# STUDIES ON CALCULATING CONVECTIVE ENERGY WITH DIFFERENT MOIST ADIABATIC PROCESSES

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## 1 INTRODUCTION

Two kinds of convective activities—dry convection and moist convection—are often mentioned when referring to severe storm forecasting. Usually dry convection only serves as a kind of adjustment or a lifting condition. Moist convection, however, can release large amounts of latent heat, thus giving it the ability of self-organization and self-development, so it can be fully developed and be maintained for a long life cycle. Thus, moist convection plays a more important role than dry convection, and severe local storms are always associated with deep-moist convection (McNulty, 1995; Schultz et al., 2000).

Moist convection is the consequence of atmospheric instability, various kinds of indices have been developed to denote this instability, and convective available potential energy (CAPE) is the most popular one in these years. CAPE theoretically represents the possible intensity that the convection may reach. It has been becoming widely used to denote the instability of atmosphere, and used in derived parameters such as bulk Richardson number (BRN; Weisman and Klemp, 1982), energy helicity index (EHI; Hart and Korotky 1991), and vorticity generation parameter (VGP; Rasmussen and Blanchard 1998). The calculation of convective energy is closely linked with moist adiabatic processes, and proper selections of the latter should be made in order to compute the former accurately and reflect these parameters in physical implications as well.

## 2 MOIST ADIABATIC PROCESSES: CALCULATIONS AND COMPARISONS

There are four general methods to deal with moist convection. The first is based on the static energy conservation. The static energy for a saturate air parcel can be expressed as:

$$E_t = C_{pd}T + L_v w_s + \varphi \qquad (1)$$

Where $C_{pd}$ is the heat capacity of dry air at constant pressure, $T$ is the absolute temperature, $L_v$ is the latent heat of condensation, $w_s$ is the mixing ratio for saturated atmosphere, $\varphi = gz$, is the geopotential height.

The second method is based on pseudo equivalent potential temperature conservation. It is popularly used nowadays, especially in China and the U.S. The express of pseudo-equivalent potential temperature is

$$\theta_e = T\left(\frac{1000}{P_d}\right)^{R_d/C_{pd}} \exp\left(\frac{L_v w_s}{C_{pd} T}\right), \qquad (2)$$

where $P_d$ is partial pressure of dry air, $R_d$ is the gas constant for dry air.

The third one, now admitted by the World Meteorology Organization, is the strict pseudo adiabatic equation:

$$(C_{pd} + C_l w_s) d\ln T - R_d d\ln P_d + d\left(\frac{L_v w_s}{T}\right) = 0, \qquad (3)$$

where $C_l$ is the specific heat for water vapor.

The last method to tackle with moist adiabatic process is reversible adiabatic process, which can be expressed as

$$C_{pd} d\ln T - R_d d\ln P_d + d\left(\frac{w_s L_v}{T}\right) + (C_l w_v + C_l w_l) d\ln T = 0 \qquad (4)$$

where $w_l$ is the specific content of liquid water and ice.

Among the four methods, condensed liquid water and ice are not considered in the lifting parcel except the last method. Eq. (2) can be derived from Eq. (3) with an assumption $C_{pd} + C_l w_s \approx C_{pd}$.

The essence of resolving the moist adiabatic processes lies in finding a series of temperature values linked with a series of pressure. Solution is somewhat difficult because of the complicated form of $w_s$. Iteration or dichotomy may be used to solve Eq. (1) to Eq. (4). For the last method, the evaluation of $w_l$ is difficult. One good but somewhat extreme method is to assume that all the condensed liquid water be kept in the parcel.

There exist some differences between the above mentioned moist adiabatic processes. In order to distinguish these differences, further explorations should be made. Emagram is employed here to display these differences. 900 mb is chosen as the lifting level

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and eleven different lifting temperatures (from –20℃ to 30℃ with an interval of 5℃) are selected. Furthermore, in order to simplify the calculations and comparisons, air parcels are assumed saturated there.

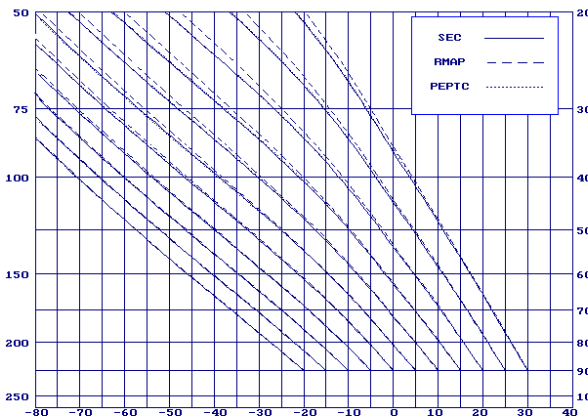


Fig.1 Intercomparisons of different methods for moist adiabatic processes: pseudo equivalent potential temperature conservation lines (dotted), static energy conservation lines (solid) and reversible moist adiabatic lines (dashed). Notice that the first two sets of lines superpose each other and cannot be easily distinguished. The ordinate is logarithmic pressure in mb. The abscissa is temperature in Celsius degree.

Figure 1 shows three sets of moist adiabatic lines corresponding to pseudo equivalent potential temperature conservation, static energy conservation, and the reversible moist adiabatic process, respectively. It can be seen that the moist adiabatic lines corresponding to the pseudo equivalent potential temperature conservation are very close to or even superpose those lines related to static energy conservation. This is to be expected since latent heats are both considered while specific heat of water vapor are both ignored in these two processes. In fact, static energy conservation and pseudo equivalent potential temperature conservation are actually different measures for nearly the same physical process.

However, things become quite different with respect to the reversible moist adiabatic lines. Although they are very close to the pseudo-equivalent potential temperature lines with respect to low lifting temperatures, obvious differences can be seen to high lifting temperatures, especially at the high levels. This can be explained by larger sensible heats accumulation due to more liquid water and relatively large temperature variations of the lifting parcel. Additionally, solidification of liquid water is ignored in reversible adiabatic process here.

Figure 2 shows three sets of moist adiabatic lines corresponding to reversible adiabatic process, pseudo equivalent potential temperature conservation and strict pseudo adiabatic equation, respectively. We can see the strict pseudo adiabatic lines lie between pseudo equivalent potential temperature lines and reversible moist adiabatic ones, just as they are expected. The temperature lapse rates of reversible adiabatic process are smaller than other moist processes for high lifting temperatures, and the lapse rates of strict pseudo adiabatic process are slightly smaller than pseudo equivalent potential temperature process. When the lifting temperatures are low, the strict pseudo adiabatic process lines are very close to pseudo equivalent potential temperature lines. Moreover, we can see the differences between reversible adiabatic lines and strict pseudo adiabatic lines are larger than that between pseudo equivalent potential temperature lines and strict pseudo adiabatic lines. Thus, pseudo equivalent potential temperature conservation is a good approximation to strict adiabatic process comparing to reversible adiabatic process.

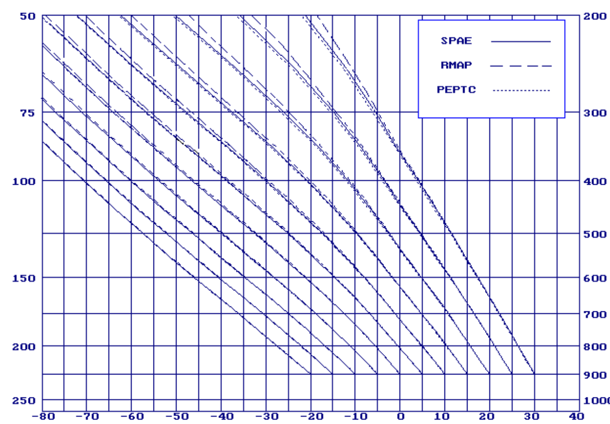


Fig. 2 As in Fig.1, but for reversible adiabatic lines (dashed), pseudo equivalent potential temperature lines (dotted) and the strict pseudo adiabatic lines (solid).

Figure 3 presents the two kinds of reversible adiabatic process, one is calculated according to ordinary reversible adiabatic process, the other is calculated according to reversible adiabatic process with liquid water solidification. In order to be acceptable, the liquid water is assumed to start solidifying below 0℃, and become totally frozen when temperature reaches –20℃. To simplify the calculation, the solidifying process is assumed linear. We can see the lapse rates are obviously smaller when considering solidification, especially to high lifting temperatures. It can be explained by large amount of heat release due to sufficient liquid water solidification. For the same reason, we can see the suddenly decrease of lapse rates with respect to solidification of liquid water between 0℃ and –20℃. This result may be invoked to interpret the behavior of a cumulonimbus whose growth suddenly increases shortly after the first traces of glaciation in its summits (Williams and Renno, 1993).

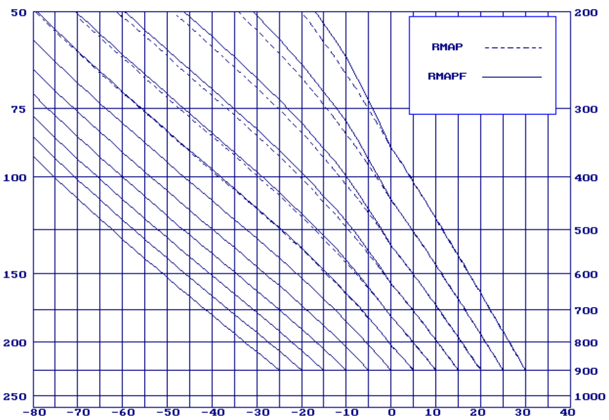


Fig. 3. Comparison of two kinds of reversible adiabatic process, one set of lines are ordinary (dashed), the others are reversible adiabatic lines with liquid water solidification (solid).

## 3 CONVECTIVE ENERGY PARAMETERS WITH LIQUID WATER DRAGGING

From the above comparison, we can see the selection of moist adiabatic process affects convective energy. By the way, the downdraft of condensed liquid water also affects the convective energy. CAPE is a theoretical result in which the gravitational pull on liquid and solid water in the cloud are not considered, thus it often overestimates the positive convective energy. Taking the gravitational effect of the condensed liquid water into consideration, CAPE then can be modified (MCAPE) as

$$MCAPE=g\int_{z_f}^{z_e}[\frac{1}{\overline{T}_{ve}}(T_{va}-T_{ve})-w_l]dz, \qquad (5)$$

where $T_{va}$ is the absolute virtual temperature of the lifting parcel, $T_{ve}$ is the absolute virtual temperature of the corresponding stratified atmosphere, $z_f$ is the height of the free convection level, $z_e$ is the equilibrium altitude, $\overline{T}_{ve}$ is the average absolute virtual temperature between $z_f$ and $z_e$, and $w_l$ is the specific content of liquid water and ice.

A crude evaluation of Eq. (5) shows that the downdraft effect of 4 g of liquid water in a 1 kg air-parcel can nearly offset the positive buoyancy generated by 1 Celsius degree of temperature difference between the air parcel and the environment.

Downdrafts are the formation mechanisms for thunderstorm wind, micro-downbursts, and low-level wind shear. When the dry cold air penetrates into the cloud cell in the midtroposphere, the liquid water in the cloud evaporates and the cloud cell becomes colder there, and downdraft flow occurs and affects the structure of the convection (Gilmore and Wicker, 1998). Supposing the downdraft flow descends along the pseudo equivalent potential temperature line, then the downdraft convective available potential energy or DCAPE (Emanuel, 1994) can be described as

$$DCAPE=g\int_{Z_{sfc}}^{Z_D}\frac{1}{\overline{T}_{ve}}(T_{ve}-T_{va})dz, \qquad (6)$$

where $z_D$ and $z_{sfc}$ are the height of the downdraft-starting level and the height of the surface level, respectively. An iso-enthalpy process presumption is suggested to acquire the primary temperature of the downdraft parcel in the midtroposphere level where dry cold air intrudes.

The modified downdraft convective available potential energy (MDCAPE ) can be used to describe the gravitational effect of condensed liquid and solid water.

$$MDCAPE=g\int_{Z_{sfc}}^{Z_D}\frac{1}{\overline{T}_{ve}}(T_{ve}-T_{va}+w_l)dz \qquad (7)$$

## 4 CASE STUDY

In order to discover the role of the condensed liquid water, a thunderstorm case with strong surface wind is analyzed with convective energy being calculated.

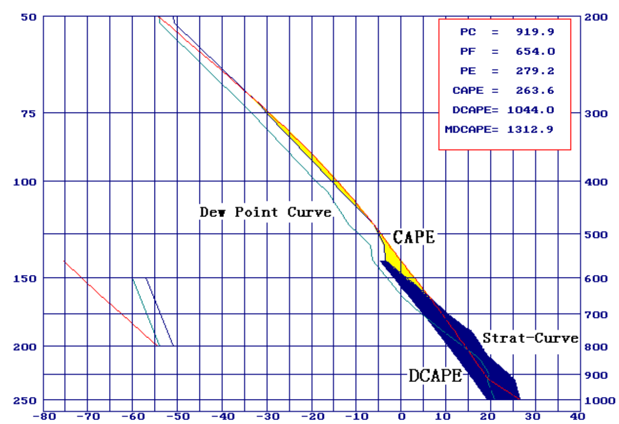


Fig. 4. Emagram for Beijing rawinsonde at 1200 UTC 3 Aug 1998. The surface observation is taken as the lift-starting point.

Figure 4 shows the Emagram of Beijing rawinsonde observation at 1200 UTC 3 August 1998. Taking the surface observation as the initial lifting level, CAPE is 263.6 J $kg^{-1}$. If the significant level (p=557.0 mb，t=-3.5℃， $t_d$=-6.3℃) is taken as the starting level for the downdraft, the wet-bulb temperature equals –4.8℃. If 10.0 g $kg^{-1}$ of liquid water is assumed to be in the parcel before the iso-enthalpy evaporation process, then the temperature reaches 18.9℃ when it descends to the surface level, with DCAPE reaching 1044.0 J $kg^{-1}$. The liquid water in the downdraft parcel evaporates gradually in the reversible moist adiabatic process to maintain the saturated state of the parcel. The remaining specific content of liquid water is 1.0 g $kg^{-1}$ when the downdraft parcel reaches the surface. The accumulated work of the gravitational effect of the downdraft liquid water reaches 268.9 J $kg^{-1}$ and the modified downdraft convection available potential energy (MDCAPE) is 1312.9 J $kg^{-1}$.

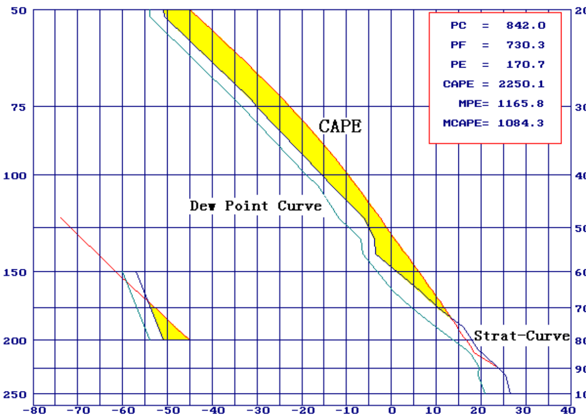


Fig. 5. As in Fig.4, using p=897.0 mb as lift-starting point.

If the most unstable layer is taken as the lifting point (Rochette, 1999), then the Emagram changes significantly with CAPE (MUCAPE; Matthe et al., 2002) dramatically increasing to 2250.1 J $kg^{-1}$(Fig. 5). The reduction energy owing to the presence of liquid water in the downdraft is 1165.8 J $kg^{-1}$ and MCAPE is 1084.3 J $kg^{-1}$. Thus we can see the downdraft effect of liquid water and ice cannot be negligible.

## 5 CONCLUSIONS

Four methods are often used to deal with the moist adiabatic process. Static energy conservation and the pseudo equivalent potential temperature conservation are nearly the same process, and their calculated results are very similar. The specific heat of water vapor is considered in the strict pseudo adiabatic equation, So the lapse rate of strict pseudo adiabatic process is slightly smaller than pseudo equivalent potential temperature conservation. If the condensed liquid water and its sensible heat are considered, the moist adiabatic process is reversible. There are little differences between each other of these four methods when the initial temperatures of lifted air parcels are low. With respect to high initial temperatures, the differences are not very obvious in lower levels of the atmosphere. When they are lifted to high levels, the reversible moist adiabatic lines are distinguished by relative smaller lapse rates of temperature due to the total sensible heats of the accumulated liquid water. When considering the heat of solidification of the large quantity of liquid water, the lapse rate is slower even more and the convective energy becomes larger correspondingly. This might be one of the reasons why the hailstorm can develop very strong. Real case analysis shows the gravitational effects of condensed liquid water are not negligible. MCAPE and MDCAPE are the modified form of CAPE and DCAPE, respectively, with the gravitational effect of the condensed liquid water. Reversible moist adiabatic processes can be invoked in convective energy calculations when considering gravitational effect of condensed liquid water.

## ACKNOWLEDGMENTS

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