation, estimates of surface fluxes are expected to be better than those provided by the traditional methods because the mean wind can be weak albeit the convective mixing is strong. The PBL-top mass entrainment is explicitly computed with a formulation that also uses the bulk TKE.

In section 2 of this paper we briefly describe general aspects of the UCLA AGCM and we explain the prognostic equations within the PBL. In section 3 we discuss the parameterization of the PBL processes. In section 4 we present results of a simulation with prescribed SST. In section 5 we discuss the results and present the conclusions.

2. Model description

The UCLA AGCM is a finite difference model that integrates the primitive equations of the atmosphere. The model’s horizontal discretization is based on the Arakawa C grid, and the vertical discretization follows Arakawa and Suarez (1983). The
parameterization of physical processes other than those of the PBL include solar and terrestrial radiation according to Harshvardhan et al. (1987 and 1989, respectively), and a prognostic version of the cumulus convection scheme by Arakawa and Schubert (1974) proposed by Pan and Randall (1998). In this scheme the original assumption of quasi-equilibrium for the cloud work function is relaxed by predicting the cloud-scale kinetic energy. We describe next the equations for the prognostic variables, with emphasis on their formulation within the PBL.

2.1 Continuous governing equations

We define \( p_b \) as the pressure at the top of the PBL. Thus the region between \( p_b \) and \( p_v \), where \( p_v \) is the pressure at the Earth surface, represents the PBL. In the free atmosphere above the PBL, we consider two regions. One between the PBL top and a tropopause level at \( p = p_t \), and the other between \( p_t \) and the model top, at \( p = P_0 \).

The definitions of the \( \sigma \) coordinate in these three regions are:

\[
\sigma = \begin{cases} 
1 + \frac{(p - p_b)}{(p_s - p_b)} & \text{for } p_s \leq p \leq p_v, \\
\frac{(p - p_v)}{(p_b - p_v)} & \text{for } p_v \leq p \leq p_b, \\
\frac{(p - p_t)}{(p_v - p_t)} & \text{for } p_t \leq p \leq p_v.
\end{cases} 
\]

(2.1a, 2.1b, 2.1c)

We currently use \( p_t = 1 \) hPa and \( p_v = 100 \) hPa.

According to these definitions, \( \sigma = -1 \) at the model top \( (p = p_t) \), \( \sigma = 0 \) at the tropopause level \( (p = p_b) \), \( \sigma = 1 \) for the PBL top \( (p = p_0) \), and \( \sigma = 2 \) at the earth surface \( (p = p_0) \). From these definitions the pressure can be obtained as

\[
p = p_s - (\sigma_s - \sigma)\pi_{\text{PBL}} \quad \text{for } \sigma \geq \sigma_b,
\]

with \( \pi_{\text{PBL}} = p_s - p_b \)

(2.2a)

\[
\text{with } \pi_{\text{imp}} = p_b - p_i
\]

(2.2b)

\[
\text{with } \pi_{\text{mer}} = p_i - p_t
\]

(2.2c)

The continuity equation can be written as

\[
\frac{\partial \pi}{\partial t} + \nabla \cdot (\pi \mathbf{v}) + \frac{\partial (\pi \sigma)}{\partial \sigma} = 0,
\]

(2.3)

where \( \pi \) is either \( \pi_{\text{PBL}} \), \( \pi_{\text{trop}} \) or \( \pi_{\text{strat}} \).

The momentum equation, for layers within the PBL, is

\[
\frac{\partial \mathbf{v}}{\partial t} + \nabla \cdot (\mathbf{v} \mathbf{v}) + f \mathbf{k} \times \mathbf{v} = -\nabla \pi \Phi - \sigma \frac{\partial \mathbf{v}}{\partial \sigma} + \frac{1}{\rho} \frac{\partial F}{\partial z},
\]

(2.4a)

where the last term is the vertical convergence of the turbulent flux of momentum. This term is discussed in section 3. In the free atmosphere, the momentum equation is

\[
\frac{\partial \mathbf{v}}{\partial t} + \nabla \cdot (\mathbf{v} \mathbf{v}) + f \mathbf{k} \times \mathbf{v} = -\nabla \pi \Phi - \sigma \frac{\partial \mathbf{v}}{\partial \sigma}.
\]

(2.4b)

The geopotential \( \Phi = gz \) is diagnosed from the hydrostatic equation,

\[
\delta \Phi = -\theta \delta \Pi,
\]

(2.5)

where \( \Pi \) is the Exner function, defined as

\[
\Pi = c_p \left( \frac{p}{P_0} \right)^{\eta}.
\]

(2.6)

The thermodynamic equation in terms of the potential temperature \( \theta \) is, within the PBL,

\[
\frac{\partial (\pi_{\text{PBL}} \theta)}{\partial t} + \nabla \cdot (\pi_{\text{PBL}} \theta \mathbf{v}) = -\frac{\partial}{\partial \sigma} \left( \pi_{\text{PBL}} \frac{\partial \theta}{\partial \sigma} \right) + \frac{\pi_{\text{PBL}} Q}{\Pi} + G_b,
\]

(2.7a)

where \( G_b \) is the contribution of the turbulent fluxes to the tendency of \( \theta \), and is discussed
in the next section, and \( Q \) is the heating rate. In the free atmosphere,
\[
\frac{\partial (\pi \theta)}{\partial t} + \nabla \cdot (\pi \theta \mathbf{v}) = -\frac{\partial}{\partial \sigma} (\pi \theta \hat{v}) + \pi Q, \tag{2.7b}
\]
where \( \pi \) is \( \pi_{\text{top}} \) or \( \pi_{\text{strat}} \) according to the respective free atmosphere regions.

The continuity equation for the total water inside the PBL predicts the water mixing ratio \( (r) \), which is the water vapor mixing ratio \( (q) \) plus the liquid water mixing ratio \( (l) \). Here we assume that PBL turbulence allows the air to hold both phases of water. No large-scale precipitation processes occur within the PBL unless the pressure at the condensation level is greater than the surface pressure (condensation level bellow the earth surface). If this occurs, large-scale precipitation is computed to make the condensation level equal to the surface level. The continuity equation for \( r \) within the PBL is
\[
\frac{\partial (\pi_{\text{PBL}} r)}{\partial t} + \nabla \cdot (\pi_{\text{PBL}} r \mathbf{v}) = -\frac{\partial}{\partial \sigma} (\pi_{\text{PBL}} r \hat{v}) - \pi_{\text{PBL}} C_i + G_i, \tag{2.8a}
\]
where \( G_i \) is the contribution of the vertical convergence of the turbulent flux of \( r \), and \( C \) is the condensation rate. For the free atmosphere,
\[
\frac{\partial (\pi q)}{\partial t} + \nabla \cdot (\pi q \mathbf{v}) = -\frac{\partial}{\partial \sigma} (\pi q \hat{v}) - \pi C_q, \tag{2.8b}
\]
where again \( \pi \) is \( \pi_{\text{top}} \) or \( \pi_{\text{strat}} \) as with the equation for \( \theta \).

2.2 Discrete equations

We discuss next the vertical discretization of the governing equations, with emphasis in the PBL. A similar discussion but for a hybrid vertical coordinate model is found in Konor and Arakawa 2001.

The atmosphere is vertically divided into layers (Fig. 1), from \( k = 1 \) (uppermost layer) to \( k = M \) (lowermost layer). We use half integer indices for labeling the layer interfaces; \( k + 1/2 \) is the interface between layer \( k \) and layer \( k + 1 \). The top of the atmosphere is the first layer interface, with vertical index 1/2, and the Earth surface is the last layer interface, with vertical index M+1/2. The lowermost free atmosphere layer has vertical index L, and uppermost PBL layer has vertical index \( L + 1/2 \). The interface between these two layers is the PBL top, defined as level B with vertical index \( L + 1/2 \). For \( p = p_b, \sigma = \sigma_b = 1 \) and for \( p = p_s, \sigma = \sigma_s = 2 \).

Horizontal velocity, temperature and water vapor mixing ratio are predicted for the layers, while vertical velocities \((D\sigma/Dt)\) are computed at the layer interfaces.

<table>
<thead>
<tr>
<th>Value</th>
<th>Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atmosphere top</td>
<td>1/2</td>
</tr>
<tr>
<td>first layer</td>
<td>3/2</td>
</tr>
<tr>
<td>Layer interface</td>
<td>k - 1/2</td>
</tr>
<tr>
<td>( \pi(D\sigma/Dt) )</td>
<td>k</td>
</tr>
<tr>
<td>( v, \theta, q )</td>
<td>Generic layer</td>
</tr>
</tbody>
</table>

Figure 1: The model layers, the layer interfaces and the vertical indexes

We discretize the mass continuity equation by
\[
\frac{\partial \pi}{\partial t} = -\nabla \cdot (\pi \mathbf{v})_k - \frac{1}{(\delta \sigma)_k} \left[ (\pi \hat{v} \hat{v} \hat{q} \mathbf{v} - (\pi \hat{v} \hat{q}) \hat{q} \mathbf{v}) \right]
\]
for \( k = 1, 2, \cdots, M \). \( \tag{2.9} \)
where \( \pi \) will be either \( \pi_{PBL}, \pi_{top} \) or \( \pi_{strat} \), according to the vertical interval in which the layer is, and

\[
(\delta \pi)_{k+1/2} = \pi_{k+1/2} - \pi_{k-1/2}. \tag{2.10}
\]

At the interface between the PBL and the free atmosphere, we compute the vertical mass flux as

\[
(\pi \dot{\sigma})_{k+1/2} = (\pi \dot{\sigma})_{B} = g(E - D - M_{B}), \tag{2.11}
\]

where we take \( E > 0 \) if there is mass entrainment into the PBL, or \( D > 0 \) if there is mass detrainment from the PBL. \( M_{B} \) is the cumulus mass detrained from the PBL to the cumulus clouds through their bases. Summation in the vertical of equation (2.9) for all the layers in the free atmosphere yields

\[
\frac{\partial \pi_{top}}{\partial t} = - \sum_{k=1}^{k_{strat}} \nabla \cdot (\pi_{strat} \dot{v})_{k} (\delta \sigma)_{k}
- \sum_{k=k_{strat}+1}^{L+1} \nabla \left( \pi_{top} \dot{v} \right)_{k} (\delta \sigma)_{k} - (\pi_{top} \dot{\sigma})_{B}, \tag{2.12a}
\]

where \( \pi_{strat} = \text{constant} \). Summation of (2.9) for the PBL layers gives

\[
\frac{\partial \pi_{PBL}}{\partial t} = - \sum_{k=M}^{L+1} \nabla \cdot (\pi_{PBL} \dot{v})_{k} (\delta \sigma)_{k} + (\pi_{PBL} \dot{\sigma})_{B}, \tag{2.12b}
\]

and summation for all layers gives

\[
\frac{\partial \pi_{S}}{\partial t} = - \sum_{k=1}^{k_{strat}} \nabla \cdot (\pi_{strat} \dot{v})_{k} (\delta \sigma)_{k}
- \sum_{k=k_{strat}+1}^{L+1} \nabla \left( \pi_{top} \dot{v} \right)_{k} (\delta \sigma)_{k}
- \sum_{k=L+1}^{M} \nabla \left( \pi_{PBL} \dot{v} \right)_{k} (\delta \sigma)_{k}. \tag{2.12c}
\]

Equations (2.12a-c) allow to predict \( \pi_{top} \), and \( \pi_{PBL} \) and therefore, also \( p_{B} \) and \( p_{S} \). On the other hand, partial summation of (2.9) also gives

\[
(\pi \dot{\sigma})_{k+1/2} = \left( \frac{\sigma_{S} - \sigma_{k+1/2}}{\sigma_{S} - \sigma_{B}} \right) (\pi \dot{\sigma})_{B}
+ \left( \frac{\sigma_{k+1/2} - \sigma_{B}}{\sigma_{S} - \sigma_{B}} \right) \sum_{k=L+1}^{M} \nabla \left( \pi_{PBL} \dot{v} \right)_{k} (\delta \sigma)_{k}
- \sum_{k=L+1}^{M} \nabla \left( \pi_{PBL} \dot{v} \right)_{k} (\delta \sigma)_{k}, \tag{2.12d}
\]

which diagnose vertical velocity in the interfaces within the PBL.

In order to obtain the discrete forms of vertical fluxes of potential temperature and the hydrostatic equation, we first define \( \theta_{k+1/2} \) as the value of \( \theta \) interpolated at the interface between layers \( k \) and \( k+1 \),

\[
\theta_{k+1/2} = \left( \frac{\Pi_{k+1} - \Pi_{k}}{\Pi_{k+1} - \Pi_{k}} \right) \theta_{k+1} + \left( \frac{\Pi_{k+1} - \Pi_{k}}{\Pi_{k+1} - \Pi_{k}} \right) \theta_{k}, \tag{2.13}
\]

The values of the Exner function at the layer are computed from its values at the layers interfaces using

\[
\Pi_{k} = \frac{p_{k+1} \Pi_{k+1/2} - p_{k} \Pi_{k-1/2}}{R/c_{p} + 1} \left( \frac{p_{k+1/2} - p_{k-1/2}}{p_{k}} \right), \tag{2.14}
\]

with

\[
\Pi_{k+1/2} = c_{p} \left( \frac{p_{k+1/2}}{p_{o}} \right)^{g_{F_{p}}}, \quad \Pi_{k} = c_{p} \left( \frac{p_{k}}{p_{o}} \right)^{g_{F_{p}}} \tag{2.15}
\]

The discretization of the vertical advection of momentum follows Arakawa and Lamb (1977). For the upper-most PBL layer,

\[
\left\{ \frac{\partial v}{\partial t} \right\}_{L+1} = \frac{1}{2} \pi_{PBL} \frac{\partial p_{PBL}}{\partial (\delta \sigma)_{L+1}} \left[ \left( v_{L+2} - v_{L+1} \right) (\pi \dot{\sigma})_{L+1} \right]
+ \left( v_{L+1} - v_{B} \right) (\pi \dot{\sigma})_{B}. \tag{2.16a}
\]

This layer is bounded by the PBL top, where the variables can have discontinuities
or jumps. We identify the values of the quantities just above the PBL top with the subscript \( B+ \), and just below with the subscript \( B- \). The term \( v_B'(\pi\hat{\sigma})_B \) in (2.16a) is defined according to an upstream advection scheme. If \( E > 0 \), \( v_B'(\pi\hat{\sigma})_B = gE v_B+ - gM_B v_B- \), where \( v_B+ \) is the wind velocity linearly extrapolated from the two next layers above the PBL. Since \( M_B \), when non-zero, always implies a detrainment from the PBL; we take \( v_B- \) as \( v \) at the layer \( L+1 \). When \( E < 0 \), \( v_B'(\pi\hat{\sigma})_B = gE v_B- - gM_B v_B- \).

For the intermediate PBL layers,

\[
(\frac{\partial}{\partial \sigma})_k \frac{dv}{d\sigma} = \frac{1}{2\pi_B'(\delta\sigma)_k} \left[ (v_{k+1} - v_k)(\pi\hat{\sigma})_k \right] + (v_k - v_{k-1})(\pi\hat{\sigma})_{k-1}, \text{ for } k = L + 2, \ldots, M - 1
\]

(2.16b)

and for the lower-most PBL layer,

\[
(\frac{\partial}{\partial \sigma})_M \frac{dv}{d\sigma} = \frac{1}{2\pi_B'(\delta\sigma)_M} (v_M - v_{M-1})(\pi\hat{\sigma})_{M-\pi}.
\]

(2.16c)

The geopotential used in the momentum equation is computed with the discrete form of the hydrostatic equation,

\[
\Phi_k = \phi_{k+1} + (\Pi_{K+1} - \Pi_{k+1}) \theta_{k+1}
\]

\[
+ (\Pi_{k+1} - \Pi_k) \theta_k \text{ for } k = 1, \ldots, K_{max} - 2
\]

\[
\Phi_{k_{max} - 1} = \phi_3 + (\Pi_3 - \Pi_{k_{max} - 1}) \theta_{k_{max} - 1}.
\]

(2.17)

This discretization maintains the constraint of global total energy conservation (Arakawa Suarez 1983).

Inside the PBL, the pressure gradient force that appears in the momentum equation is computed in a way similar to that proposed by Konor and Arakawa (2001) for the hybrid vertical coordinate,

\[
-\nabla_p \Phi_k = -\nabla \Phi_k - \frac{1}{\pi} (\phi_{k+1} - \phi_k) \left[ \Phi_k - \phi_{k+1} \right] (v_{p,b})
\]

\[
- \frac{1}{\pi} (\phi_{k+1} - \phi_k) \left[ \phi_{k+1} - \phi_k \right] (v_{p,b})
\]

\[
\Phi_{k+1} - \phi_k \left[ \phi_{k+1} - \phi_k \right] (v_{p,b})
\]

(2.18)

Equations (2.19a,b,c) show the discretization of the vertical convergence of potential temperature within the PBL.

\[
\frac{\partial}{\partial \sigma} (\pi_B \theta)_{L+1} =
\]

\[
\frac{1}{(\delta \sigma)_L+1} \left[ \theta_{L+1}(\pi\hat{\sigma})_{L+1} - \theta_B(\pi\hat{\sigma})_B \right],
\]

(2.19a)

where \( \theta_B(\pi\hat{\sigma})_B \) is defined in the same way as \( v_B'(\pi\hat{\sigma})_B \).

\[
\frac{\partial}{\partial \sigma} (\pi_B \theta)_{k} =
\]

\[
\frac{1}{(\delta \sigma)_k} \left[ \theta_{k+1}(\pi\hat{\sigma})_{k+1} - \theta_{k+2}(\pi\hat{\sigma})_{k+1} \right] - \theta_{k+2}(\pi\hat{\sigma})_{k+1}, \text{ for } k = L + 2, \ldots, M - 1
\]

(2.19b)

and

\[
\frac{\partial}{\partial \sigma} (\pi_B \theta)_{M} =
\]

\[
- \frac{1}{(\delta \sigma)_M} \theta_{M+1}(\pi\hat{\sigma})_{M+1} - \theta_B(\pi\hat{\sigma})_B.
\]

(2.19c)

The convergence of the vertical moisture flux is discretized within the PBL by

\[
\frac{\partial}{\partial \sigma} (\pi\theta)_{L+1} =
\]

\[
\frac{1}{(\delta \sigma)_L+1} \left[ \theta_{L+1}(\pi\hat{\sigma})_{L+1} - \theta_B(\pi\hat{\sigma})_B \right],
\]

(2.20a)

\[
\frac{\partial}{\partial \sigma} (\pi\theta)_{k} =
\]

\[
\frac{1}{(\delta \sigma)_k} \left[ \theta_{k+1}(\pi\hat{\sigma})_{k+1} - \theta_{k+2}(\pi\hat{\sigma})_{k+1} \right] - \theta_{k+2}(\pi\hat{\sigma})_{k+1}, \text{ for } k = L + 2, \ldots, M - 1
\]

(2.20b)
and
\[
\left[ \frac{\partial}{\partial \sigma} (\pi \mathbf{\hat{o}}) \right]_M = -\frac{1}{\left( \delta \sigma \right)_M} r_{M-1} (\pi \mathbf{\hat{o}})_{M-1/2},
\]

(2.20c)

where \( r_B(\pi \mathbf{\hat{o}})_B \) is computed in the same way as \( v_B(\pi \sigma)_B \) and \( \theta_B(\pi \sigma)_B \). \( r_{k+1/2} \) is the value of \( r \) at the layer interfaces, interpolated from the adjacent layers, with
\[
r_{k+1/2} = \left( r_{k+1} + r_k \right) / 2.
\]
This interpolation formula is valid only within the PBL.

3. Parameterization of PBL processes

The PBL parameterization proposed here is based on Randall and Schubert (2004), in which the bulk (vertically integrated) PBL turbulent kinetic energy (TKE) is predicted and used for the computation of the surface fluxes of moisture, sensible heat and momentum. TKE is also used for an explicit formulation of the mass entrainment rate at the PBL top. Turbulent fluxes within the PBL are considered to be associated with eddies of two different typical length scales: convective eddies, with a length scale of the same order of magnitude as the PBL depth, and small-scale eddies, resulting from the three-dimensional turbulent cascade. In this work we take advantage of the multi-layer simulation of the PBL for computing explicitly the effects of both types of turbulent fluxes in the tendencies of the prognostic magnitudes.

The computation of turbulent fluxes associated with convective eddies assumes that the departure from well-mixed profiles of the thermodynamic variables, particularly moist static energy and water content, is small. Hence, turbulent fluxes associated with these eddies have uniform vertical convergence within the PBL. On the other hand, computation of small-scale eddies turbulent fluxes is based on a K-type formulation. Since small-scale eddies are affected by the large-scale turbulence, we will use a formulation of the local diffusion coefficients based on bulk properties, (non-local approach). The formulation of the diffusion coefficient follows the work by Holtslag and Boville (1993).

We assume that the PBL has a thin layer at the bottom with near-constant vertical turbulent fluxes, and may have a convectively unstable mixed layer above it. There are three basic regimes in the PBL functioning: a), the cloud-free, convectively unstable regime, in which TKE in the PBL is driven mainly from an unstable surface layer, b) the collapsed mixed layer regime, typical of cloud-free, night time over land, in which a cold earth surface results in a stable surface layer, and TKE is only generated by wind shear and locally dissipated near the surface, and c), cloud-topped regime, in which TKE is generated by convective instability due to radiative cooling near the PBL top (Lilly 1968). In the following, we discuss the computation of turbulent fluxes due both convective and small-scale eddies.

3.1 Turbulent fluxes and their contribution in the prognostic equations

If \( \psi \) is either \( v, r, \) or \( \theta \) or \( h \), the moist static energy defined by \( h = \Pi \theta + \Phi + L q \), with \( q = r \) for non-saturated air, and \( q = q^* = r - 1 \) for saturated air, the turbulent flux of \( \psi \), defined by
\[
F_{\psi} = -\rho \left( w' \psi' \right),
\]
where \( w' \) is the turbulent vertical velocity. Note that we define the turbulent fluxes as positive when downward, according to the direction of increasing \( \sigma \).

For \( h, r \) and \( \theta \) we consider \( \tilde{F}_{\psi} = \bar{F}_{\psi} + \tilde{F}_{\psi} \), where \( \tilde{F}_{\psi} \) is the flux due to convective eddies, and \( \bar{F}_{\psi} \) is the flux due to small-scale turbulent eddies. For \( \theta \), the turbulent fluxes due to both convective and small-scale eddies are computed from the respective turbulent fluxes of \( r \) and \( h \) in such a way that they depend on whether the air is unsaturated or saturated. For unsaturated air,
\[
F_{\theta} = \frac{1}{\Pi} \left[ F_h - LF_q \right],
\]
(3.1)

where \( L \) is the latent heat of water and \( F_q \) is equal to \( F_r \). For saturated air,
\[ F_\theta = F_\theta \left[ \Pi(1 + \gamma) \right] \quad \gamma = \frac{L}{c_p} \left[ (\partial q^*/\partial T)_p \right]. \]

(3.2)

The contribution of the turbulent fluxes to the tendencies of each prognostic variable \( \psi \) is

\[ G_{\psi_k} = -g \frac{F_{\psi_{k+1}^r} - F_{\psi_k^r}}{(\Delta \sigma)_k}, \quad k = L + 1, \ldots, M. \]

(3.3)

### 3.1.1 Turbulent Fluxes due to Convective Eddies

Fluxes of \( h \) and \( r \) are computed at the Earth's surface using a modified version of the bulk aerodynamic formulas proposed by Deardorff (1972) discussed below. At the top of the PBL, turbulent fluxes are computed taking into account the discontinuity or "jump" between the values of \( h \) and \( r \) at a level in the free atmosphere slightly above the PBL top, "B+" level, and those at a level slightly below the PBL top, "B" level. When we have turbulent mass entrainment \( E > 0 \), the turbulent fluxes of these variables at the PBL top are

\[
\begin{align*}
(F_r)_{B} &= E(f_{r_B} - r_{B-}) \quad \text{(3.4)} \\
(F_h)_{B} &= E(h_{B+} - h_{B-}) + (\Delta R)_h
\end{align*}
\]

where \((\Delta R)_h\) is the radiative cooling near the PBL top when the PBL is cloud toped. If \( E < 0 \), we ignore the terms affected by \( E \). At the internal interfaces of the layers, we compute the fluxes from linear interpolations in sigma between the values at the PBL top and the Earth's surface,

\[
\begin{align*}
(F_{\psi})_{m+1}^r &= \frac{\sigma_{m+1}^r - \sigma_B}{\sigma_S - \sigma_B} (F_{\psi})_S \\
&+ \frac{\sigma_S - \sigma_{m+1}^r}{\sigma_S - \sigma_B} (F_{\psi})_B, \quad \text{(3.5)}
\end{align*}
\]

where \( \psi \) is \( h \) or \( r \). For \( \theta \), the turbulent fluxes are computed from the fluxes of \( h \) and \( r \) or the flux of \( h \) according to whether the air is unsaturated or saturated as in (3.1) and (3.2).

### 3.1.2 Turbulent Fluxes due to Small-Scale Eddies

Let \( \psi \) be the horizontal velocity \( v \), the water mixing ratio \( r \) or the moist static energy \( h \). We compute the contribution of the diffusion due to small-scale eddies to the tendency of these variables as

\[
\frac{\partial \psi}{\partial t} = \left[ \text{other terms} \right] + \frac{1}{\rho} \frac{\partial F_{\psi}}{\partial z}, \quad \bar{F}_{\psi} = \rho K \frac{\partial \psi}{\partial z}, \quad \text{(3.6)}
\]

where \( K \) is the turbulent diffusion coefficient to be discussed later, and \( \rho K \frac{\partial \psi}{\partial z} \) is the turbulent flux of \( \psi \) due to small-scale eddies, \( \bar{F}_{\psi} \). The discrete form of (3.6) is

\[
\frac{\psi_k^{n+1} - \psi_k^n}{\Delta t} = \frac{1}{\rho_k \Delta z_k} (F_{\psi_{k+1}^r} - F_{\psi_{k-1}^r}), \quad k = L + 1, \ldots, M, \quad \text{(3.7)}
\]

where

\[
\bar{F}_{\psi_{k+1}^r} = \frac{\rho_k \psi_k K_{k+1}^r}{\Delta z_k} (\psi_k^{n+1} - \psi_k^{n-1}). \quad \text{(3.8)}
\]

Note that we use the time level \( n+1 \) for computing the fluxes of the right hand side of the discrete prognostic equations and, therefore, we have an implicit time scheme. We assume

\[
\bar{F}_{\psi_{\text{kinop}}} = 0, \quad \bar{F}_{\psi_{\text{max}}} = 0. \quad \text{(3.9)}
\]

Considering that the contribution to the tendencies is in the flux form, and using the restriction (3.9), it is possible to prove that mass weighted averages of the diffused quantities are conserved, that is,
\[
\sum_{k=\text{k trop}}^{k_{\text{max}-1}} \psi_k^n \rho_k \Delta z_k = \sum_{k=\text{k trop}}^{k_{\text{max}-1}} \psi_k^n \rho_k \Delta z_k.
\]

(3.10)

In this way we diffuse the momentum, the water mixing ratio, and the moist static energy. Potential temperature is diffused by computing the vertical convergence of its turbulent diffusion, which in turn is computed from the fluxes of h and r given by (3.1) and (3.2). Depending on whether air is unsaturated or saturated:

\[
\left( F_0 \right)_{k+1/2} = \frac{1}{\Pi_{k+1/2}} \left[ \left( F_h \right)_{k+1/2} - \text{L} \left( F_q \right)_{k+1/2} \right]
\]

(3.11a)

if the air is unsaturated, \( F_q = F_r \). If the air is saturated,

\[
\left( F_0 \right)_{k+1/2} = \frac{1}{\Pi_{k+1/2}} (1 + \gamma_{k+1/2}) \left( F_h \right)_{k+1/2}.
\]

(3.11b)

3.2 Bulk turbulent kinetic energy equation

We assume that TKE within the PBL is generated by the buoyancy and wind shear effects. The prognostic equation for the TKE is

\[
\frac{\partial e_{\text{PBL}}}{\partial t} = \frac{g}{(\delta p)_{\text{PBL}}} (B + S - D)
\]

\[-\frac{e_{\text{PBL}}}{(\delta p)_{\text{PBL}}} g + \frac{e_{\text{PBL}}}{\pi_{\text{PBL}}} \nabla \cdot (\pi_{\text{PBL}} v),
\]

(3.12)

where \( e_{\text{PBL}} \) is the TKE, \( B \) is the generation rate of TKE due to buoyancy, \( S \) is the generation rate due to shear, and \( D \) is the dissipation rate. The other terms that contribute to TKE tendency are the loss of TKE due to the entrainment of non-turbulent air form the free atmosphere (the second term of the right hand side), and the contribution of horizontal mass convergence (the last term of the right hand side). Let us define \( B \) as

\[
B = \int_{p_B}^{p_S} \frac{k F_{vs}}{p} \, dp,
\]

(3.13)

were \( k = \frac{R}{c_p} \), and \( F_{vs} \) is the turbulent flux of dry virtual static energy defined as

\[
F_{vs} = (w' s' \gamma), s_v = \Pi \theta_v + gz,
\]

with \( \theta_v = T \left( \frac{p}{p_0} \right)^{\kappa} \),

and \( T_v = T (1 + 0.608 q - 1) \)

(3.14)

In order to compute \( F_{vs} \) we consider whether the air is saturated or unsaturated. In a cloud free PBL, air is not saturated. In the case of a cloud-topped PBL, we define \( p_c \) as the pressure of the condensation level. Below the condensation level (\( p > p_c \)), the air is not saturated, while above \( (p < p_c) \) it is. If the air is not saturated, (in a cloud free PBL, or in the sub-cloud layer of a stratus covered PBL), we compute

\[
F_{vs} = \Pi F_\theta + 0.608 c_p T F_q,
\]

(3.15a)

where \( F_\theta \) and \( F_q \) are the turbulent fluxes associated with convective eddies discussed earlier. When the air is saturated (above the condensation level in a cloud-topped PBL),

\[
F_{sv} = \beta F_h - c_p T F_q, \beta = \frac{1 + 1.608 c_p T/L}{1 + \gamma}
\]

(3.15b)

Since we assume that \( F_h \) and \( F_r \) are linear in \( \alpha \) within the PBL, \( F_{sv} \) is also be linear in the case of the cloud-free PBL. In the case of the cloud-topped PBL, it is linear both below and above the condensation level although it is discontinuous at that level. For a cloud-free PBL, we use

\[
B = k \left( \frac{p - p_B (F_{sv} + F_{svb})}{p + p_B} \right),
\]

(3.16a)
where we have approximated the pressure in the integrand of (3.13) by its vertical mean value over the PBL. For the cloud-topped PBL,
\[
B = \kappa \left( \frac{(p_C - p_B)(F_{svC_+} + F_{svB})}{p_C + p_B} \right) + \kappa \left( \frac{(p_S - p_C)(F_{svS} + F_{svC_-})}{p_S + p_C} \right),
\]
(3.16b)

where \( F_{svC_+} \) is the flux of virtual dry static energy immediately above the condensation level computed (as \( F_{svB} \)) from (3.15b), while \( F_{svC_-} \) is the flux of virtual dry static energy immediately below the condensation level, computed (as \( F_{svS} \)) from (3.15a). We approximate the pressure that divides the integrand in equation (3.13) by its mean values at the cloud layer and the sub cloud layer. Note that in the case of a cloud-topped PBL, computation of \( B \) (and hence of \( F_v \)) includes the radiative cooling effects at the PBL top. When the PBL is not cloud topped, the main driving of the TKE buoyancy generation is the flux of \( F_v \), from the surface. A more detailed discussion can be found in Konor and Arakawa (2001).

The shear production of TKE is defined by
\[
S = \int_{z_s}^{z_t} F_v \frac{\partial v}{\partial z} \, dz.
\]
(3.17)

We compute this term from
\[
S = \alpha \rho S \| F v \parallel v_s + \frac{1}{2} \mathbf{E}(v_{B+} - v_{B-})^2,
\]
(3.18)

where \( F v \) is the flux of momentum at the earth surface, \( v_{B+} \) and \( v_{B-} \) are the velocities immediately above and bellow the PBL top, respectively. \( \alpha \) is a coefficient that tends to zero when the PBL thickness grows Here we assume that TKE generated near the Earth’s surface is locally dissipated so that the first term of the right hand of (3.18) is relevant only when the PBL is thin. The dissipation term is computed from
\[
D = C \rho_{PBL} (e_{PBL})^3,
\]
(3.19)

where the coefficient \( C \) is taken as 1.0.

### 3.3 Formulation of the surface fluxes

The surface fluxes of momentum, temperature and moisture are determined from an aerodynamic formula, which is modified version of that proposed by Deardorff (1972). Our formulation considers both the square root of the bulk TKE and the mean large-scale PBL velocity to determine the velocity scale. With this formulation, the surface fluxes are expected to be better estimated compared to the traditional methods, since the mean wind can be weak while the convective mixing is strong. The fluxes of momentum, temperature and moisture are computed as follows:
\[
F_u = \rho C_u C_T \left( u_m, \alpha_1 \sqrt{\frac{e_{PBL}}{T_{mean}}}, \theta_{mean} \right),
\]
\[
F_v = \rho C_v C_T \left( u_m, \alpha_2 \sqrt{\frac{e_{PBL}}{T_{mean}}}, \theta_{mean}, \theta_{G}, q_{G}, q_M, k \right),
\]
\[
F_q = \rho C_q C_T \left( u_m, \alpha_3 \sqrt{\frac{e_{PBL}}{T_{mean}}}, \theta_{mean}, \theta_{G}, q_{G}, q_M, k \right)
\]
(3.21)

where \( C_u \) and \( C_T \) are coefficients that depend on the bulk Richardson number, the PBL thickness and the surface roughness length, as in Deardorff (1972), \( \theta_g \) is the potential temperature at the Earth surface and \( q_g \) is the saturation moisture at the Earth’s surface temperature and pressure. \( k \) is a coefficient of water availability of the terrain. This coefficient is one in water surfaces, and close to zero in arid terrains. \( u_m \) is the module of the lower-most layer velocity. We are currently using \( \alpha_1 = 12.5 \), and \( \alpha_3 = 9.5 \).

### 3.4 Entrainment formulation

The entrainment formulas used in our parameterization are proposed by Randal and Schubert (2004). The formulas recognize whether the PBL is cloud-topped by stratus or not, and whether TKE is above the allowed minimum \( TKE_{\text{min}} \) or below.

If TKE is above the \( TKE_{\text{min}} \) value, which is taken as \( 0.01 \text{m}^3/\text{s}^2 \), we consider that there
is a positive turbulent mass entrainment E. If the PBL is cloud free, it is computed as

\[
E = \left( \frac{2kC}{1 - k} \right) \frac{\rho_{PBL} e_{PBL} \sqrt{e_{PBL}} - e_{min} \Pi_B \theta_{PBL}}{g(\Delta s_v)_B},
\]

(3.22)

where k and C are parameters, taken as 0.2 and 1.0, respectively, PB is the Exner function at the B level, \(\rho_{PBL}\) and \(\theta_{PBL}\) are the density and the potential temperature vertically averaged over the PBL and \((\Delta s_v)_B\) is the difference of the virtual static stability computed at the level B, minus the level B-

In the case of a cloud-topped PBL,

\[
E = \left[ b_1 \rho_{PBL} e_{PBL} \sqrt{e_{PBL}} - e_{min} \Pi_B \theta_{PBL} + b_2 \rho_{PBL} \Delta s_v - \Delta s_{v, crit} \right] / \left[ \Pi_B \theta_{PBL} + b_2 g(\Delta s_v - \Delta s_{v, crit})_B \right],
\]

(3.23)

where \(\rho_{PBL}\) and \(\theta_{PBL}\) are vertically averaged in the sub-cloud layer, \(b_1\) and \(b_2\) are parameters taken as 0.4 and 0.8, respectively, \(\beta\) is computed from (3.15b) for the B–level, and \((\Delta s_{v, crit})_B\) is defined by:

\[
(\Delta s_v)_{crit} = \left( \frac{L - 1.608 \Pi_B \theta_{B*} (q_T (P_b/P_b) - q_{B*})}{1 + \gamma_{B*}} \right)
\]

(3.24)

When the prognostic equation (3.12) for TKE forecasts a value lesser than TKE\(_{min}\), TKE\(_{min}\) is taken instead, and the PBL is considered to detrain mass at a constant rate, that corresponds to 250 hPa in 3 hours.

### 3.5 Computation of the small-scale eddies diffusion coefficient

For the computation of the diffusion coefficient due to small eddies, we use a scheme based in Holstlag and Boville, (1993), modified in order to use the bulk TKE.

In a cloud-free PBL, the diffusion coefficient for as magnitude \(\psi\) is

\[
K_{\psi} = w_h \frac{1 - \frac{z}{(\delta z)_{PBL}}}{2},
\]

(3.25)

where \(\alpha\) is a scale factor, \(k\) is the Von Karman constant, \(w_h\) is a scale velocity to be defined bellow, \(z\) is the height above the Earth’s surface of a generic PBL point, and \((\delta z)_{pBL}\) is the PBL total height thickness. Note that the formula is based on \(w_h\) and \((\delta z)_{pBL}\), which are non-local parameters. In the case of a cloud topped PBL, we consider this formula to be valid in the sub-cloud layer, while in the cloud layer \(K_{\psi}\) has a constant and large value (currently taken as 20 m\(^2\)/s).

For the computation of \(w_h\), we examine whether the PBL is convectively unstable or stable in terms of the sign of the turbulent flux of \(\theta_v\) in the Earth’s surface.

For unstable conditions:

If \(z < 0.1(\delta z)_{pBL}\): for moist static energy and water mixing ratio,

\[
w_h = \frac{u^*}{\phi_h}, \quad \phi_h = \left( 1 - 15 \frac{z}{L} \right)^{-\gamma}\]

(3.26a)

where \(u^*\) is the friction velocity, \(u^* = (F_{uv}/\rho_{PBL})^{1/2}\). For momentum (u and v),

\[
w_m = \frac{u^*}{\phi_m}, \quad \phi_m = \left( 1 - 15 \frac{z}{L} \right)^{-\gamma}\]

(3.26b)

for \(z > 0.1(\delta z)_{pBL}\)

\[
w_h = \sqrt{e_{PBL}}, \quad w_m = Pr \sqrt{e_{PBL}}
\]

(3.26b,c)

where \(Pr\) is the Prantl number, is a function of the ratio of the buoyancy to shear TKE production terms given by equations (3.13) and (3.17). This function is 1.0 for B/S=0 and 0.6 for B/S >=10.0, in between these two values of B/S we currently interpolate linearly.

For the stable case, if \(z < 0.1(\delta z)_{pBL}\)
\[ w_h = \frac{u_s}{\phi_h}, \quad \phi_h = \min\left(1 + 5 \frac{z}{L}, 5 + \frac{z}{L}\right) \]  
(3.27a)

and \( w_m = w_h \). If \( z > 0.1(\delta z)_{PBL} \).

\[ w_h = \beta \sqrt{e_{PBL}} \]  
(3.27b)

with

\[ \beta = \min\left(1, \frac{u_s}{z \phi_h \sqrt{e_{PBL}}}\right), \]

\[ \phi_h = \min\left(1 + 5 \frac{0.1(\delta z)_{PBL}}{L}, 5 + 1 \frac{0.1(\delta z)_{PBL}}{L}\right) \]  
(3.27c)

and again \( w_m = w_h \).

4. Results of a short climate simulation

In this section we present selected results from a simulation with prescribed SST, that was extended for two years. The resolution used was 5° in latitude by 4° degrees in longitude, there were used 14 vertical layers in the free atmosphere and 4 in the PBL. The simulation started with conditions representative of a November 15th.

We focus on those magnitudes that are specially affected by the new PBL parameterization; PBL thickness, stratus incidence and the net short wave radiation a latent heat fluxes at the earth surface.

Figure 2 shows the monthly average of PBL thickness in terms of pressure for the second January of the simulation, (2a) and for the second July. (2b).

In order to have some insight of the daily cycle of PBL thickness, Fig. 3a shows the PBL top height evolution, measured as pressure from the earth surface, of a point at 5°E, 22°N, (in the Sahara desert) and the potential temperature within the PBL during 72 hours of July. It is found a strong diurnal cycle, with a collapsed mixing layer after sunset. Fig. 3b also shows time evolution during 72 hours of July, for a point at 135°W, 30°N (representative of the region of marine stratus in front of California coast), the potential temperature within the PBL and the condensation level. The diurnal cycle is not clearly noticeable in this diagram. Note that the potential temperature is nearly constant when there is the PBL is not cloud topped or in the sub-cloud layer, this is consistent with moist static energy and moisture near vertically homogeneous. In the cloud layer, moisture reduces with height and hence potential temperature increases.
Figure 3. a: Simulated PBL height, as function of local time, at 22°N, 5°E, and potential temperature during 72 hours of July. PBL height is measured in terms of pressure form the earth surface. Contour interval for potential temperature is 2°K. B: same as a, but at 30°N, 135°W. Condensation level is shown as a dash line.

Now we focus our attention in the incidence of stratus clouds inside the PBL. Figure 4 shows the quarterly averages, for December-February, March-May, June-August and September-November, of the fraction of time with stratus incidence computed from observations by Klein and Hartman (1993), for four oceanic regions of relevant stratus incidence: a region in front of California Coast, a region around Canarias Islands, a region in front of Peruvian coast and a region in front of Namibian Coasts. Klein and Hartman analysis are shown as asterisks. The figure also shows, for each of the same regions, as a line, the simulated running three months averages, for our second year of simulations.

Figure 5 shows the global field of the stratus incidence for January (5a) and July (5b). Their patterns compares reasonably well with those reported by Klein and Hartman for December-February and June-August respectively.

Figure 6 shows the net downward short wave radiation for the second simulated January (6a), for the second simulated July (6b), and the same flux obtained from NASA SRB Analysis averaged for the Januarys.
from 1984 to 1991 (6c) and for the Julys from 1983 to 1990 (6d).

Fig. 6. a: Downward net short wave heat flux at surface, for the second simulated January. Contour interval is 30 Watts/m². b: Same as a, but for the second simulated July. c: Same as a, but from NASA SRB analysis, and for the Januarys averaged from 1984 to 1991. d: Same as c, but for Julys averaged from 1983 to 1990.

Figure 7 shows the upward latent heat flux the second simulated January (7a), for the second simulated July (7b), and the same flux obtained from COADS analysis averaged for the Januarys from 1979 to 1993 (7c) and for the Julys from 1979 to 1993 (7d).

Fig. 7. a: Latent heat flux, second simulated January. Contour interval is 30 Watts/m². b: Same as a, but for the second simulated July. c: Same as a, but from COADS analysis, and for the Januarys averaged from 1979 to 1993. d: Same as c, but for July. (c and b are from http://wwwt.emc.ncep.noaa.gov/gmb/noor/indest/COADS/lh/lh.htm)
5. Summary and Conclusions

We have presented a new parameterization of the PBL for use in general circulation models of the atmosphere. This new parameterizations maintains the advantages of the one presented in Suarez et al. (1983), specially the fact that a modified definition of the s coordinate makes the PBL top to be a coordinate surface, which allows for an explicit computation of the PBL top discontinuities and a more direct account of its effects. In the new PBL parameterization, the entrainment formulas are explicit and make use of the PBL TKE, they are based on the work by Randall et al. (2004). It also considers reviewed bulk aerodynamic formulas for the computation of the surface fluxes, that use both the mixed layer mean velocity and the TKE. The formulation in multiple layers allows for the existence of vertical shears of velocities, (not shown in this work).

The behavior of PBL thickness in terms of monthly means and diurnal cycle over land seems reasonable in the preliminary climatic simulation presented here. The incidence of stratus coverage, the short wave and latent heat fluxes in the earth surface may be considered as encouraging with respect of the potential of this model for being used in coupled simulations with OGCMs. Coupled simulations with regional POP OGCM, for the tropical Pacific basin, and with MIT global OGCM are in processes and aimed to be presented at the conference.

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