# ON THE SPATIAL SCALING OF A COMPLEX ADAPTIVE SYSTEM

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#### 1. Objectives

We intend to show that measured and modeled averages of latent heat flux over large areas can be scaled, if we know how the ensemble spatial variance of the independent variables depend on the spatial resolution. The question arises that one often has mean fields of temperature, water vapor concentration, and net radiation, which may or may not produce the correct ensemble average latent heat flux. The spatial ensemble average of a product may be expressed as:

$$\langle xy \rangle_{space} = \langle x \rangle \langle y \rangle_{space} + r\sigma_x \sigma_y$$

where r is the correlation coefficient, and  $\sigma$  is the spatial standard deviation. The above relationship may be applied to the independent variables of the Penman-Monteith equation.

As our starting point, we take the geophysical tutorial Daisyworld (Watson and Lovelock 1983), modified for moist, dark and dry, bright daisies, or fictional plant functional types. We then move on to a spatially explicit form of Daisyworld with real plant functional types, and examine its spatial scaling. This paper adds a new element to Daisyworld: land-surface biophysics in the radiative energy balance – we consider the additional effects of surface and aerodynamic resistance on the surface energy balance.

#### 2. Methods

The composition of grid cells is determined by who lives in the cell, the options being a plant type or bare ground. In the one-dimensional Daisyworld with moist and dry daisies, the surface energy balance is coupled to an equilibrium planetary boundary layer (PBL), which in turn feeds back to the surface energy balance. Stomatal resistance is prescribed. The equilibrium boundary layer is modeled according to:

$$\frac{dZ_i}{dt} = \frac{H_v}{Z_i \rho c_p \gamma}$$

where  $Z_i$  is the PBL depth,  $H_v$  is a virtual surface heat flux (a linear combination of the latent and sensible heat fluxes),  $\rho$  is the density of air,  $c_p$  is the heat capacity of air, and  $\gamma$  is the psychometric constant.

Competition between daisies in the onedimensional Daisyworld is modeled using Watson and Lovelock's (1983) original coupled differential equations:

$$\frac{dn_{dark}}{dt} = n_{dark} \left( n_{bare} * \beta - n_{dark} * P_{death} \right)$$

where  $n_{dark}$  and  $n_{bare}$  are the total area of dark daisies and bare ground, respectively,  $\beta$  is the areal growth rate of daisies, and  $P_{death}$  is the probability of mortality.

Spatial competition between plant functional types is modeled using a cellular automaton, which is discretized in time, space and state (van Bloh et al. 1999). The scheme is based on an eight-cell Queen's neighborhood (as opposed to a four-cell Rook's neighborhood). Given bare ground, the probability of colonization in a uniform neighborhood is:

$$p_{decid} \left( n_{ever} = 0 \middle| n_{decid} \neq 0 \right) = 1 - (1 - \beta)^{n_{decid}}$$

where  $p_{type}$  refers to the probability of colonization by a functional type, n refers to the number of neighbors, and  $\beta$  refers to the area growth rate. Among unlike neighbors, the general probability P is composed as:

$$P_{decid}\left(n_{decid}, n_{ever} \neq 0\right) = \frac{p_{decid}}{\sum_{i=type} p_i}$$

An essential part of Daisyworld is the parabolic response of photosynthesis to temperature.

## 3. Results and Discussion

The zero-dimensional mean-field model, coupled to an equilibrium PBL, yielded trade-offs of colonized ecosystem area with respect to stomatal resistance (Figure 1). This result shows that plants which "live fast and die young" with respect to stomatal resistance win competitively against their more conservative neighbors. This is exactly the situation at a Fluxnet site in the Mediterranean ecosystem of lone, California (38:25:54 North,120:57:58 West), where spendthrift *Quercus douglassi* outcompete their evergreen neighbors. Deciduous oaks at lone have pronounced seasonality in photosynthetic capacity, which correlates with stomatal conductance.

The spatially explicit model demonstrates what occurs when an explicit feedforward effect is programmed into the life-cycle of plants. In an instructive accident, the first author temporarily dealt with the uncertainty in the plant death rate by linking it to the plant growth rate, resulting in equilibrium between the growth rate and the death rate. This resulted in a feedforward effect in which the excessive death rate randomized the spatial distribution of plants (Figure 2). The results are less clumped than one would expect from this model, and do not compare well with reality.

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Figure 1. Trade-offs in colonized areal extent between moist dark, and dry bright plant functional types.



Figure 2. Occupancy of deciduous (white) and evergreen (black), with equilibrium between the growth rate and the death rate.

The fact that the climatological forcing in this version of the model lacked stochasticity also worked to determine the patterns of spatial organization (Fernandez-Illescas and Rodrigues-Iturbe 2004).

To assess how well our results correspond to reality, we plotted the common log of the spatial variance against the common log of the spatial resolution. The result is similar to a power spectrum. However, the total variance has individual components which beg to be understood:

$$z(i) = f(i) + s(i) + \varepsilon$$

where z(i) is the state variable as a function of location, f(i) is a secular trend in space, s(i) is the contribution of local spatial autocorrelation, and  $\varepsilon$  is the contribution of aleatory uncertainty. From this perspective, the slopes in Figure 3 may represent the contributions of longwave stochasticity. Stochasticity does not only occur at local spatial scales; it also occurs at large spatial scales.

Our results show more longwave variance than a Poisson distribution. Our results also show "windows" of scaling for which multiple slopes are indicated. In comparison, the regression between the variance and the resolution of NDVI for the oak savanna field site yielded exponents of  $\sim$  -0.4.



Log<sub>10</sub> of the Spatial Resolution

Figure 3. Spatial scaling of Daisyworld.

In future work, stochasticity in climatological forcing will influence the spatial scaling of oak-savanna ecosystems (Fernandez-Illescas and Rodrigues-Iturbe 2004). Drought-induced mortality will be modeled after Martinez-Vilalta et al. (2002). The surface energy balance and photosynthetic temperature response functions will both be corrected for nonlinearities in surface temperature.

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