5.7 PARAMETERIZATION OF MOMENTUM FLUXES IN A PBL MASS-FLUX MODEL

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1. INTRODUCTION

In 2001, we (Lappen and Randall, 2001a-c; hereafter, LR-a,b,c), presented a "higher-order mass-flux model" called ADHOC, which represents the PBL's large eddies in terms of an assumed joint distribution of the vertical velocity and scalars such as potential temperature or water vapor mixing ratio. ADHOC uses the equations of higher-order closure to predict selected moments of the assumed distribution, and diagnoses the parameters of the distribution from the predicted moments. Once the parameters of the distribution are known, all moments of interest can be computed. This version of ADHOC was incomplete, in that the horizontal velocity components and the "pressure terms" involving covariances between pressure and other variables were not incorporated into the "assumed distribution" framework. Instead, the vertical flux of horizontal momentum and the pressure terms were parameterized using standard methods.

This talk will describe an updated version of ADHOC (ADHOC2) that includes consistent representations of the momentum fluxes and pressure terms. We assume idealized geometries for the PBL's coherent structures,, consistent with the mass-flux framework. This means that we move beyond assumed *probability* distributions, and towards assumed *spatial* distributions. In particular, we consider idealized versions of two commonly occurring coherent structures, namely unsheared plumes (cylindrical geometry with the cylinder's axis perpendicular to the ground; Fig. 1) and sheared rolls (homogeneity in one horizontal direction; Fig. 4). We use the assumed geometries to derive velocity fields. Covariances such as momentum fluxes are then constructed directly, by spatial integration. The expressions that we obtain for these higher moments contain unknown parameters related to the geometry of the circulations. These include the radii of the updraft and downdraft for the unsheared plume case, and the tilt, orientation angle, and cross-roll width of the roll circulation. We provide a method for diagnosing these parameters using quantities that are available in ADHOC2. To our knowledge, this is the first time that a PBL parameterization has been used to diagnose such parameters. Tests of the new parameterization show encouraging agreement with statistics computed from large-eddy simulations.

2. MOMENTUM FLUXES

The mass-flux approach has been used to parameterize momentum transports by deep cumulus convection (e.g., Wu and Yanai, 1994), but with little in the way of supporting tests. To our knowledge, the only study that had investigated the use of standard mass-flux formulae to represent momentum fluxes in a PBL model is that of Brown (1999). In his shallow-cumulus study, he found that the representation of momentum fluxes with an assumed-joint distribution was poor compared with using the same approach for scalar fluxes. Here, we have better luck using an assumed *spatial* distribution as discussed below.

2.1 Axisymmetric free convection

Consider an ensemble of axisymmetric convective plumes in the absence of a mean flow. Obviously in this case there is no vertical flux of horizontal momentum. Plumes in free convective PBLs have been extensively investigated using both observations (e.g., Willis and Deardorff, 1974) and LES (e.g., Schumann and Moeng, 1991).

To analyze the circulation associated with a plume, we adopt cylindrical coordinates, with radial coordinate r. In , the inner cylinder of radius $r = R_i(z)$ (the subscript stands for "inner") represents a convective "draft[1]" across which the vertical velocity is horizontally uniform, while the annulus between the inner and outer cylinders represents the compensating draft of the opposite sign. The radius of the outer cylinder, i.e., the total diameter of the plume, is denoted by R_o, where the subscript stands for "outer". This is depicted in Fig. (1). In this case, we will define the inner (outer) draft to be the updraft (downdraft). The fractional area occupied by the updraft (inner draft) is

$$\sigma_i(z) = \left[\frac{R_i(z)}{R_o}\right]^2 . \tag{1}$$

We assume that the vertical velocity and thermodynamic variables are horizontally uniform within the inner cylinder and the surrounding annulus, but in general they are discontinuous across $r = R_i(z)$. The radial velocity and pressure must vary radially, as discussed below.

We also assume that R_o is independent of height and time, and that the plumes are "closely packed." We use the mass-flux quantities of vertical velocity, along with the continuity equation to work out the radial dependence of the radial velocity component. We assume that there is no velocity component in the azimuthal direction. We also derive the boundary conditions that apply across $r = R_i(z)$. Once we determine the radial velocities, we work out expressions for the momentum fluxes. In addition, we propose a method to determine the height-independent radius of the plume, R_o.

Over the updraft, we can use $w = w_i$ and radially integrate the anaelastic continuity equation,

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1. Here "draft" can refer to either an updraft or a downdraft.

$$\frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{1}{\rho_0}\frac{\partial}{\partial z}(\rho_0 w) = 0\,, \tag{2}$$

to get an expression for the radially velocity as a function of r. This gives

$$v_r(r) = -\frac{r}{2\rho_0}\frac{\partial}{\partial z}(\rho_0 w_i) \text{ for } r < R_i\,. \tag{3}$$

We can do the same thing over the downdraft (using $w = w_o$) and get

$$r v_r(r) = \frac{1}{\rho_0}\frac{\partial}{\partial z}(\rho_0 w_o)\left(\frac{R_o^2}{2} - \frac{r^2}{2}\right) \text{ for } R_i < r \le R_o\,. \tag{4}$$

Here we have used the boundary conditions

$$v_r(R_o) = 0 \text{ and } v_r(0) = 0\,. \tag{5}$$

Suppose that $w_i(z)$, $w_o(z)$, $R_i(z)$, and R_o were known. Then, as outlined above, $v_r(r, z)$ could be determined from the continuity equation. The perturbation pressure field could then be determined, using methods described in the next Section. ADHOC gives us values of $w_i(z)$, $w_o(z)$, and the updraft area fraction, $\sigma(z)$. If we know either $R_i(z)$ or R_o, we can determine the other using Eq. 1. The problem is that both $R_i(z)$ and R_o are actually unknown. Next, we present a method to determine R_o, *keeping in mind that it must be independent of height*.

Our approach starts from the observation that the large-eddy kinetic energy per unit mass in the horizontal part of the motion, $e_H = 0.5 v_r^2$, will tend to increase as R_o increases. This ideas suggests that we can determine R_o from e_H. We write

$$e_H = \frac{1}{\pi R_o^2}\int_0^{2\pi}\left(\int_0^{R_o}\frac{1}{2}v_r^2 r\,dr\right)d\phi = \frac{1}{R_o^2}\left(\int_0^{R_o} v_r^2 r\,dr\right). \tag{6}$$

Substituting the solution for $v_r(r)$ derived above, and using (1), we find that

$$e_H = R_o^2\left\{\left[\frac{1}{4\rho_0}\frac{\partial}{\partial z}(\rho_0 w_i)\right]^2\sigma^2\right\}$$
$$+\left[\frac{1}{2\rho_0}\frac{\partial}{\partial z}(\rho_0 w_o)\right]^2\left[-\frac{1}{2}\ln(\sigma) - (1-\sigma) + \frac{1}{4}(1-\sigma^2)\right]\,. \tag{7}$$

ADHOC2 determines $\sigma(z)$ using the methods of LR, and it also determines $(e_H)_M$, which is the vertical average of e_H through the depth of the PBL. Using the (height-independent) value of $(e_H)_M$, we can diagnose the (height-independent) diameter of the plumes, R_o. We can then use 1 to diagnose $R_i(z)$. We can then solve for the two-dimensional distribution of the radial velocity using Eqs. 3-4. The kinematic structure of the plume is thus fully determined.

Using the LES model of Khairoutdinov and Randall (2003), we performed a simulation of Wangara day 33, which was a clear-convective day. To test the parameterized expressions for $v_r(r, z)$, we vertically integrate Eq. 7 and use the LES results to diagnose σ, e_H, $\frac{\partial w_i}{\partial z}$, and $\frac{\partial w_o}{\partial z}$ to obtain $R_i(z)$ and a height-independent value of R_o as described above. This method yields $R_o = 1208$ m.

We used our numerical results in Eqs. 3-4, with $R_o = 1208$ m, to determine the distribution of the radial velocity. The resulting radial and vertical velocities are denoted by the arrows plotted in Fig. (2). The longest arrows in the plot represent a particle speed of approximately 2 m s^{-1}. The dashed lines in the figure represent the height-dependent updraft-downdraft boundaries. The diagnosed radial velocity field shows convergence down low and divergence up high. Due to the fact that R_i is a function of z, we see an expected jump across the updraft-downdraft edge.

The results of the simulation were then used to test our formula for e_H (Eq. 7), to see how sensitive it is to R_o. We use the LES value of $\sigma(z)$ in Eq. 1, to determine $R_i(z)$ for different values of R_o. We then use the LES values for $w_i(z)$, and $w_o(z)$ in Eq. 7 and calculate e_H. In Fig. (3), the results are compared with the profile of e_H as diagnosed from the LES results. The best overall agreement near the surface and the top of the PBL occurs for $R_o = 900$ m, while the best agreement near the mid-level of the PBL is for $R_o = 1300$ m. The agreement with the diagnostic estimates of R_o given above is encouraging.

Finally, we note that the vertical momentum flux ($\overline{w'v'_r}$) can be determined using a formula analogous to Eq. 6.

2.2 Rolls

Next, consider idealized “roll” circulations, which are horizontally uniform in one direction. Our approach that is broadly similar to that used, in the preceding section, to analyze plumes. Key differences are that rolls are expected to occur in the presence of significant shear of the horizontal wind, and they are expected to transport horizontal momentum vertically. We simulated the roll case of Glendening (1996; G96) with the LES model described by Khairoutdinov and Randall (2003). All results in this Section are compared with this LES run.

To represent rolls, we adopt Cartesian coordinates and assume alternating updrafts and downdrafts, aligned at an angle η from the y-axis (Fig. 4).

Let $x = x_0(z)$ denote the boundary between one particular downdraft and one particular updraft, with the updraft on the side of the larger values of x (Fig. 4). The “opposite” wall of the updraft is at $x = x_0(z) + L_u(z) \equiv x_1(z)$, so that the updraft occupies the region $x_0(z) < x < x_1(z)$. A neighboring downdraft occupies the region $x_1(z) < x < x_0(z) + L = x_1(z) + L_d(z) \equiv x_2$, where L is the total width of the roll, i.e.,

$$L = L_u + L_d\,, \tag{8}$$

and $L_d(z)$ is the width of the downdraft. The fractional area occupied by the updraft is

$$\sigma(z) = \frac{L_u(z)}{L} = \frac{x_1(z) - x_0(z)}{L}. \qquad (9)$$

We assume that

$$\frac{\partial L}{\partial z} = 0. \qquad (10)$$

Here again we integrate the continuity equation to obtain the horizontal velocities $u(x)$ seperately in the updraft and downdraft regions ($w = w_u$ and $w = w_d$ respectively). This results in

$$u(x) = u(x_0 + \varepsilon) - \left(\frac{x - x_0}{\rho_0}\right)\frac{\partial}{\partial z}(\rho_0 w_u) \text{ for } x_0 < x < x_1 \qquad (11)$$

and

$$u(x) = u(x_0 + \varepsilon) - \frac{L_u}{\rho_0}\frac{\partial}{\partial z}(\rho_0 w_u) - (w_u - w_d)\frac{\partial x_1}{\partial z} - \left(\frac{x - x_1}{\rho_0}\right)\frac{\partial}{\partial z}(\rho_0 w_d) \quad \text{for } x_1 < x < x_2. \qquad (12)$$

To obtain 11-12, we used 9 and the fact that the mass flow rates are continuous across $x = x_0$ and $x = x_1$.

The next step is to work out u', the departure of u from its horizontal average, $\bar{u}$, which can be determined by the integral

$$\bar{u} = \frac{1}{L}\int_{x_0}^{x_0 + L} u\, dx = \frac{1}{L}\left(\int_{x_0}^{x_0 + L_u} u\, dx + \int_{x_1}^{x_1 + L_d} u\, dx\right). \qquad (13)$$

We substitute Eqs. 11-12 into Eq. 13 and subtract the result from Eqs. 11-12 to get

$$\begin{aligned} u' &= \frac{L_d}{L}\left(\frac{L_u}{2}\right)\frac{1}{\rho_0}\frac{\partial}{\partial z}[\rho_0(w_u - w_d)] \\ &+ \frac{(w_u - w_d)}{2}\left[\left(\frac{\partial x_0}{\partial z}\right) - \left(\frac{\partial x_1}{\partial z}\right)\left(\frac{L_u - L_d}{L}\right)\right] \\ &+ \left(\frac{x - x_0}{L}\right)\left\{-L_d\frac{1}{\rho_0}\frac{\partial}{\partial z}[\rho_0(w_u - w_d)] + (w_u - w_d)\left(\frac{\partial x_1}{\partial z} - \frac{\partial x_0}{\partial z}\right)\right\} \\ &\text{for } x_0 < x < x_1 \end{aligned} \qquad (14)$$

and

$$\begin{aligned} u' &= -\left(\frac{L_u}{L}\right)\left(\frac{L_d}{2}\right)\frac{1}{\rho_0}\frac{\partial}{\partial z}[\rho_0(w_u - w_d)] \\ &+ (w_u - w_d)\left[\left(\frac{1}{2} - \frac{L_u}{L}\right)\left(\frac{\partial x_0}{\partial z}\right) - \frac{1}{2}\left(\frac{\partial x_1}{\partial z}\right)\right] \\ &+ \left(\frac{x - x_1}{L}\right)\left\{L_u\frac{1}{\rho_0}\frac{\partial}{\partial z}[\rho_0(w_u - w_d)] + (w_u - w_d)\left(\frac{\partial x_1}{\partial z} - \frac{\partial x_0}{\partial z}\right)\right\} \\ &\text{for } x_1 < x < x_2. \end{aligned} \qquad (15)$$

The vertical flux of u momentum can then be determined by Eqs. 14-15 and

$$\begin{aligned} \rho_0\overline{w'u'} &= \frac{\rho_0}{L}\left(\int_{x_0}^{x_0 + L_u} w_u u'\, dx + \int_{x_1}^{x_1 + L_d} w_d u' dx\right) \\ &= \rho_0(w_u - w_d)^2\left(\frac{L_u L_d}{2L^2}\right)\left[\left(\frac{\partial x_0}{\partial z}\right) + \left(\frac{\partial x_1}{\partial z}\right)\right] \\ &= \rho_0(w_u - w_d)^2\sigma(1 - \sigma)\lambda \quad , \end{aligned} \qquad (16)$$

A similar process can be done to determine the variance of the u momentum. According to 16, the momentum flux is different from zero only when the *tilt* of the updrafts and downdrafts, defined by

$$\lambda \equiv \frac{1}{2}\left[\left(\frac{\partial x_0}{\partial z}\right) + \left(\frac{\partial x_1}{\partial z}\right)\right], \qquad (17)$$

is different from zero. Using mass-flux formulas, we can write 16 as

$$\overline{w'u'} = (\overline{w'w'})\lambda. \qquad (18)$$

In model that predicts both $\overline{w'u'}$ and $\overline{w'w'}$ as functions of height, we can use 18 to diagnose the tilt. From 9, we see that

$$\frac{\partial x_1}{\partial z} - \frac{\partial x_0}{\partial z} = L\frac{\partial \sigma}{\partial z}. \qquad (19)$$

Using 18-19, we obtain

$$\frac{\partial x_0}{\partial z} = \frac{\overline{w'u'}}{\overline{w'w'}} - \frac{L}{2}\frac{\partial \sigma}{\partial z}, \qquad (20)$$

and

$$\frac{\partial x_1}{\partial z} = \frac{\overline{w'u'}}{\overline{w'w'}} + \frac{L}{2}\frac{\partial \sigma}{\partial z}. \qquad (21)$$

If L, $\sigma(z)$, $\overline{w'u'}$ and $\overline{w'w'}$ are known, we can diagnose $\partial x_0/\partial z$ and $\partial x_1/\partial z$ from Eqs. (20) and (21). ADHOC2 predicts the latter 3 quantities and we have developed a method to determine L (using a method similar to that used to diagnose R_o in the clear convective case; see Lappen and Randall, 2004).

Using this value of L and the LES values of $\overline{w'u'}$, $\overline{w'w'}$, and $\sigma(z)$, we plot the tilt in Fig. (5). Here, $\partial x_0/\partial z$ is the tilt of the wall to the left side of the updraft, while $\partial x_1/\partial z$ is the tilt of wall on the right side of the updraft (see Fig. 4). The tilt ranges between 0% and 10% throughout the PBL. Plots of observed and numerically simulated rolls show this to be a reasonable number (G96).

At this point, we have a complete picture of the roll including tilt, and circulation. We can use Eqs. (11)-(12) along with LES updraft and downdraft vertical velocities to get the total wind vector. Figure 5 shows the diagnosed roll structure.

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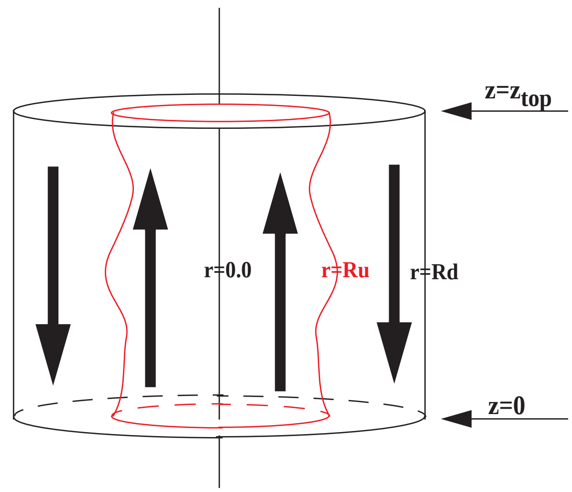

Figure 1: *Idealization of the clear convective geometry. The inner and outer cylinders are concentric circles. R_u (R_d) is the distance from the updraft center to the outer edge of the updraft (downdraft). Note that R_u to varies with height.*

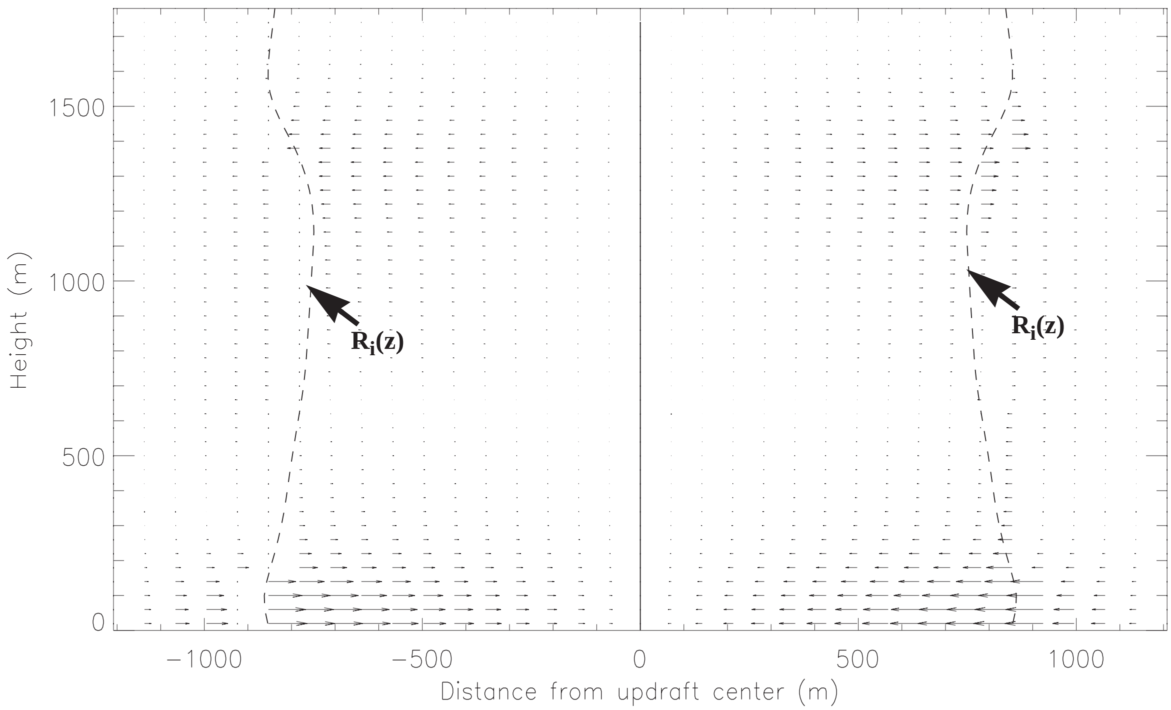

Figure 2: *Parameterized radial velocity obtained for the Wangara case. The solid line is the updraft center, while the dashed lines represent the updraft-downdraft border. The longest arrows shown (near the bottom) are approximately 2.0 m/s.*

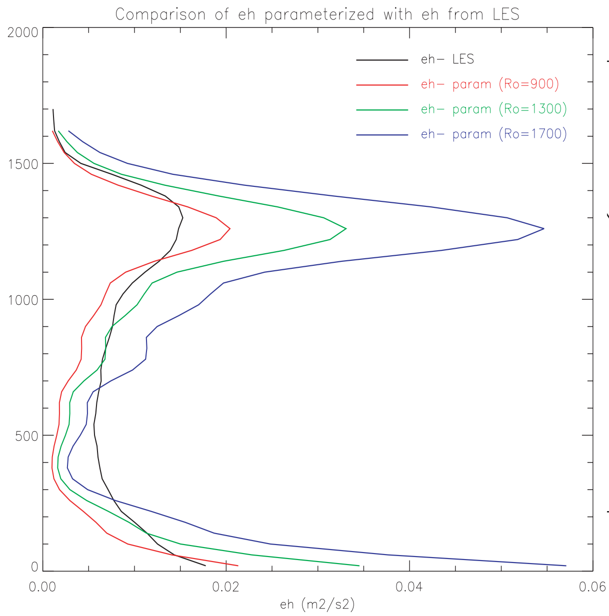

Figure 3: *Comparison of the LES and parameterized (Eq. 7) horizontal TKE for different values of* R_o.

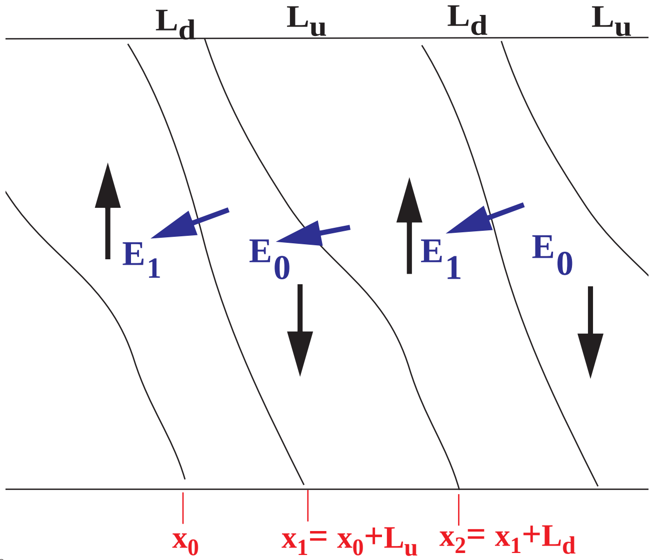

Figure 4: *Cross-sections through the roll: (a) A vertical cross section-- L_u and L_d are the widths of the updraft and downdraft r; E_0 and E_1 represent mass entrained across a draft edge.*

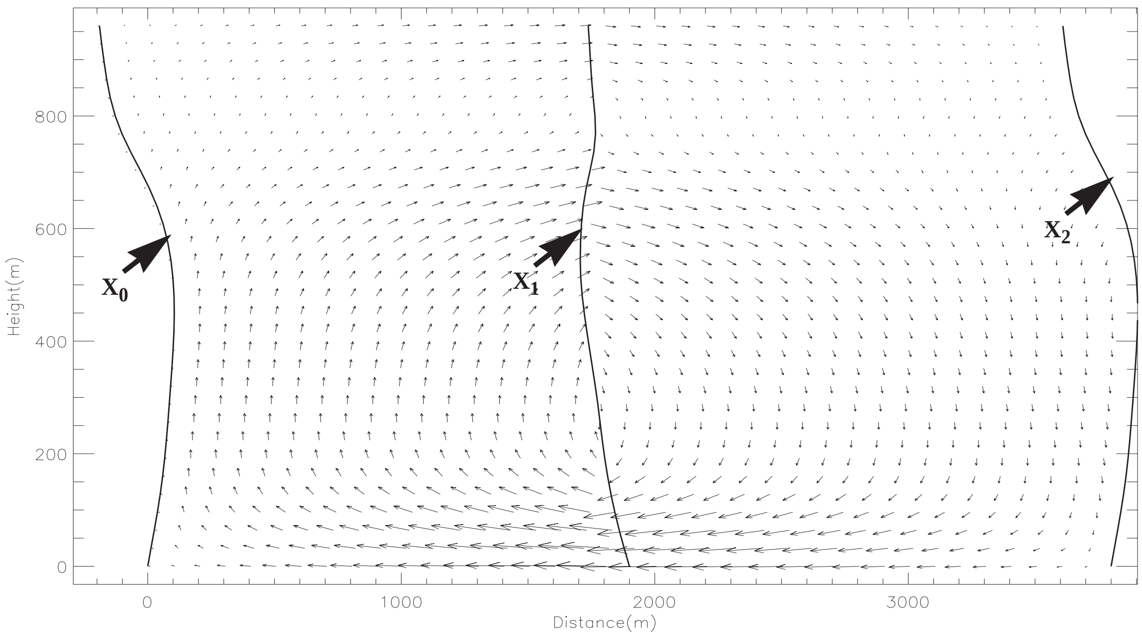

Figure 5: *Picture of the parameterized roll and its parameterized circulation.*