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APPLICATION OF WAVELET ANALYSIS TECHNIQUES TO THE STUDY OF TURBULENCE AND GRAVITY WAVES USING AIRCRAFT DATA AND MESOSCALE-MODEL FIELDS

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1. INTRODUCTION

The nature of the relationship between gravity waves and turbulence has been a topic of considerable theoretical investigation for several decades. However, confirmation of theoretical predictions from direct observations is very much missing. Customary spectral techniques applied to data do not lend themselves well to a proper understanding of the highly intermittent and nonstationary nature of interactions between waves and turbulence. Using 25-Hz aircraft data taken through a turbulent upper-level jet stream in the atmosphere, we show here that wavelet transformation techniques offer a better understanding of such temporally evolving interactions. The frequency-time domain analyses of the in-situ aircraft data suggest a close relationship between turbulence intensity and gravity-wave activities. Our results indicate that clear-air turbulence is strongly associated with a spectrum of gravity waves associated with a strong tropopause-folding event. Upon applying the wavelet technique to the polarization relationship associated with gravity waves and to a Stokes parameter analysis, we show that gravity waves and turbulence possess distinctive polarization signatures, and that the intermittency of gravity-wave generated turbulence in the atmosphere is closely related to the changing polarity of gravity waves.

The wavelet transformation technique has also been adapted to the study of data from the 20-km Rapid Update Cycle (RUC) model, which is used operationally at the National Center for Environmental Prediction (NCEP) as guidance for predicting the occurrence of turbulence. Further use of wavelet techniques in analyzing mesoscale model data shows promise for determining with great precision time-varying gravity wave propagation characteristics and wave-wave interactions.

2. TIME-FREQUENCY ANALYSIS OF GRAVITY WAVES AND TURBULENCE

Given a set of time-series data $f(t)$, one can use wavelets to transform this dataset into both time and frequency space. The transformation can be expressed mathematically as

$$T(\omega,t)=\left\langle \psi_{\omega,t},\, f\right\rangle = \int \psi^*_{\omega,t}(\tau)\, f(\tau)\, d\tau \,,$$

where T represents the transformation coefficient, $\psi^*_{\omega,t}(t)$ is a locally supported transformation kernel (asterisk denotes the complex conjugate), and the angle brackets denote an inner product.

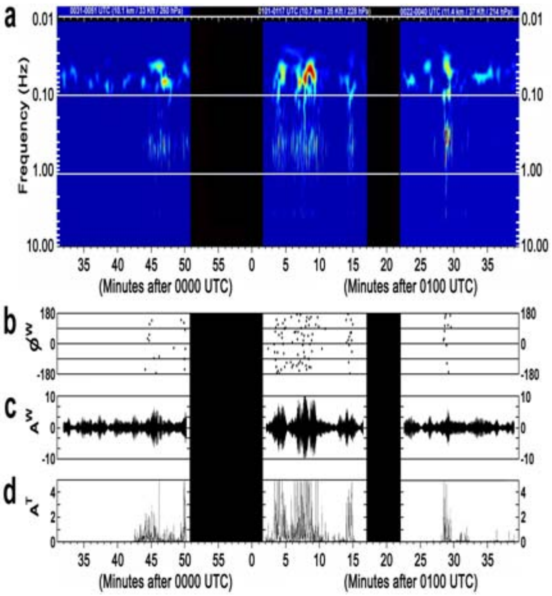

Fig. 1: a) Time-frequency display of wavelet analysis of aircraft vertical acceleration data at 10.1, 10.7, and 11.4 km flight altitudes (cm s^{-2}); b) Phase of gravity waves (degrees) at which maximum turbulence intensity occurred for turbulence > 0.5 cm^2 s^{-4}; c) Gravity waves reconstructed from wavelet analysis in the 0.07 frequency band; and d) Turbulence intensity (cm^2 s^{-4}) reconstructed from wavelet analysis in the

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0.65 Hz frequency band. Background noise level of wavelet amplitudes is depicted by blue, with increasing intensity shown by yellow and red shading. Black segments indicate times when the aircraft was going through maneuvers (primarily changes in altitude) that invalidated the measurements.

Figure 1 shows the wavelet analysis of vertical acceleration field (ACINS) observed by the NOAA G-IV aircraft during the SCATCAT field experiment (see details in Koch et al. 2004). Fig. 1a presents the wavelet power of ACINS as function of both time and frequency. One can see that two types of signals are apparent in this time-frequency analysis. The fast signals occurred in the spectral region approximately greater than 0.1 Hz with a typical spatial scale on an order of a few hundred meters (converted using the constant aircraft speed of 230 m s^{-1}).

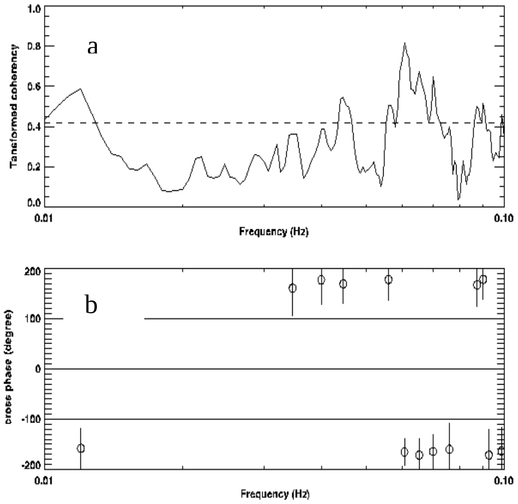

Fig. 2: Cross spectra for the zonal (u) and meridional (v) velocities: a) transformed coherency between u and v; and b) the phase-difference between u and v. Peaks of the coherency spectrum above the dashed line are 95% statistically significant, and the error-bars in the phase diagram represent 95% confidence intervals for the computed phase difference (at the center of each circle).

This type of signal displayed some intermittency patterns, and did not show a strong frequency selection. The time during which these signals occurred is consistent with the time of the MOG (moderate-or-greater) turbulence report by the pilots. Therefore, from their scales, intermittency patterns, and occurrence time, we may confidently determine that these signals are of the clear-air turbulence type. The slow signals occurred mostly in the spectral region of 0.01–0.1 Hz with the spatial scale on the order of a few kilometers to tenth of kilometers. In contrast to the fast signals, the slow signals showed strong frequency selections. By analyzing their polarization signatures (see next two sections), we determine that these slow signals are clearly related to the gravity-wave activities.

With further examination of this time-frequency diagram, one can see that the occurrence of the fast signals (turbulence) seems to be closely related to the strong surges of the slow signals (gravity waves). This picture provides a direct observational evidence for the interaction of turbulence and gravity waves, which is not available from the traditional spectral analysis.

3. RETRIEVAL OF GRAVITY WAVES USIING CROSS-SPECTRAL ANALYSIS AND WAVELET TRANSFORMATION

In the real world, gravity waves generally possess transient (local) characteristics, i.e., the wave amplitude and phase can be functions of time and space.

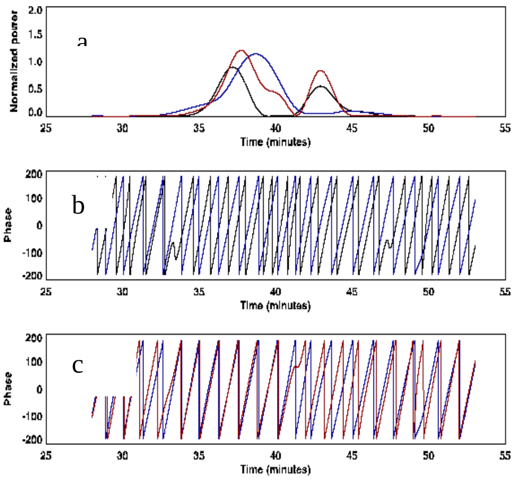

Fig. 3: Wavelet transformations of u (black curve), v (blue curve), and θ (red curve) for a monochromatic gravity wave (frequency = 0.012 Hz). a) Normalized power (squared amplitude); b) Phase for u and v; c) Phase for v and θ. Time is minutes after 0000 UTC 18 February 2001.

To completely reconstruct a wave, one not only needs to exactly determine wave frequency (wavenumber), but also needs to locate the time-spatial variability of wave amplitude and phase

(Lu et al. 2004a). Therefore, we propose a combined use of the cross-spectral method, which, in conjunction with gravity-wave polarization theory, determines a wave frequency or wavenumber (an example is given in Fig. 2a and 2b), and wavelet analysis, which determines the wave's amplitude and phase as functions of time and space (an example is presented in Fig. 3a-3c), to fully discern a gravity wave from a complex set of data.

Fig. 1c shows a band of monochromatic gravity waves reconstructed using this proposed method. Fig. 1b displays the phases of these waves as functions of time. Fig. 1d shows the intensity of turbulent events retrieved from the same aircraft data.

4. STOKES PARAMETER ANALYSIS USING WAVELET-BASED CROSS SPECTRA

For partially polarized waves, the Stokes parameter analysis provides an averaged view of wave polarization properties, including the coherency, the degree of polarization, the major axis orientation angle of the polarization ellipse, and the phase difference angle. Because of the nature of dual-space tractability of wavelet transformation, one could develop a wavelet-based cross-spectral analysis scheme, which is ideal for the calculation of the set of Stokes parameters.

We applied this wavelet-based cross-spectral analysis package to a set of zonal (u) and meridional (v) winds from the aircraft in-situ observations. In Fig. 4, we plot the degree of polarization (panel a) and major axis (panel b) in the time-averaged view. From this figure one can see that there exist two distinctively different spectral regions, which correspond to gravity waves and turbulence, respectively. In the gravity-wave spectrum, ranging from 0.01 to approximately 0.2 Hz, polarization displayed a strong frequency selection. The degree of polarization is high (over 60% for instance) for only three waves (with frequency around 0.01, 0.06, and 0.15 Hz). Corresponding to these three waves, the horizontal wave vector (the orientation of major axis) consistently points to -50^{o} off the zonal direction. In the turbulence spectrum, which is in the region greater than 0.2 Hz, the polarization is very high persistently over the whole spectrum (excluding the transition region). The horizontal wave vector had an abrupt shift about 90^{o} clockwise from that of the gravity wave, to about $30^{o} - 40^{o}$ of the zonal direction.

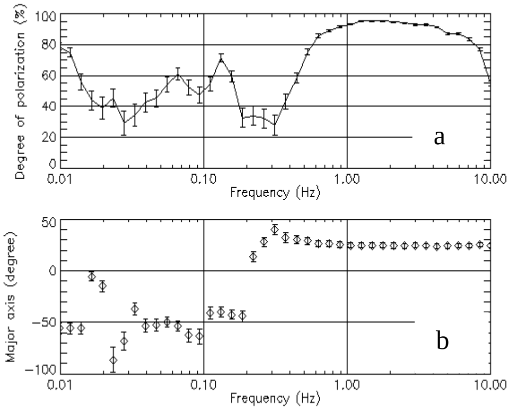

Fig. 4: The degree of polarization and the orientation of the major axis (points to the direction of horizontal wave vector) as functions of frequency, computed from the set of Stokes parameters. The error bars indicate confidence intervals of the analyses (plus/minus two times the standard deviation of the time mean divided by the sample size).

Figure 5a and 5b show the coherency and phase difference spectra. The coherency spectrum looks very similar to the spectrum of degree of polarization (Fig. 4a). The gravity-wave phase spectrum presents a clear polarization feature: all phase angles are clustered around $\pm 180^{o}$. The turbulence phase spectrum, all at about 0^{o}, indicates a completely different phase relationship between u and v (in phase) compared with that for gravity waves (out of phase).

Localization of polarization properties of gravity waves and gravity wave-induced turbulence in time and space can be achieved by wave-averaging auto-spectra and cross-spectra of wavelet transformed horizontal winds. This technique has been discussed in detail in Lu et al. (2004b).

5. GRAVITY-WAVE ANALYSIS USING 2D SPATIAL WAVELET

In order to accurately determine mesoscale gravity-wave propagation as well as wave scale

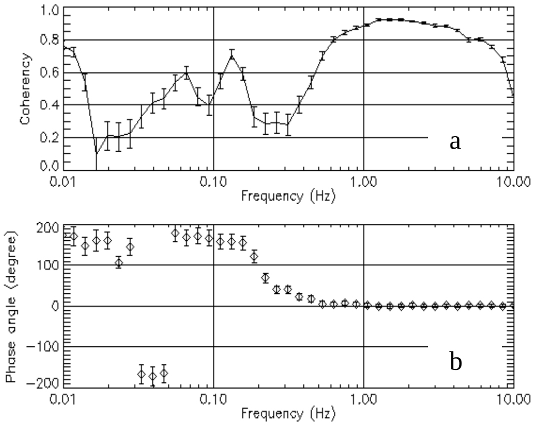

Fig. 5: Same as Fig. 4, except for the coherency and the phase difference between *u* and *v*.

interaction, we also developed a 2D spatial wavelet package. The application of this 2D wavelet scheme for mesoscale model data is able to display clear wave propagation patterns and separate waves of various scales.

Figure 6 shows the wavelength-spatial analysis of vertical velocity from NOAA Rapid Update Cycle (RUC) mesoscale model output. The analysis indicated that there exists a train of gravity waves of 300-km scale associated with an upper-level jet streak and a tropopause-folding event. With a series of different wavelength-scale analyses, one could construct a wavelet-based Hovmöller diagram to display an episode of mesoscale gravity-wave activity. Such a wavelet-based Hovmöller diagram can display both wave propagation features as well as wave scale interaction.

6. SUMMARY

In this paper, we have demonstrated several application studies using the wavelet technique to analyze aircraft observations and mesoscale model data, in relation to gravity waves and clear-air turbulence problems. The results from these studies indicate that the wavelet is a powerful analysis tool, especially for a set of data representing phenomena such as turbulence and gravity waves that are highly intermittent and nonstationary in nature. The time-frequency analysis provided by the wavelet transformation gives direct evidence of interaction between turbulence and gravity waves in the upper troposphere. The combined use of the cross-spectral method and wavelet transformation can retrieve a set of gravity waves with temporal and spatial variability. Wavelet-based cross-spectra are ideal for the calculation of the set of Stokes parameters, which lead to a better understanding of polarization properties of gravity waves and gravity wave-induced turbulence. The application of a 2D spatial wavelet transformation to mesoscale model data showed promising results in extracting gravity waves with accurate wave propagation and scale interaction information.

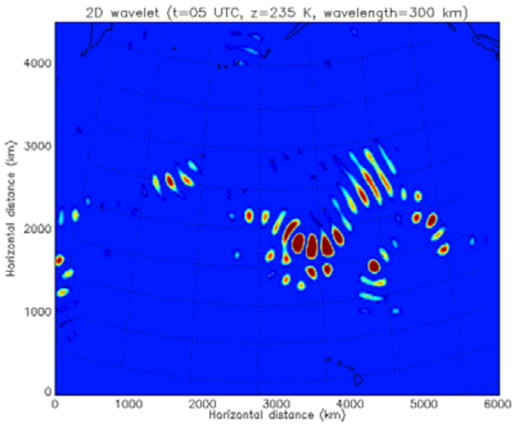

Fig. 6: 2D wavelet analysis of vertical velocity field from RUC model simulation of SCATCAT case.

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