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PRESUMABLE CAUSE OF TORNADO EVOLUTION.

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INTRODUCTION.

We have obtained the dependences of period of eutectic pattern on a velocity of interface, a mechanism of forming of cellular front under a crystallization of melts, intercoupling between dependence of period of eutectic pattern on a velocity of crystallization with a number of atomic layers of interface in papers Gus'kov, Orlov [1-4]. These outcomes have common fault - model, used in these operations, does not explain physical reasons of origin of the interface instability, which reduces in the listed appearances. Reason is that classical model of a directional crystallization [5, 6] is valid proposition only in range 0 – 0.25_1 (fig. 1) of an initial concentration of a mixture, which is range of a unlimited solubility of components. Now there is no theory, which would explain behavior of the system in range 0.25 – 0.75 of the restricted solubility. In represented paper we try to explain behavior of the system in the range of the restricted solubility.

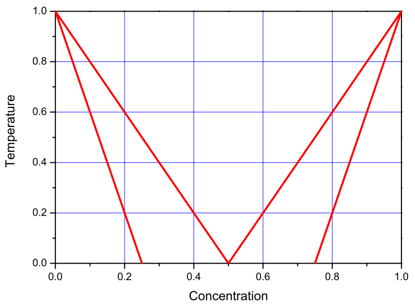

Fig. 1. Example of phase diagram.

The classical model of crystallization [5] considers stationary distribution of concentration in a liquid phase under condition of densities equality of liquid and solid phase. In this case the condition of conservation of mass flow is satisfied, if an initial concentration of the melt is equal to concentration of the solid phase [5]. In the field of the restricted solubility the concentration of the solid differs from initial concentration of the melt. The requirement of a constancy of mass flows of components in an one-dimensional case leads that the velocities of the mixed fluid and separate components should differ from solving of classical model. The arising additional diffusion flows lead to additional pressure between components of the mixture, which is described by the Darsy equation.

$$\frac{dp_i}{dz} = J_i = \rho_i \left(w_i - w \right) \quad (1)$$

here J_i is diffusion flow, ρ_i is density, w_i is velocity of considered component, w is velocity mixture. In bulk phases the pressure between components is counterpoised by a mutual influence of diffusion flows of each of components. On interface in the range of the restricted solubility of such equilibrium will not be, since the pressure on the part of the solid phase will not vary, but in the liquid phase the pressure will vary owing to a modification of velocities of components and mixture. Let's write down expression for a diffusion flow [8]

$$J_i = -\rho D \nabla C_i \quad (2)$$

where D is diffusion coefficient. Let's substitute the equation (2) in the diffusion equation, which in this case becomes form

$$D\Delta C_i - \frac{w^2}{D} C_i = -\frac{wN_i}{\rho D} \quad (3)$$

N_i is mass flow considered component.

STATIONARY REGIMES OF DIRECTIONAL PHASE TRANSITIONS.

We consider three stationary regimes of directional phase transitions. First regime is an equilibrium regime. An interface velocity in this regime is so small, that both phases are in equilibrium. The concentration distribution in both phases is a constant value, and value of concentrations is determined experimentally by equilibrium phase diagram. The second stationary regime is directional phase transition in the field of a unlimited solubility of components. The interface velocity is equal of such value, that the diffusion in one of phases can be neglected. The theory of such phase transitions is considered in many monographs, for example in Pfann [9], Barton a.o. [10] for solid - liquid of the phase transition, Sedunov [11] for liquid - gas of the phase transition. In this paper we consider a possibility of quantitative description of the stationary regimes in the field of the restricted solubility of components. Let's C(0) is component concentration on the liquid interface, C_{sol} is component concentration in the solid. We want to find a position $C(0)$ and C_{sol} on the phase diagram for a nonequilibrium system with arbitrary given initial concentration and interface velocity. We have a diffusion equation, which yields of concentration distribution basically at any boundary conditions. Initially in any stationary regime (except for equilibrium) the interface concentration is not defined. The classical theory of phase transition supposes, that in the stationary regime $C(0)$ and C_{sol} are interrelated by the equilibrium phase diagram by a requirement of equality of temperatures on boundary [9-11].

To find the concentration distribution, Burton [11] had used the classical solution of the stationary diffusion equation for an infinity interval

$$C_i(z) = \left[C_i\left(0\right) - C_i\left(\infty\right) \right] \exp\left(-\frac{V_s z}{D} \right) + C_i\left(\infty\right) \quad (4)$$

An obligatory requirement for use of the classical solution is the requirement of equality of components density of fluid and solid mixtures. All this allows to find concentration and temperature on interface of the nonequilibrium system by geometrical build-up from the requirement of an equilibrium of the system on interface in a quasi-equilibrium regime. Thermodynamic sense of the equilibrium state of a system is equality of chemical potentials of the mixtures component on the interface. The classical theory of phase transition [9, 10, 11] guesses, that in nonequilibrium stationary regime on interface the equilibrium value of concentration is fixed. This value, as was shown above, is associated with the concentration $C(0)$ and C_{sol}. Hence equality of chemical potentials of mixtures components is used here too. We keep in the common theory the same requirement for arbitrary densities of components and arbitrary concentration on exterior boundary.

The theory of phase transition [9-11] considers the concentration distribution in a diffusion layer close to interface. The concentration value is set on outside boundary of this layer. The thickness δ of the diffusion layer depends on a hydrodynamic velocity of the fluid [9, 10] in case of the solid - liquid phase transition or of the velocity of gas [11] in case of the fluid - gas phase transition. The solution of the diffusion equation (2) looks like

$$C_i(z) = \left\{\left[\frac{N_i}{\rho w} - C_i(0) + \left(C_i(\delta) - \frac{N_i}{\rho w}\right)\exp\left(\frac{w}{D}\delta\right)\right]\times\right.$$
$$\exp\left(\frac{w}{D}z\right) + \left[\frac{N_i}{\rho w} - C_i(\delta) + \left(C(0) - \frac{N_i}{\rho w}\right)\times\right.$$
$$\left.\left.\exp\left(\frac{w}{D}\delta\right)\right]\exp\left(\frac{w}{D}(\delta - z)\right)\right\}\left[\exp\left(\frac{2w}{D}\delta\right) - 1\right] + \frac{N_i}{\rho w} \tag{5}$$

The solution of the diffusion equation does not give any information on the value of interface concentration C(0). The Darcy equation gives allocation of partial pressure. On interface this pressure depends on concentration, interface velocity and mass flow the component. We write down the solving of Darsy equation in the form

$$p_i(0) = p_i\left(C_i(0), V_S, N_i\right)$$

However this dependence does not give the interdependence of the interface concentration and the phase diagram. In the equilibrium $V_S = 0$, $N_i = 0$ and the Darcy equation gives an integration constant, which is equal to fractional pressure of equilibrium regime

$$p_{ieq} = p_{ieq}\left(C_i(0), 0, 0\right), \tag{6}$$

Together with these solving we consider dependence of the chemical potential on temperature, pressure and concentration. The common pressure is constant; therefore chemical potential depends on temperature and concentration. The value of concentration on interface is set only by equilibrium condition of the system in stationary regime, which relates the chemical potential of the component on the interface to the chemical potential of the component in the solid. The equality of chemical potentials of phases of the component at temperature of phase transition looks like

$$\mu_{Sol}\left(T_e, C_{Sol}(0)\right) = \mu_{Liq}\left(T_e, C_{Liq}(0)\right)$$

In this equation we know dependence

$$T_e\left(C_{Sol}(0), C_{Liq}(0)\right) = 0 \tag{7}$$

Let's consider the classical solving of the diffusion equation (4). We do not know the value of concentration on interface $C(0)$. In an experiment is set only the concentration $C(\infty)$. The Darcy equation gives solving

$$p_{iClas}(0) = p_{iClas}\left(C_i(0), V_{S\_Clas}, C_i(\infty)\right) \tag{8}$$

$C(\infty)$ here is obtained for the considered solving from the ration

$$\frac{N_i}{\rho w} = C_i(\infty)$$

The dependence (8) relates fractional pressure to concentration on interface. The equations (7) and (8) contain three unknowns p_i (0), $C(0)$ and C_{sol}, therefore for a solving of the problem there should be one more equation. We have found such equation. According to definition of the chemical potential it depends only on temperature, pressure and concentration.

$$\mu_{Sol}\left(T_e, C_{Sol}(0), P\right) = \mu_{Liq}\left(T_e, C_{Liq}(0), P\right)$$

here P is external pressure. We want to note, that the chemical potential does not depend on fractional pressure the component. This property of the chemical potential specifies distinction between properties of fractional pressure and component concentration. The system becomes nonequilibrium, when the interface begins to move. The movement of interface leads to a variation of the diffusion flow. The variation of the diffusion flow leads to a variation of fractional pressure. If the velocity of the mixture w and its density ρ are constants, then the distribution of fractional pressure depends on the component velocity w_i and on his concentration $C_i = \rho_i / \rho$. To associate the variation of fractional pressure with interface concentration we guess existence of a one-dimensional boundary layer close to the interface. Owing to exterior actions in the layer there is an fractional overpressure, which differs from the equilibrium one. In liquid bulk the fractional overpressure is compensated by relative influence of the liquid components. On the interface the fractional overpressure should be compensated by fractional pressure of the solid component. We guess that in the nonequilibrium stationary regime the fractional pressure on the interface has the equilibrium value. The concentration on the interface in this case should vary. It becomes equal to a value, which correspond to other equilibrium state of the system.

STATIONARY REGIMES IN THE RANGE OF THE UNLIMITED COMPONENTS SOLUBILITY.

The equilibrium regime is a regime at a small interface velocity. In this regime the diffusion has time to reduce both phases in the equilibrium state. The classical solving (4) can be transformed to equilibrium by a requirement of equality of interface concentration and concentration in the infinity point. In this case the sum in brackets is equal to zero and for the equilibrium regime we obtain expressions

$$C_i(z) = \frac{N_i}{\rho w}$$

and (6). Let's remark that in one-dimensional stationary regimes always $N_i = const$.

Let's consider distribution of concentration and of fractional pressure on an infinite interval at $V_s \neq 0$. The solving of the equations (1) and (3) in this case look like

$$C_i(z) = \left[C_i(0) - \frac{N_i}{\rho w} \right] \exp\left(\frac{wz}{D} \right) + \frac{N_i}{\rho w}$$

$$p_i(z) = -\frac{\rho}{\kappa} \left[C_i(0) - \frac{N_i}{\rho w} \right] \exp\left(\frac{wz}{D} \right) + p_{ieq}$$

At interface the fractional expression of pressure has form

$$p_i(z) = -\frac{\rho}{\kappa} \left[C_i(0) - \frac{N_i}{\rho w} \right] + p_{ieq} \quad (9)$$

To balance this pressure between rigid and fluid phases we should change concentration Ci(0) so that the bracket in expression (9) converted to a zero. Hence we obtain for concentration the equation

$$C_i(0) = \frac{N_i}{\rho w}$$

Hence concentration on boundary in a fluid of an equilibrium condition should be equal to concentration an infinite point of a classical condition. But the concentration at the infinite point of the nonequilibrium regime is equal to concentration in the solid according to the equation of balance of mass flow. We have obtained the value of interface concentration of the nonequilibrium regime.

The Burton a.o. theory [10] uses the classical diffusion equation. This theory guesses that the one-dimensional diffusion layer is located close to interface. The boundary condition in [10] is set on a finite interval. Concentration distribution looks like

$$C_i(z) = \left[C_i(0) - C_{iSol} \right] \left[\exp\left(-\frac{V_s z}{D} \right) - \exp\left(-\frac{V_s \delta}{D} \right) \right] + C_i(\delta)$$

This expression coincides with the classical solution (4) qualitatively. It contains the same exponential curve and constant item. If we shall substitute δ = ∞ in this solving, it becomes equal to a classical solving (4). Hence value of parameters in this case can be found as well as for the classical solving.

STATIONARY REGIMES IN THE RANGE OF THE RESTRICTED COMPONENTS SOLUBILITY

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Let's consider the calculation of values $C_i(0)$ and C_{Sol} for solving (5) in the field of a restricted solubility. Initial the expression for concentration on interface is

$$C_i(0) = \left[2\left(C_i(\delta) - \frac{N_i}{\rho w} \right) \exp\left(\frac{w}{D}\delta \right) + \left(\frac{2N_i}{\rho w} - C_{iun}(0) \right) \exp\left(\frac{2w}{D}\delta \right) + C_{iun}(0) \right] \times \quad (10)$$

$$\left[\exp\left(\frac{2w}{D}\delta \right) + 1 \right]^{-1}$$

This equation is obtained as follows. We consider two stationary regimes. The concentration $C_{iun}(0)$ regime I is in the interval of the unlimited solubility 0 - C_{eut} (for example in the interval 0 – 0.5 for the phase diagram of the fig. 1). The concentration $C_i(0)$ regime II is in the interval of the restricted solubility C_{eut} - 1 (0.5-1 for the phase diagram of the fig. 1). When the system goes out the equilibrium, the values of component concentration at first are increasing in the range of the unlimited solubility up to C_{eut}, and then go in range of the restricted solubility, in which $C_i(0) > C_{eut}$. The corresponding equilibrium values of component concentration in the solid in both regimes I and II are in the concentration interval of the left-hand line of a liquidus (0 - 0.25 for the phase diagram of the fig. 1). All parameters of the regime I we find how it is described in the previous item. To find parameters of regime II we use equality of concentrations in the solid in both regimes. We guess that if the concentrations in the solid on the interface are equal, then the fractional pressure on the interface of regimes I and II are equal also. It follows from the conservation law of the impulse flow, because the Darcy equation is an approximation of the conservation equation of the impulse flow [12]. We equate fractional pressure of the component on the interface of regimes I and II. The obtained equation gives expression (10) for concentration on the interface of the regime II. The interface velocity V_s, thickness of the diffusion layer δ and concentration on exterior boundary $C_i(\delta)$ are given independent parameters in this problem . Physical properties depending from a composition of a material are the density of fluid and solid mixtures and diffusivity. The velocities of mixtures depend on the common given interface velocity V_s. Three parameters $C_i(0)$, $C_{iun}(\delta)$ and C_{Sol} are unknowns in this problem.

The equation (10) describes relative movement of components of the mixed fluid in the diffusion layer. This movement gives rise a additional diffusion flow, which leads to additional fractional pressure of components in the regime 2. On interface this additional pressure is equalized by the change of components concentration. The value of this concentration is obtained from the equilibrium condition of solid and fluid on the interface. The obtained here stationary regimes are unstable and can not be observed in experiments. They lead to arise of nonuniform structures. Special cases of these structures can be for example eutectic pattern or spontaneous condensation resulting in formation of atmospheric vortex.

The area of a supercooled liquid is formed in bulk liquid phases before interface according to the obtained solving. This area is similar to the classical concentration supercooling [5,6]. It is formed as a result of the value of component concentration before interface more than eutectic concentration. Such supercooled layer can be cause of a plate-like crystallization, i.e. alternation of a solid plates located of parallel of the interface. At condensation of gas this layer leads to the accelerated transition of the interface, which in a result restricts sizes of nascent drops or snowflakes.

Figures 2 and 3 has shown concentration distribution and interface movement velocity at D=10^{-8} m/s, $\rho = \rho_{un}$, stationary interface velocity V_s = 10^{-5} m/s, δ = 0.001 m, and for phase diagram of figure 1. Concentration on outside of diffusion layer δ is 0.3 (1), 0.42 (2), 0.45 (3), 0.47 (4), 0.5 (5). According the calculations the interface is stable in the conditions of curve 1, and it is unstable in conditions 2 and 3. It is explained to that in the conditions 1 value C(δ) is less, than the concentration of the quasi-equilibrium regime I in the same point. The equilibrium distribution coefficient for all curves shown on figures 2 and 3 is less than one. But the curves 1 correspond to slope of the liquidus m < 0, and curves 2 - 5 of m > 0. Hence, as shown 666 at parameters of the curve 1 the interface is stable, and at parameters of curves 2-5 the interface is unstable. These results explain physical sense of the instability, found in papers [1-4]. We remark that stationary regime of curves 2 - 5 can not be observed in experiments because it is unstable. If on the interface the concentration will increase, temperature of phase transition of the fluid will increase too. The increase of temperature of interface leads to increase of the kinetic supercooling and to increase of the interface movement velocity. But the increase of the interface movement velocity leads to the further increase of concentration on the interface. The similar unstable process will be the result and at spontaneous decrease of concentration. Reason of instability is the rise of a dispersion of the components velocity of the mixed fluid and, as a corollary, additional diffusion flows.

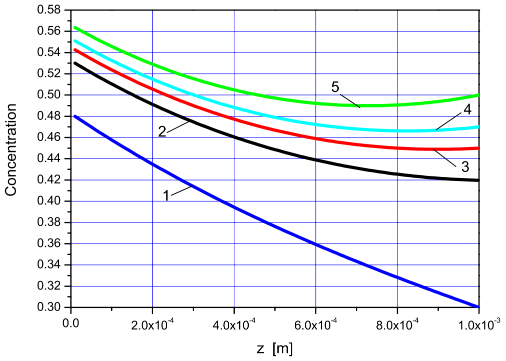

Figure 2.

UNSTABLE CONDENSATION IN ATMOSPHERE.

The one-dimensional movement of the components with different velocities in the range of the restricted solubility can exist only in a limited range of the system parameters. Really, the partial velocities of components are equal to the velocity of the mixture in the regime of the unlimited solubility on any velocity of interface. The situation varies in the range of the restricted solubility. Padding diffusion flows arise in this range. Estimation of existence of laminar one-dimensional stream of monopropellant liquid is the Reynolds number. We can not estimate range of existence of the one-dimensional laminar stream of components of the mixture with different partial velocities because the hydraulic theory of mixtures now does not exist. It is possible to do estimates of the existence range of such stream only on the basis of experimental observations. Narrow interface velocity range, in which the eutectic pattern is formed, demonstrates that the range of existence of the one-dimensional components stream of the fluid with different partial velocities is small. Intuitively this outcome is clear. The fluids have a large density and consequently the partial velocity of components can not be large - streams become three-dimensional. However there are phenomenons, in which the considered streams can be rather intensive. The condensation of a fluid in an atmosphere is such phenomenon. Now we shall show, that the considered instability well explains the mechanism of formation of whirlwinds in the atmosphere - of cyclones, hurricanes, and tornado. Most striking example is of tornado formation. We now shall show, that the requirements for a possibility of rise of unstable condensation are fulfilled, and that the offered mechanism of tornado formation does not contradict any singularities of this phenomenon.

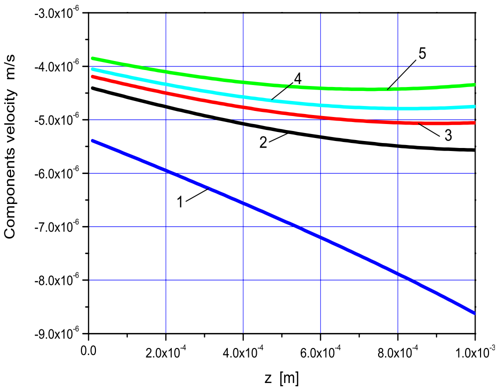

Figure 3.

The unstable condensation can arise if the following requirements are fulfilled. 1. The atmosphere in an area of condensation is the mixture of components, which have the restricted solubility. The composite chemical composition of the atmosphere is described for example in the monographic [13]. The condensation of drops is considered there as condensation of composite solutions. 2. The processes of condensation should be described by the transport equations. Agrees [13] drops measure from 1 μm in clouds, almost up to 1 mm in a shower. The kinetic exposition is applied only to drops by the size of unities micron

[11]. 3. In the area of the diffusion layer there should be the boundary on which the particular value of concentration is supporting during condensation. Hence conditions which are described by the boundary value problem of the diffusion on a finite interval should exist. This boundary value problem was solved above. Agrees [11] this requirement also is fulfilled. During relative movement of the drop and air the Prandtl number is close to unit. Hence diffusion and hydrodynamic layers practically coincide. From here follows, that the value of the component concentration equal to concentration of the atmosphere is supported at distance of hydrodynamic layer in limits of the diffusion layer during relative movement of the drop and air. 4. The size of drops, which are formed as a result of unstable growth, should have restrictions, which give the size observed in experiment. Precomputations similar [1] for parameters of an atmosphere have shown, that the growth increment (the calculations are made in an approximation of flat boundary) $\delta \sim 10^6$ ([1]). If to guess, that the growth velocity is restricted to the sound velocity, the range estimate of the drop sizes gives 1 mm. Let's remark, if not it is taken into account of atmosphere hydrodynamics, that theoretically velocity of movement of air is restricted to the sound velocity. We obtain process to similar vacuum explosion. In contrast to chemical explosions, in the inside area of drops shaping, in this case the pressure will be cushioned by a restricted size of drops and by emergent hydrodynamic flows between them. 5. Comparison with other known mechanisms of rise tornado. Agrees [14] the physical mechanism tornado completely yet is not ascertained. The basic hindrance to an explanation of the mechanism of tornado rise is the impossibility to explain an energy source necessary for existence tornado. This energy on existing estimates should have magnitude approximately 3.5 MW. We have set the requirement, that during condensation such energy is got. Then the estimation of velocities of front of spontaneous condensation area of drops was done (not front of phase transition, but the front of atmosphere area, in which happens condensation). According to this estimation the velocity of area boundary of spontaneous condensation should be equal ~ 2.3 m/s. It is the small velocity compared with observable velocities of movement of the atmosphere. This velocity also is small in comparison with restriction obtained in item 4. Hence phenomenon of condensation has enough energy for maintaining tornado.

Now we want to show, that the offered theory explains observable during the action of tornado the facts. These facts are enumerated in [14]. Basic of them the following are. Tornado is lowered from the top downward and cause damage. According to our theory the unstable condensation develops in clouds. It spontaneously seizes the next areas of the atmosphere and works as a vacuum core. This core absorbs the atmosphere containing water vapor. The restrictions on movement direction of area of negative pressure are not present. 2. Tornado forms the core circled by whirlwind in a horizontal section. 3. Constantly there is a considerable swing pressure between the core and peripheral part of tornado. The offered theory explains phenomenon of items 2 and 3. Hence any of the basic observable singularities tornados does not contradict the surveyed mechanism of tornado rise.

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