4.1 ENSEMBLE-DERIVED BACKGROUND ERROR COVARIANCES FOR OPERATIONAL NWP

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1. INTRODUCTION

In this study several approaches for obtaining more accurate background error covariances for atmospheric data assimilation are explored. Experiments are conducted by replacing the covariances in the operational three-dimensional variational analysis system (3D-var) at the Canadian Meteorological Centre. In the current system, these covariances are computed using the so-called NMC method that is known to suffer from several deficiencies. The background error correlations are also assumed to be horizontally homogeneous and isotropic.

For this study, random samples of background error are generated by simulating (using a Monte Carlo approach) the error generated at each stage of the forecast-analysis process. The 3D-var is used to perform the analyses using perturbed observations and background field and consequently the approach is termed the "perturbed 3D-var". As part of the approach two different methods for simulating model error are examined: (1) additive random error drawn from a specified Gaussian distribution and (2) random perturbations to the tendencies of the physical parametrizations of the forecast model. For both approaches a simple adaptive tuning procedure is employed to ensure the simulated background error variances are consistent with observation error variances and innovation variances.

One of several strategies for estimating the full covariance matrix from a relatively small number of error samples is then employed. Approaches include the use of a wavelet representation and a spatially localized ensemble representation of the correlations. Both allow the usual assumptions of homogeneity and isotropy to be relaxed to some extent.

Finally, a simple approach for the on-line tuning of the background error variances is employed. This is similar to the approach used to tune the model error variances in the perturbed 3D-var, except that the tuning is applied directly within the main analysis system without the need to run a parallel perturbed forecast-analysis cycle. The result is a slowly varying estimate of the background error variances obtained from the innovation variances computed at each analysis time.

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2. PERTURBED 3D-VAR

The extended Kalman filter provides a means of continually evolving the error covariances of the estimated state in a sequential forecast-analysis system. According to the extended Kalman filter, the background error covariances are given by

$$\mathbf{B} = \mathbf{M}\mathbf{P}^a\mathbf{M}^T + \mathbf{Q}, \tag{1}$$

where $\mathbf{B}$ is the background error covariance matrix, $\mathbf{M}$ is the linearized forecast model, $\mathbf{P}^a$ is the covariance matrix for the error in the previous analysis and $\mathbf{Q}$ is the model error covariance matrix that accounts for the additional error induced by errors in the forecast model. An accurate statistical description of the model error is not available and remains a major challenge for most data assimilation approaches (Dee 1995). The analysis error covariances are given by

$$\mathbf{P}^a = (\mathbf{I} - \mathbf{K}\mathbf{H})\,\mathbf{B}. \tag{2}$$

where $\mathbf{K} = \mathbf{B}\mathbf{H}^T\left(\mathbf{H}\mathbf{B}\mathbf{H}^T + \mathbf{R}\right)^{-1}$ is the Kalman gain matrix. For realistic problems, the solution of Eq. (1) and (2) is computationally infeasible due to the high dimensionality of the problem. Instead of manipulating the full covariance matrices, one common approach is to approximate the probability distributions by an ensemble of random samples drawn from the distribution. This is the approach taken for the ensemble Kalman filter (EnKF). A simpler approach based on Monte Carlo simulation and similar to that described by Houtekamer *et al.* (1996) was recently used to recompute the stationary background error covariances in the variational analysis system at the European Centre for Medium-Range Weather Forecasts (Fisher and Andersson 2001). For the present study a similar approach, the perturbed 3D-var, is employed and is compared with the NMC method as currently used in the operational system.

In the perturbed 3D-var approach, as in the EnKF, an ensemble of forecast-analysis experiments are conducted with perturbed observations and background states, but with the analyses performed using the 3D-var with the operational background error covariances. Also, instead of attempting to compute flow-dependent error statistics, the approach is used to estimate the stationary component of the error statistics over a period of several weeks. Due to the pooling of samples over time, only a small number of perturbed forecast-analysis experiments are required in addition to an unperturbed experiment. Differences between the 6 hour forecasts from the perturbed and unperturbed experiments are then computed and used to represent samples of background error. The specification of the model error covariances used to compute perturbations to the background states remains the biggest challenge. To partially overcome this difficulty, an adaptive tuning procedure is used. The tuning approach is based on a comparison between the innovation statistics from the unperturbed experiment and the simulated innovations from the perturbed experiment. The approach is simpler than that proposed by Dee (1995) and examined by Mitchell and Houtekamer (2000) in the EnKF context. The simplification is attained by assuming the model error covariances are proportional to the current operational background error covariances and therefore only the scaling factors applied to these covariances must be determined (as suggested by Mitchell and Houtekamer 2000). The scaling factors for wind components and temperature are computed independently for each of the model's 28 vertical levels and for each of three latitude bands. The tuning procedure guarantees that the horizontally averaged innovation variances simulated by the perturbed experiment equal the true innovation variances. An alternative approach of representing the model error by randomly perturbing the tendencies from the physical parametrizations is also currently begin explored.

Two perturbed 3D-var experiments were run over the period November 25, 2003 to December 31, 2003 to obtain a total of 136 error samples. With these error samples a new background error covariance matrix was computed using the same method to model the correlations as in the operational system. Results from an assimilation cycle experiment using these newly computed background error covariances over the same period produce 6 hour forecasts of significantly improved quality over the Northern extra-tropics as compared to using the operational background error covariances (Fig. 1). These statistics were computed versus the radiosonde observations over the period December 3-25, 2003 at 12 hour increments (total of 45 cases). Verification of forecasts of up to 5 days and initialized with the analyses of each experiment will be shown in the conference presentation.

3. CORRELATION MODELING

In the currently operational analysis system the horizontal correlations are homogeneous and isotropic and the vertical correlations are horizontally homogeneous for the independent variables: Ψ, χ', T', $log(Q)$ and p_s' (primes represent unbalanced components). Latitudinal dependencies in the geostrophic and Ekman balance operators do allow some spatial variations in the correlations for the "full" variables, but these are quite limited. Consequently, several alternative approaches are being examined for es-

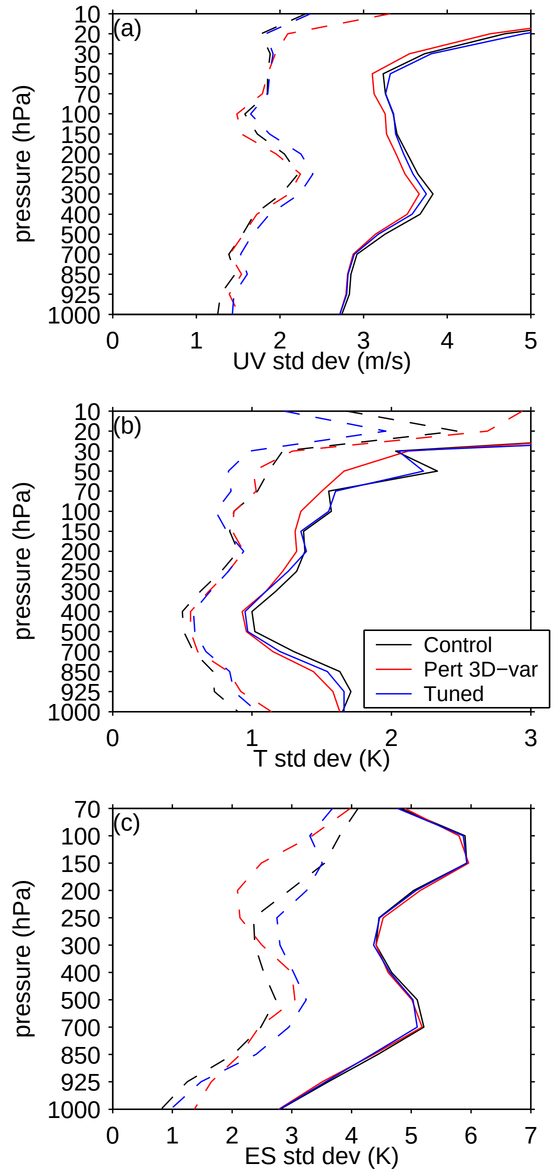

Figure 1: Standard deviation of the difference between 6 h forecasts and radiosonde observations (solid curves) and between the analyses and observations (dashed curves) for the Northern extra-tropics for (a) wind components, (b) temperature, (c) dew-point depression. Results are shown for the control experiment using the currently operational background error covariances, the experiment using covariances computed from the perturbed 3D-var error samples, and the experiment using the operational covariances with variances that are tuned on-line.

timating the full correlation matrix from the O(100) samples of background error obtained with the perturbed 3D-var approach. It is possible to use the simple sample estimate, however it quickly becomes evident that a much larger number of error samples must be used to reduce the estimation error to an acceptable level and to be able to fit the observations (Buehner 2004). Two approaches that reduce estimation error without increasing the number of error samples are now described.

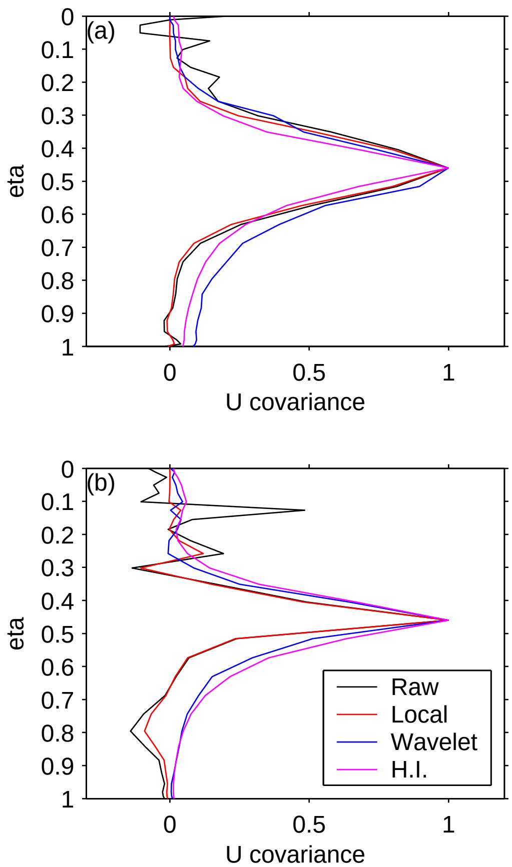

Figure 2: Vertical covariances for zonal wind at 460 hPa, 180° longitude and (a) 60° N or (b) at the equator. The covariances are normalize to have a maximum value of 1. H.I. denotes the homogeneous and isotropic representation of the correlations used in the operational system.

3.1 *Sample Estimate with Localization*

The most obvious problem with using the sample estimate for the correlations is the presence of large spurious correlations at very large separation distances where the correlations are expected to be small. To overcome this problem, a procedure for spatially localizing the correlations was proposed by Gaspari and Cohn (1999) and examined in the context of an EnKF by Houtekamer and Mitchell (2001) and Hamill *et al.* (2001). An efficient approach for implementing horizontal and vertical localization in a variational context with preconditioning is described by Buehner (2004). This approach attempts to leave the correlations corresponding to relatively short separation distances unaffected, while suppressing those for large separation distances.

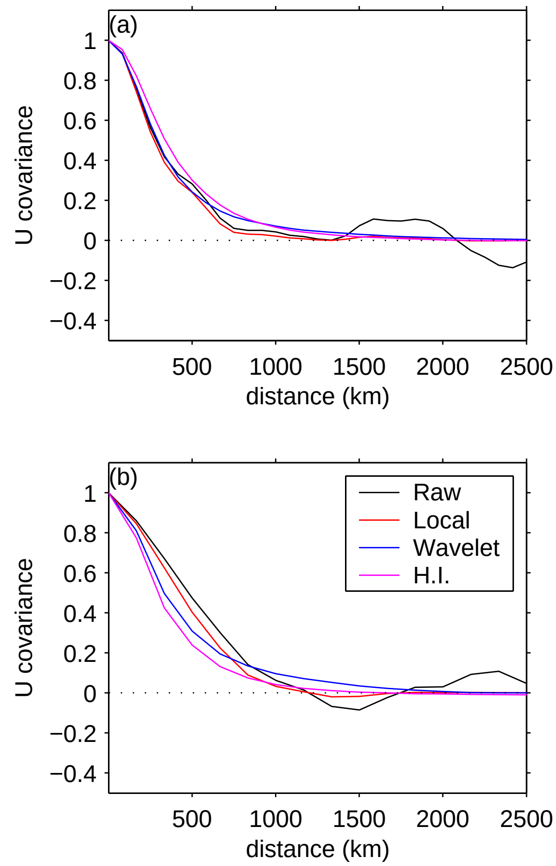

Figure 3: Covariances in the zonal direction for zonal wind at 460 hPa, 180° longitude and (a) 60° N or (b) at the equator. The covariances are normalize to have a maximum value of 1.

3.2 *Wavelet Expansion*

An approach based on a non-orthogonal wavelet expansion on the sphere was introduced by Fisher and Andersson (2001). The wavelet functions are both band-limited in spectral space and spatially localized in grid-point space. Using the wavelets as a set of basis functions, the error samples are first transformed into wavelet space. This gives a measure of the error associated with the range of horizontal scales represented by a particular wavelet for the region surrounding each grid-point. Therefore, the correlations depend on both horizontal scale and spatial location. By assuming zero correlations between the error for different wavelets and between horizontal grid-points, an efficient representation of the full correlation matrix can be obtained. The bandwidth for each wavelet is usually specified so that in grid-point space the basis functions for each range of scales is simply a dilated version of a mother wavelet. However, for the largest scales the wavelets have global coverage and therefore global scale correlations appear due to the difficulty in estimating such correlations. To obtain more localized correlations it is simply necessary to impose a minimum bandwidth when defining the wavelet functions. The wavelet expansion used for the present study span the spectral bands centered about the total wavenumbers 0,2,4,8,12,16,20,24,36,52,75,108. Note that the constant bandwidth for scales between wavenumber 4 and 24 means that the correlations for these scales will have a similar horizontal localization. This has a similar effect on the correlations as the use of the localization procedure described in Section 3.1.

3.3 *Horizontal and Vertical Structure*

To demonstrate the effect of using the different approaches for modeling the correlations, the results from a series of single observation experiments are presented. First the vertical covariances for zonal wind is shown in Fig. 2 for a zonal wind observation located at 180° longitude and either on the equator (Fig. 2a) or at 60° N (Fig. 2a). As expected, the covariances are similar for the two locations when the homogeneous and isotropic correlations are employed (they are not identical due to meridional variations in the background error variances for Ψ and χ). The effect of the vertical localization is evident by comparing the unmodified sample estimate of the correlations (Raw) and the localized correlations (Local). Both of these covariance functions exhibit a generally broader structure at the extra-tropical location and a sharper structure at the tropical location as compared to the homogeneous and isotropic correlations. The wavelet-based correlations (Wavelet) exhibit a similar, but less marked, difference when compared to the homogeneous correlations.

Similarly, the horizontal correlations (zonal direction only) for zonal wind are shown in Fig. 3 for the same two locations. In the horizontal the sample estimate and localized correlations are sharper at the extra-tropical location and broader at the tropical location as compared with the homogeneous and isotropic correlations. Again, the wavelet-based correlations exhibit a structure that is somewhat in between the others, but at the tropical location exhibits more larger correlations than the localized or homogeneous and isotropic correlations beyond about 1000 km.

4. ON-LINE TUNING OF BACKGROUND ERROR VARIANCES

A simple approach for efficiently tuning the background error variances was developed for the operational 3D-var analysis system. The approach is similar to that used for tuning the model error variances in the perturbed 3D-var experiments described above. By making the assumption that the observation error variances are correct, it becomes straightforward to compute the background error variances as

$$\sigma_b^2 = \sigma_d^2 - \sigma_o^2, \tag{3}$$

where σ_b^2 is the background error variance projected into the space of the observations, σ_d^2 is the innovation variance and σ_o^2 is the observation error variance. In the first tests, only radiosondes data are used since the observation error variances are probably most accurate for this data type and the observed quantities have close correspondence with the analysis variables (i.e. winds, temperature, and humidity). Because the spatial distribution of the radiosonde network is sparse and not uniform the variances are computed over only three regions: Northern extra-tropics, Southern extra-tropics and the tropics. In addition, to obtain background error variances that are vertically and temporally smooth additional filtering was performed on the variances. Figure 4 shows the time-mean vertical profile and vertical-mean time series of the scaling coefficients.

Currently, we are exploring how to use TOVS brightness temperature observations for this tuning since they have better global coverage. However, first a new estimate for the TOVS observation error variances must be computed since currently the observation error variances and corresponding background error variances (projected into brightness temperature space using a randomization approach) are grossly inconsistent with the innovation variances for several channels. The approach of Desroziers and Ivanov (2001) is now being considered to recompute the TOVS observation error variances.

This approach was applied in an assimilation experiment using the operational background error covariances over the same period defined in Section 2. The preliminary results for the Northern extra-tropics show a significant impact from the tuning on the fit of the analyses to the data (Fig. 1, dashed curves, Control vs. Tuned). The effect of fitting the observations less closely in the troposphere and more closely above about 100 hPa is consistent with the computed tuning coefficients that are shown in Fig. 4. Figure 1 also shows the impact on the fit of the 6 hour forecasts to the observations (solid curves). This shows a slight improvement from the tuning in the troposphere and some degradation in forecast quality in the stratosphere. The reasons for this difference are still under investigation. Overall, the impact on the 6 hour forecasts of wind and temperature is less than the positive impact from using the background error covariances estimated from the perturbed 3D-var error samples discussed in Section 2.

More detailed results will be shown during the conference presentation including the impact of this tuning procedure on the resulting 5 day forecasts.

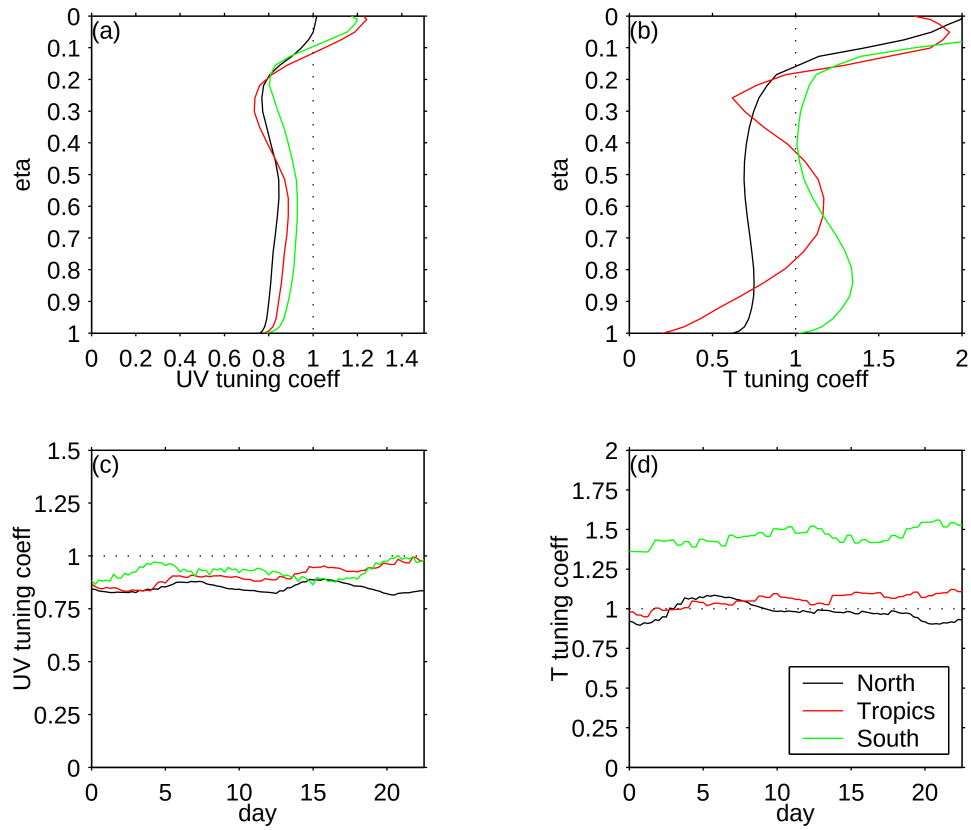

Figure 4: Tuning coefficients for the background error variances computed from innovation and observations error variances. The coefficient are shown as a vertical profile of the time-mean for (a) winds and (b) temperature and as a time series of the vertical mean for (c) winds and (d) temperature. They are also computed separately for the two extra-tropical regions and the tropics.

5. CONCLUSIONS

The preliminary results presented here suggest that improvements to the currently operational 3D-var analysis system can be obtained by replacing the background error covariances with covariances computed using the perturbed 3D-var approach. The examination of alternative approaches for modeling background error correlations in order to relax the assumptions of homogeneity and isotropy are continuing.

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