P2.5 MEASUREMENT ERRORS USING A GEONOR WEIGHING GAUGE WITH A CAMPBELL SCIENTIFIC DATALOGGER

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1. INTRODUCTION

The Geonor weighing gauge uses a vibrating wire to measure the weight of precipitation in a bucket. A square law formula then relates the frequency of the vibrating wire to the amount of precipitation. This paper will discuss the errors contributed by the Geonor transducers and by the datalogger. It is assumed that the gauge is using three transducers. Additional errors that can occur when just one transducer is used are briefly mentioned in the conclusion. Also, noise, primarily caused by wind pumping, is not discussed here but left to a paper on algorithms. Both the CR23x and CR10x dataloggers are considered.

2. VIBRATING WIRE TRANSDUCER

2.1 *Fundamentals*

The transducer consists of a wire with magnetic properties under tension and two magnets placed nearby. One magnet "plucks" the wire to cause oscillation and the other magnet detects the vibration. Because the output is continuous, the two magnets must be part of an oscillating circuit providing positive feedback to the plucking magnet. For the oscillation to be the natural frequency of the wire, care is taken that the other components of the circuitry cause no forcing. This is the method presented in DiBiagio (2003) of the Norwegian Geotechnical Institute and is likely the method used in the Geonor.

The resonant frequencies of a wire under tension are

$$f^2 = n^2F/4L^2\mu, \quad n = 1, 2, - - - \qquad 1.$$

where L is length of the wire, μ is the mass/unit length and n = 1 for the fundamental resonant frequency.

For a weighing gauge, the parameter being measured is the increase in weight beyond a reference weight comprising the empty bucket and the mechanism for holding said bucket. From Bakkehoi (1985), the relationship between strain and frequency is

$$f^2 - f_0^2 = \varepsilon Eg/4L^2\rho C \qquad 2.$$

where ε is strain, E is Young's modulus of elasticity, g is gravity, ρ is density of the wire and C is a constant of the device. In this case, f_0 is stated to be the zero strain frequency. However, that would also mean the force, F, and the frequency, f, are both zero. Equation 2 is therefore modified to be

$$f^2 - f_0^2 = (\varepsilon - \varepsilon_0)Eg/4L^2\rho C \qquad 3.$$

Young's modulus is

$$E = \text{stress/strain} = \sigma/\varepsilon = (F/A)/(\Delta L/L) \qquad 4.$$

where A is the cross sectional area of the wire and stress has units of pressure and strain is dimensionless.

Now using εE = F/A and ρA = μ,

$$f^2 - f_0^2 = (g/C)(F - F_0)/4L^2\mu \qquad 5.$$

which is consistant with Equation 1 and where C has dimensions of acceleration.

By taking the difference in frequencies, the gauge is measuring the increase in force beyond the reference force. Then $F - F_0$ can be replaced by F = Wg, where W is the weight that has been added to the system. Equation 4 becomes

$$W = (4L^2\mu C/g^2)(f^2 - f_0^2) \qquad 6.$$

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where the constant has dimensions of $Kg*sec^2$. To state Equation 5 as P centimeters of precipitation, the constant must have dimensions of $cm*sec^2$ and C must have dimensions of $cm*m/Kg*sec^2$.

Tunbridge (1988) state that the following two equations are in general use for the gauge

$$M_i = K(f^2 - f_0^2) \quad 7.$$

$$M_i = A(f - f_0) + B(f - f_0)^2 \quad 8.$$

Equation 8, which has become more commonly used, may be put into the form

$$M_i = B(f^2 - f_0^2)[1 + (A - 2Bf_0)/B(f + f_0)] \quad 9.$$

where M_i is the measurand and $|(A - 2Bf_0)/B(f + f_0)| < 0.15$ for $f \geq f_0$. The constants, B and K, derive from the $4L^2\mu C/g^2$ and have dimensions of $cm*sec^2$.

2.2 *Temperature Sensitivity*

The wire will have a temperature sensitivity given by

$$L = L_a[1 + c(T - T_a)] \quad 10.$$

where c is the linear coefficient of expansion and T_a is the ambient temperature. Geonor uses the same bright polished music wire as DiBiagio (2003), a steel alloy wire of propietary composition. For steel and iron, the coefficient of linear expansion is 0.000012/C°. The mass per unit length, μ, will change inversely as the change in length of the wire

$$\mu = \mu_a[1 - c(T - T_a)] \cong \mu_a/[1 + c(T - T_a)] \quad 11.$$

Therefore, Equation 5, expressed as precip, becomes

$$P = (4L_a^2\mu_a C/g^2)[1 + c(T - T_a)](f^2 - f_0^2) \quad 12.$$

The change in precip reading, $\Delta P = P - P_a$, over the range from -50°C to 50°C is shown in Figure 1. Note that the total change is 0.12% of the reading which is reasonably consistant with the 0.001%FS(reading)/°C specified by Geonor.

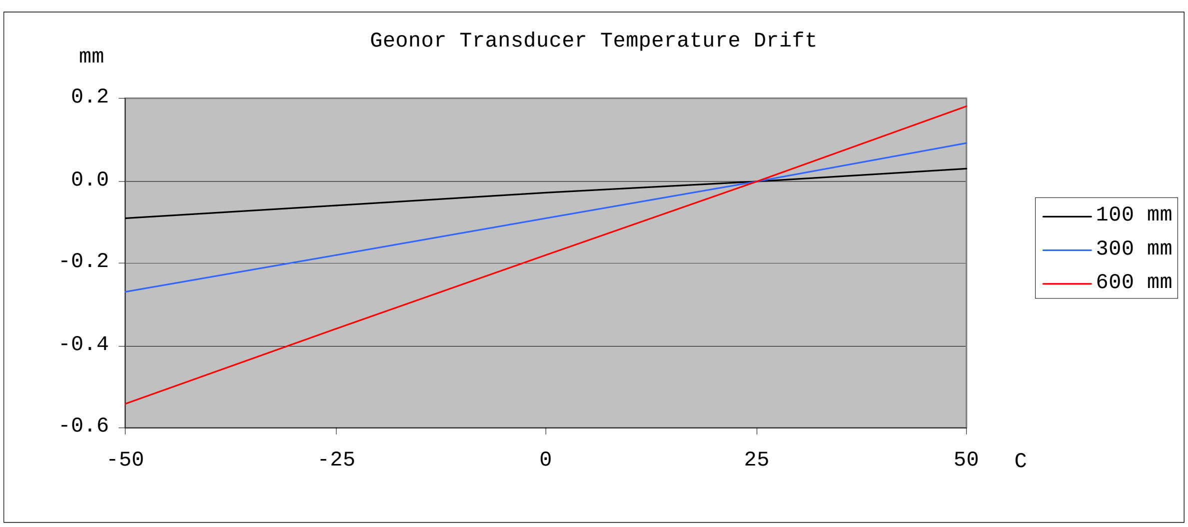

Figure 1.

2.3 *Zero Drift*

In DiBiagio's (2003) study, eight wires have been operated continuously and two wires intermittantly for 27 years without failure. The wires were clamped in steel fixtures at different levels of tension. Seven wires were stressed below 15% of their yield strength and three were stressed above 21% of their yield strength. The results are shown in Figure 2.

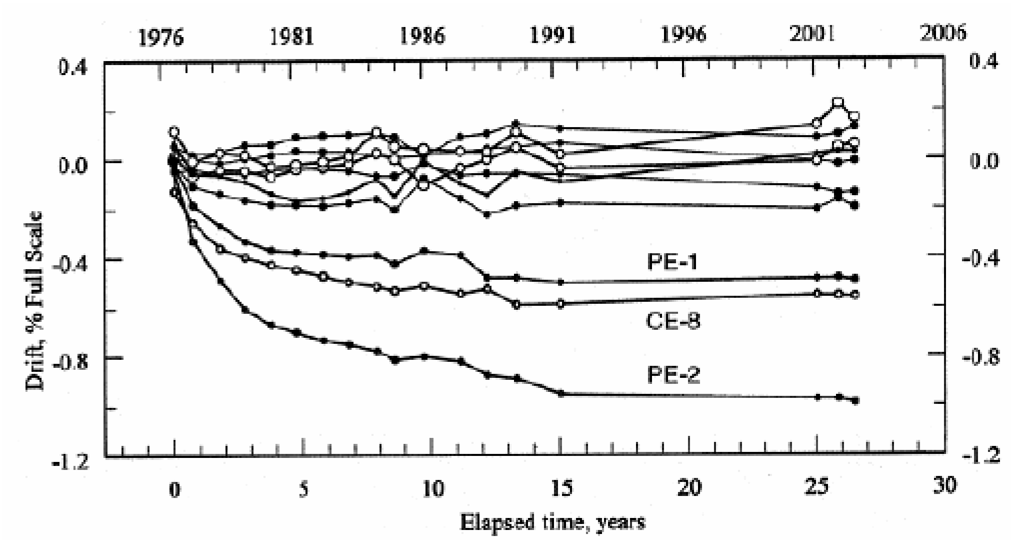

Figure 2.

The more highly stressed wires completed nearly all of their drift in the first 15 years because the tension was not maintained. As the wires stretched, the tension reduced and the frequency dropped. The more lightly stressed wires drifted very little and tended to wander back and forth.

Drifting of the more lightly stressed wires was small enough that it was overshadowed by temperature effects. The wires experienced a temperature range of 14°C to 28°C and demonstrated a linear variation with positive slope of 0.0086%FS(reading)/°C of the reading, much stronger than shown in Figure 1. DiBiagio (2003) points out that the coefficients of expansion for the wire and the steel block to which it is clamped are not the same.

Figure 3 shows the drift as a function of percent of yield strength. A practical weighing gauge transducer should not allow the wire tension to exceed 10% of the yield strength.

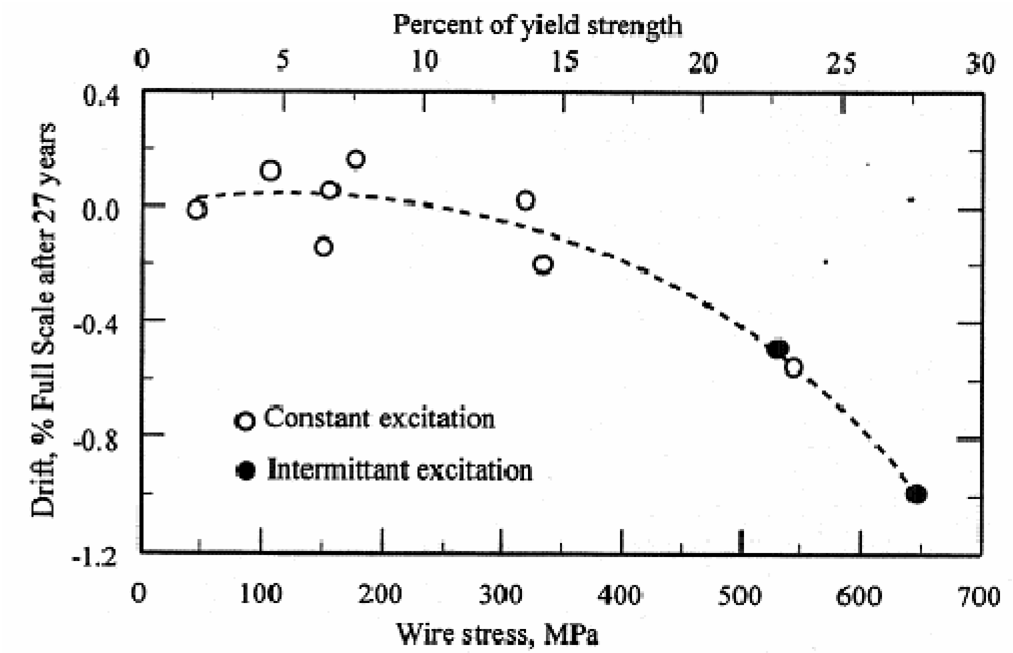

Figure 3.

3. DATALOGGER MEASUREMENT

Two ways to measure a time interval or frequency using a Campbell Scientific data logger are the P27 and P3 instructions. The cpu clock frequencies are 9.8 MHz for the CR23x and 4.92 MHz for the CR10x. In each case, the system clock is the clock divided by 4.

3.1 *P27 – Period Average*

This instruction measures the period, T, of the signal in microseconds. The user programmer inputs the number of periods, N, to be measured. The execution time of the instruction will be NT plus a setup time. The start and stop occur entirely within and are controlled by the P27 instruction.

Normally, the resolution would be determined by the system clock period, T_c. However, the manuals give a value of 12 ns for the CR23x and 35 ns for the CR10x. This is because Campbell Scientific years ago developed a unique propietary method to greatly improve resolution. These resolutions are determined by Root Sum Square analysis. A worst case analysis gives 17 ns for the CR23x and 50 ns for the CR10x.

The measured interval will then be the system clock period times the number of periods, MT_c, plus the difference of two small correction times, t_1 and t_2, and the errors of the correction times, δt_1 and δt_2.

Swenson (2003) give the cpu clock accuracy as ± 0.01%. Then the actual interval will be

$$NT = MT_c + t_1 - t_2 \pm 0.0001{*}MT_c \pm (\delta t_1 - \delta t_2) \pm 0.5t_r \quad 13.$$

The measured period is $T_m = (MT_c + t_1 - t_2)/N$ and the error is

$$\frac{T - T_m}{T_m} = \frac{\pm 0.0001{*}\, MT_c \pm (\delta t_1 - \delta t_2) \pm 0.5t_r}{(MT_c + t_1 - t_2)}$$

$$\approx \pm 0.0001 \pm 0.5t_r/NT \quad 14.$$

where $t_1 - t_2$, calculated by the logger operating system, ranges between 0 and T_c. Then $t_1 - t_2 \lll MT_c$, $\pm(\delta t_1 - \delta t_2) \approx 0$ and $NT \approx MT_c$. The second term is the error of resolution, $E_r = 0.5t_r/NT$, which is intended to be a small percentage of the total error.

When measuring a Geonor weighing gauge, the period, T, will range from about 930 μs (1075 Hz) to about 345 μs (2900 Hz). Then, for N > 25, E_r will range from less than 0.0001075% to less than 0.000303% for a CR10x and from less than 0.00003656% to less than 0.000103% for a CR23x.

The error in period [and frequency since 1/(1 ± x) ≈ (1 – ± x) for small x] will be 0.01% + E_r. For a minutely average of 12 samples, the clock error remains the same but the error of resolution is reduced by an order of magnitude. The measurement error of a minutely average is 0.01%.

3.2 *P3 – Pulse*

This instruction counts all pulses or periods in the execution interval, T_e, in this case, 5 seconds. Accuracy is determined by the accuracy at which the execution interval is repeated. Swenson (2003) give the accuracy as ± (0.00005T_e + 0.05 ms) over a -40° C to 80° C temperature range.

The execution interval then becomes

$$T_e \pm (0.00005*T_e + 0.05\ \text{ms}) = (N + 0.5)T \pm xT \qquad 15.$$

where x ranges from 0 to 0.5. Note that T_e = 5000 ms and the execution interval error is 0.3 ms.

The measured period is $T_m = T_e/N$ and, using a worst case value for x, the error is

$$\frac{T - T_m}{T_m} = \pm \frac{0.00005*T_e + 0.05\ \text{ms}}{T_e} - \frac{1 \pm 1}{2N}$$

$$= 0.5[\pm 0.00012 - (1 \pm 1)/N] \qquad 16.$$

where second order effects have been dropped.

For a Geonor, the value of N will range from 5376.3 for T = 930 μs to 14,493 for T = 345 μs and the error will range from 0.0246% to 0.0129%.

3.3 *The Geonor Reading*

For the Geonor, the equation is

$$P = A(f - f_0) + B(f - f_0)^2 \quad \text{cm} \qquad 17.$$

which, for a small error in frequency, becomes

$$P_e = A(f \pm \delta f - f_0) + B(f \pm \delta f - f_0)^2 \quad \text{cm} \qquad 18.$$

which gives the following error equation

$$P_e - P = [\pm A \pm 2B(f - f_0)]\delta f + B\delta f^2 \quad \text{cm} \qquad 19.$$

Typical parameters are: A = 0.0167, B = 0.0000091 and f_0 = 1050 Hz. Then for T ranging from 930 μs to 345 μs, the error will range from 0.01844 mm to 0.1461 mm for P27 and from 0.04538 mm to 0.1884 mm for P3.

The data are presented in Figure 4. It can be clearly seen that there is little difference in the errors using P27 versus P3. P27 error is symmetric about zero whereas P3 error is offset slightly to the negative.

Choosing between P27 or P3 is therefore based on other operating characteristics. P27 requires the program to stop and wait while the measurement is made but uses an analog channel which are in sufficient supply. P3 permits the program to continue while the measurement is being made but there are an insufficient number of pulse channels.

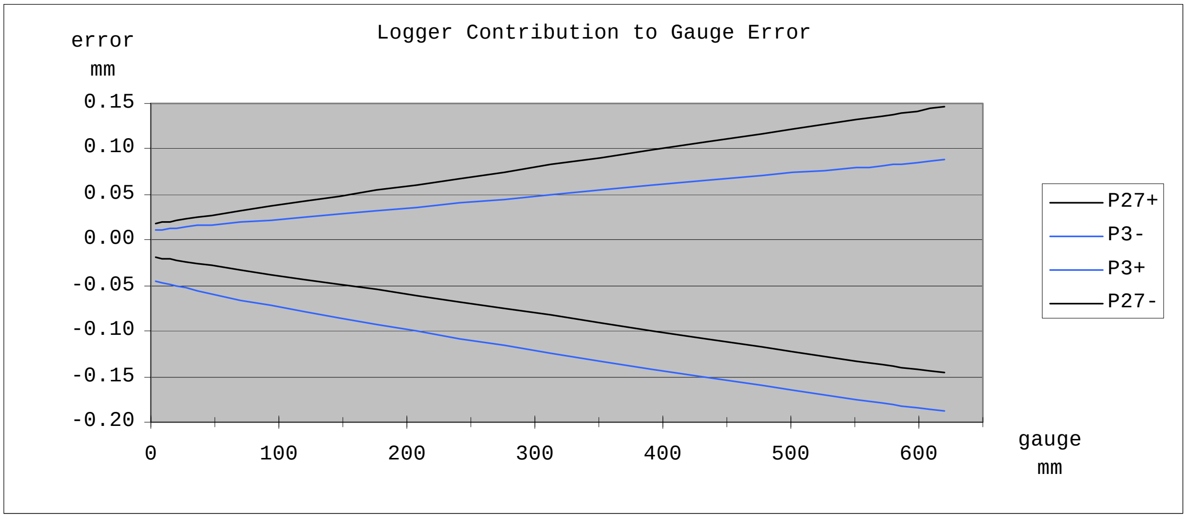

Figure 4.

4. TRANSDUCER-DATALOGGER ERROR

The combined Geonor-logger error of measurement, shown in Figures 5 and 6, is a combination of Figures 1 and 4. Because the temperature effects are known, correction can be applied which will reduce the worst case error to less than ± 0.2 mm.

Figure 5.

Figure 6.

5. CONCLUSION

This paper has concerned itself with a Geonor weighing gauge that is level with three transducers, connected to a Campbell Scientific datalogger and is measuring in calm conditions. It was shown that the gauge has little zero drift but a noticeable temperature sensitivity. Because the temperature sensitivity is known, it can be compensated in the algorithm. The logger error is completely unknown and cannot be compensated.

Since temperature affects the transducer by changing the length of the wire, gauges with one transducer have the additional problem that leveling is also affected. Additionally, sun shining on one side of an installation may also have an effect on leveling. These are installation problems which are not considered in this discussion.

6. REFERENCES

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